Scaling Relations between Gas and Star Formation in Nearby Galaxies

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Abstract. High resolution, multi-wavelength maps of a sizeable set of nearby galaxies have made it possible to study how the surface densities of H I, H₂ and star formation rate ($\Sigma_{\rm HI}, \Sigma_{\rm H2}, \Sigma_{\rm SFR}$) relate on scales of a few hundred parsecs. At these scales, individual galaxy disks are comfortably resolved, making it possible to assess gas-SFR relations with respect to environment within galaxies. $\Sigma_{\rm H2}$, traced by CO intensity, shows a strong correlation with $\Sigma_{\rm SFR}$ and the ratio between these two quantities, the molecular gas depletion time, appears to be constant at about 2 Gyr in large spiral galaxies. Within the star-forming disks of galaxies, $\Sigma_{\rm SFR}$ shows almost no correlation with $\Sigma_{\rm HI}$. In the outer parts of galaxies, however, $\Sigma_{\rm SFR}$ does scale with $\Sigma_{\rm HI}$, though with large scatter. Combining data from these different environments yields a distribution with multiple regimes in $\Sigma_{\rm gas} - \Sigma_{\rm SFR}$ space. If the underlying assumptions to convert observables to physical quantities are matched, even combined datasets based on different SFR tracers, methodologies and spatial scales occupy a well define locus in $\Sigma_{\rm gas} - \Sigma_{\rm SFR}$ space.

Keywords. galaxies: evolution, galaxies: ISM, radio lines: ISM, radio lines: galaxies

1. Introduction and the Global Star Formation Law

Great progress has been made towards an understanding of star formation (SF) on small scales in the Milky Way, but many open questions remain about its connection to large scale processes: what sets where SF occurs in galaxies and how efficiently gas is converted into stars? How important are global, galaxy-scale environmental parameters as opposed to small-scale properties of the interstellar medium (ISM)? What is the role of feedback in regulating SF? To address such questions, theoretical modeling and simulations need to be constrained by comprehensive observations.

Both observations and theory have focused on the relationship between the star formation rate (SFR) and the gas density, for which a tight power-law relationship was observed in a large number of galaxies by Kennicutt(1998). Such a relationship was first suggested many decades ago by Schmidt(1959), who studied the distributions of atomic gas and stars in the Galaxy. Over the following decades, similar studies targeted individual Local Group galaxies, e.g., M33 (Madore et al. (1974), Newton(1980)), the Large Magellanic Cloud (Tosa & Hamajima(1975)), and the Small Magellanic Cloud (Sanduleak(1969)). Kennicutt(1989) carried out the first comprehensive extragalactic study targeting a large sample of nearby galaxies and Kennicutt(1998) followed up this work, focusing on measurements averaged across galaxy disks. In a sample of 97 nearby normal and starburst galaxies, he found a close correlation between the galaxy-average total gas surface density ($\Sigma_{\rm gas} = \Sigma_{\rm HI} + \Sigma_{\rm H2}$) and the galaxy-average SFR surface density ($\Sigma_{\rm SFR}$). Following this work, it has become standard to study the relationship between gas and

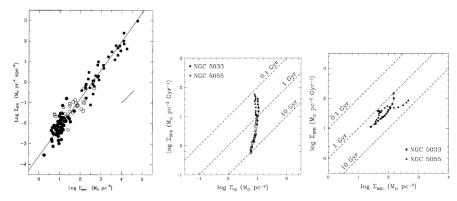


Figure 1. Left: Kennicutt(1998) found a strong correlation over many orders-of-magnitude between global averages of $\Sigma_{\rm SFR}$ and $\Sigma_{\rm gas} = \Sigma_{\rm HI} + \Sigma_{\rm H2}$ for a large sample of nearby normal and starburst galaxies. He derived a power law index $N \approx 1.40$, implying more efficient SF for galaxies with higher average gas columns. Middle and Right: Surface densities of gas and SFR measured in radial profiles by Wong & Blitz(2002) for two exemplary nearby spirals. The middle panel shows $\Sigma_{\rm SFR}$ versus $\Sigma_{\rm H1}$, the right panel $\Sigma_{\rm SFR}$ versus $\Sigma_{\rm H2}$. Wong & Blitz(2002) found no correlation between atomic gas and SFR, whereas molecular gas and SFR scale with one another.

star formation via surface densities, which are observationally more easily accessible than volume densities.

