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The outer gas disk of our Galaxy (and many others) is warped, bending away from the plane defined by the inner disk. The bend begins just outside the solar circle; gas at longitudes $\ell \approx 80^{\circ}$ reaches highest above the plane, while material at $\ell \approx 260^{\circ}$ lies below it. Shortwavelength ripples are superposed. The orbit of a free particle inclined to the galactic disk precesses at a rate which depends on galactocentric radius; warped structures will tend to do the same, winding the warp into a tight spiral. The large-scale galactic warp has no sense of spirality, and is not obviously of recent origin - why then has it survived