Kennicutt (1998) found $\Sigma_{\rm SFR} = A \times \Sigma_{\rm gas}^N$, with intercept A and power law index N — a relationship that is variously referred to as the "star formation law," "Schmidt-Kennicutt law," or "Schmidt Law." Kennicutt (1998) derived $N \approx 1.40$. Because the ratio $\Sigma_{\rm SFR}/\Sigma_{\rm gas}$ describes how efficiently gas is converted into stars (and is thus often referred to as the star formation efficiency, SFE), this super-linear power law index implies that systems with higher average gas surface densities more efficiently convert gas into stars (left panel, Figure 1). This measured value is close to N=1.5, which is expected if the free-fall time in a fixed scale height gas disk is the governing timescale for SF on large scales. Other studies working with disk-averaged, global measurements found N to be in the range of $\sim 0.9-1.7$ (e.g., Buat et al. (1989), Buat(1992), Deharveng et al. (1994)).

The availability of high-resolution maps of CO emission, the standard tracer of molecular gas, made it possible to follow up the work of Kennicutt (1998) with studies focusing on azimuthally-averaged radial profiles of gas and SF. Resolving galaxies in this way makes it possible to look at how gas and SF relate within individual galaxy disks, opening up a wide range of environmental factors to explore. Wong & Blitz(2002) used BIMA SONG data (Helfer et al. (2003)) to study 6 nearby spirals, Boissier et al. (2003) explored a larger sample of nearby spirals, Heyer et al. (2004) studied the Local Group galaxy M33, and Schuster et al. (2007) explored the gas-SF relation in M51. These studies derived power law indices in the range $N \approx 1-3$, leaving it unclear whether a single relation relates gas and SF when galaxy disks are spatially resolved. Further disagreement centered on the relationship of SF to different types of gas — HI, H₂, and total gas. Intuitively, one might expect a stronger correlation between SF and the cold, molecular phase, rather than the atomic phase. Wong & Blitz(2002) indeed found a much stronger correlation of $\Sigma_{\rm SFR}$ with the molecular gas, $\Sigma_{\rm H2}$ (compare Figure 1). However, Kennicutt(1998) and Schuster et al. (2007) both found a better correlation of Σ_{SFR} with the total gas, Σ_{gas} , than with $\Sigma_{\rm H2}$.

2. Recent Advances: Gas and Star Formation on sub-kiloparsec Scales

One reason that different studies returned such different results were the wide range of SFR tracers employed in the various analyses. Furthermore, different studies employed widely varying methods to correct the observed UV and H α intensities for the effects of extinction by dust. Because the correction factor is usually $\gtrsim 2$, the adopted methodology makes a large difference. A large step forward in this field came from the *Spitzer* space telescope. As part of SINGS (*Spitzer* Infrared Nearby Galaxies Survey, Kennicutt *et al.* (2003)), *Spitzer* obtained high-resolution IR maps of a large sample of nearby galaxies. Calzetti *et al.* (2005) and Calzetti *et al.* (2007) demonstrated the utility of these maps to trace recently formed stars obscured on small scales, particularly when used in combination with H α — a tracer of unobscured star formation.

At the same time the VLA large program THINGS (Walter et al. (2008)) obtained the first large set of high resolution, high sensitivity 21-cm line maps for the same sample of galaxies. Following shortly thereafter, the IRAM large program HERACLES mapped CO emission for an overlapping sample of nearby galaxies (first maps in Leroy et al. (2009)). The result was, for the first time, a matched set of sensitive, high spatial resolution maps of atomic gas, molecular gas, embedded and unobscured star formation for a large sample of nearby galaxies. The resolution of the maps allowed hundreds of independent measurements per galaxy, leading to significantly improved statistics and the ability to carefully isolate regions with specific physical conditions.

Bigiel et al. (2008) combined these data to compare H I, H₂, and SFR across a sample of 7 nearby spiral galaxies. Figure 2 shows the results of this analysis: the left panel shows $\Sigma_{\rm HI}$, the middle panel $\Sigma_{\rm H2}$, and the right panel $\Sigma_{\rm gas} = \Sigma_{\rm HI} + \Sigma_{\rm H2}$ versus $\Sigma_{\rm SFR}$ (derived from a combination of far UV and 24 μ m emission). Galex far UV emission was chosen to trace the recent, unobscured SF because of the low background in the FUV channel and the large field-of-view of the GALEX satellite. In these plots, H I and H₂ show distinct behaviors: the atomic gas shows no clear correlation with the SFR, whereas the molecular gas exhibits a strong correlation. As a result, the composite total gas-SFR relation is more complex than a single power law. In the combined (total gas) plot in the right panel, one can clearly distinguish where the ISM is H I dominated (low gas columns, steep relation) form where it is H₂ dominated (high gas columns, roughly linear correlation).

If a power law is fit to the molecular gas distribution in the middle panel, one obtains $N \approx 1.0$. This can be restated as a constant ratio $\Sigma_{\rm SFR}/\Sigma_{\rm H2}$, which means that on average each parcel of $\rm H_2$ forms stars at the same rate. Leroy et al. (2008) searched for correlations between $\Sigma_{\rm SFR}/\Sigma_{\rm H2}$ and a number of environmental variables — ISM pressure, dynamical time, galactocentric radius, stellar and gas surface density — and found little or no variation across the disks of 12 nearby spirals. On the other hand many of these same environmental variables do correlate strongly with the $\rm H_2$ -to-H I ratio. The combined conclusion of these two studies was that the average depletion time in the molecular gas (i.e., $\Sigma_{\rm H2}/\Sigma_{\rm SFR}$) of nearby spirals is fairly constant at $\sim 2.0\,\rm Gyr$, while the abundance of molecular gas is a strong function of environment.

Blanc et al. (2009) carried out a similar experiment to Bigiel et al. (2008). They sampled the inner part of M51 with 170 pc diameter apertures and estimated local SFR surface densities from H α emission. Their integral field unit observations allowed for accurate estimates of internal extinction and corrections for contamination by the AGN and diffuse ionized gas. They used a Monte-Carlo approach to incorporate upper limits into their power law fit. Their results are in good agreement with Bigiel et al. (2008) regarding M51 in particular as well as regarding the general conclusions they reached: a

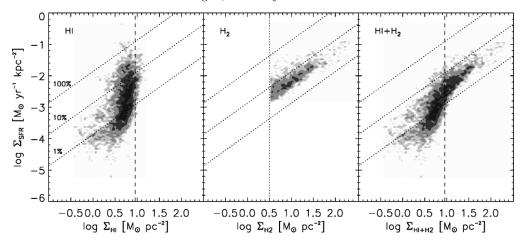


Figure 2. $\Sigma_{\rm SFR}$ versus $\Sigma_{\rm HI}$ (left), $\Sigma_{\rm H2}$ (middle) and $\Sigma_{\rm gas}$ (right panel) for pixel-by-pixel data from 7 nearby spirals at 750 pc resolution. The contours represent the density of sampling points (pixels), where darker colors indicate a higher density. The H_I distribution saturates at about $10~{\rm M}_{\odot}~{\rm pc}^{-2}$ and shows no correlation with the SFR (left panel). Gas in excess of this surface density is predominantly molecular. The middle panel illustrates the correlation between H₂ and SFR, which can be described by a power law with slope $N\approx 1.0$. This implies a constant H₂ depletion time of $\sim 2~{\rm Gyr}$. The total gas plot (right panel) illustrates the different behavior of H_I and H₂ dominated ISM at low and high gas column densities, respectively.

virtual absence of correlation between SFR and ${\rm H\,{\sc i}}$, a strong correlation between SFR and ${\rm H\,{\sc i}}$ and a molecular gas depletion time that shows little variation with molecular gas column.

Recently, Rahman et al. (2010) explored the impact of different SFR tracers and the role of possible contributions from diffuse emission and different sampling and fitting strategies on the relationship between $\Sigma_{\rm H2}$ and $\Sigma_{\rm SFR}$. They found that the SFR derived for low surface brightness regions is extremely sensitive to the underlying assumptions, but that at high surface brightness the result of a roughly constant H_2 depletion time is robust.

Even more recently, Schruba *et al.* (in prep.) combined the HERACLES and THINGS data to coherently average CO spectra as a function of radius. With this approach they are able to trace molecular gas out to $1.2\,\mathrm{r}_{25}$, allowing them to study the relation between H_2 and SFR where H I dominates the ISM. They demonstrate that the tight correlation between H_2 and SFR crosses seamlessly into the H I-dominated outer disk (left panel Figure 3).

With so much effort expended measuring SFRs and gas densities over the years, it is interesting to ask whether the literature largely agrees. The right panel in Figure 3 shows a collection of literature measurements along with the data from Bigiel et al. (2008). After matching underlying assumptions about how to derive physical quantities from the observables, the literature data populate a well-defined locus in $\Sigma_{\rm SFR}$ – $\Sigma_{\rm gas}$ space.

With some consensus emerging on the broad distribution of data in SFR-H₂ space, attention is turning to the origin of the intrinsic scatter in the SFR-H₂ ratio. Schruba et al. (2010) looked at this as a function of spatial scale in M33 and showed that scatter in the CO-to-H α ratio increases dramatically once a resolution element contains only a single star-forming region (i.e., H II region or giant molecular cloud). This occurs at scales of ~ 150 pc in M33 but should be a function of the environment studied. They interpreted

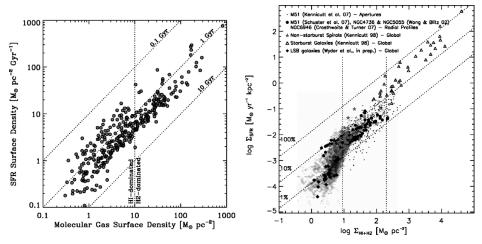


Figure 3. Left: $\Sigma_{\rm SFR}$ versus $\Sigma_{\rm H2}$ from Schruba et al. (in prep.). They apply a stacking analysis to the HERACLES CO data to probe the H₂-SFR relation far into the regime where $\Sigma_{\rm H1} > \Sigma_{\rm H2}$. The correlation between $\Sigma_{\rm SFR}$ and $\Sigma_{\rm H2}$ extends smoothly into the H I-dominated parts of galaxies out to $1.2\,\mathrm{r}_{25}$. Right: Comparison between different datasets using different methodologies from Bigiel et al. (2008). The contours are identical to the right panel in Figure 2 and the overplotted symbols come from studies using a variety of SFR tracers and methodologies. All datasets have been adjusted to match the same set of assumptions when converting observables to physical quantities. The composite sample occupies a well-defined locus in $\Sigma_{\rm SFR} - \Sigma_{\rm gas}$ space.

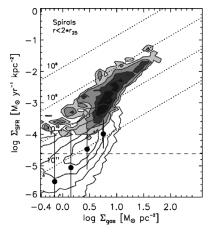
this finding as a result of the evolution of these regions, so that information on the life cycle of giant molecular clouds is embedded in the $\Sigma_{\rm SFR}$ - $\Sigma_{\rm H2}$ relation, especially at high resolutions.

Another avenue of investigation is the role of the host galaxy in driving the scatter in the SFR-H₂ ratio. Leroy et~al.~(2008) and Bigiel et~al.~(2008) found little environmental dependence of this quantity in large spiral galaxies, but with the completion of the HERACLES survey (47 galaxies spanning from low-mass dwarfs to large spirals) we can now apply a similar analysis to a much wider sample of galaxies. Bigiel et~al.~(in~prep.) and Schruba et~al.~(in~prep.) explore how scatter in the SFR-to-H₂ ratio breaks into scatter within galaxies and scatter among galaxies. They find that scatter among galaxies dominates the relation, with a clear trend evident so that less massive, more metal poor, later type galaxies show systematically higher ratios of SFR-to-H₂. Young et~al.~(1996) saw this trend using integrated FIR-to-CO ratios. These new analyses show that once these galaxy-to-galaxy variations are removed, galaxies exhibit a very tight internal H₂-SFR relation.

3. Scaling Relations beyond the Optical Disks

HI maps reveal atomic gas out to many optical radii in spiral galaxies and over the past few years, GALEX UV observations have revealed widespread SF in the outer parts of many galaxies (e.g., Thilker et al. (2005), Gil de Paz et al. (2007), Bigiel et al. (2010b)). These outer disks have fewer heavy elements, less dust, and lower stellar and gas surface densities than the inner parts of spiral galaxies. Contrasting the gas-SFR relationship in outer disks with that in the inner parts of normal galaxies gives a chance to assess the impact of these parameters on SF.

In the left panel of Figure 4 we show the results of a pixel-by-pixel analysis of outer galaxy disks: the open contours show $\Sigma_{\rm SFR}$ versus $\Sigma_{\rm gas}$ for a sample of 17 spiral galaxies



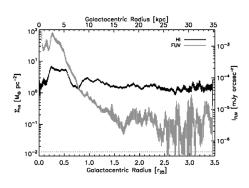


Figure 4. Left: $\Sigma_{\rm SFR}$ versus $\Sigma_{\rm gas}$ for the outer (open contours) and inner (filled contours) parts of nearby spiral galaxies (Bigiel et al. (2010a)). The combined distribution reveals multiple regimes: at large radii and low surface densities, $\Sigma_{\rm SFR}$ scales with $\Sigma_{\rm gas} \approx \Sigma_{\rm HI}$. Over most of the area in disks, $\Sigma_{\rm SFR}$ is a very steep function of $\Sigma_{\rm gas}$, with the H₂-to-H I ratio being the key determinant of $\Sigma_{\rm SFR}$. At high column densities, the gas is predominantly molecular and correlates well with $\Sigma_{\rm SFR}$. At even higher column densities, a steepening of this relation, meaning and increasing efficiency of SF, may accompany the transition from galaxy disks to starbursts. Right: H I and far UV radial profiles for M83 out to almost 4 optical radii r₂₅. The inferred H I depletion time is about a Hubble time at large radii, much longer than the molecular gas depletion times measured in many nearby spirals.

at 15" resolution (corresponding to physical scales between 200 pc and 1 kpc) from Bigiel et al. (2010a). SFRs are estimated from GALEX far UV emission and $\Sigma_{\rm gas}$ is estimated from HI emission alone, assuming a negligible contribution from molecular gas on \sim kpc scales in outer galaxy disks. For comparison, the filled contours show sampling data from the star forming disks of 7 spiral galaxies from Bigiel et al. (2008) (compare the right panel in Figure 2 above). In the outer disks (open contours) $\Sigma_{\rm SFR}$ scales with $\Sigma_{\rm gas}$, i.e., $\Sigma_{\rm HI}$, though the scatter in $\Sigma_{\rm SFR}$ for a given H I column is large. This is an interesting difference compared to the inner parts of spiral galaxies, where the H I showed no clear correlation with $\Sigma_{\rm SFR}$ (compare Section 2).

The HI-FUV relation observed for outer disks suggests two things. First, that at large radii the availability of HI may be a bottleneck for star formation. Even if stars form directly from H₂, molecular clouds must be assembled from H_I and this will only be possible in regions with enough H_I to assemble these clouds. Second, in the inner parts of galaxies many physical conditions important to the H I-H₂ conversion change while $\Sigma_{\rm HI}$ remains approximately fixed, but in the outer parts $\Sigma_{\rm HI}$ varies while other environmental conditions show comparatively little variation. As a result, $\Sigma_{\rm HI}$ transitions from being a relatively unimportant driver for SF in the inner parts of galaxies to a key quantity at large radii. This is apparent from the right panel of Figure 4, where we use extremely deep FUV data from GALEX and HI data from THINGS to trace SF and HI out to almost four optical radii in the nearby spiral M83 (Bigiel et al. (2010b)). The H_I depletion time (HI-to-SFR ratio) inferred from the radial profiles in this plot is approximately constant at large radii: it is about a Hubble time, i.e., much longer than the $\sim 2\,\mathrm{Gyr}$ molecular gas depletion time scale observed in the inner parts of galaxies. This suggests relatively fixed conditions for molecular cloud formation and that assembling H_2 , rather than forming stars out of H_2 , is the rate-limiting process for SF in outer disks.

4. The Composite Star Formation Law

The combined distribution (inner and outer disks) in the left panel of Figure 4 can be divided into different parts according to gas column density, each part describing the relation between gas and SFR in a particular regime in a typical spiral galaxy disk. For low gas columns (outer disks), the SFR scales with gas column, though with significant scatter. At smaller radii and higher gas columns — corresponding to much of the area inside the star-forming disk — the distribution becomes much steeper. In this regime, knowing $\Sigma_{\rm gas}$ alone is not enough to predict $\Sigma_{\rm SFR}$ with any accuracy. Across this regime the H₂-to-H_I ratio is varying steadily as a function of other environmental quantities. At yet smaller radii and higher gas columns, the dominant phase of the ISM transitions from atomic to molecular and a strong correlation emerges between $\Sigma_{\rm gas}$ and $\Sigma_{\rm SFR}$.

There is good observational evidence (e.g., Kennicutt(1998), Gao & Solomon(2004), Greve et al. (2005), Bouché et al. (2007)) that at higher gas columns, the relation steepens further, so that the SFR-per-H₂ ratio is higher in starburst galaxies than in normal galaxy disks. This may drive the frequent observation of $N \approx 1.5$ in starburst galaxies. However, it must be emphasized that there is currently a lack of data probing normal disk galaxies and starbursts in a self-consistent way, so the details at the high end of this relation remain uncertain.

5. Conclusions

With vast improvements in the data available for nearby galaxies some consensus is beginning to emerge on how different parts of galaxies populate the $\Sigma_{\rm SFR}$ - $\Sigma_{\rm gas}$ parameter space. The role of environmental quantities other than gas surface density alone are beginning to become clear and different relations are emerging for different types and parts of galaxies. When viewed in detail the composite relation may not be a simple power law, but it contains key information to constrain theories and to benchmark simulations.

Challenges remain, too. The determination of star formation rates at low surface brightness is still difficult. The use of CO to trace H_2 underpins almost all of this work but the CO-to- H_2 conversion factor remains imprecisely calibrated as a function of environment. Finally, the fundamental units of star-formation, individual molecular clouds, remain largely observationally inaccessible beyond the Local Group — a situation that will not change until ALMA begins its full operations.

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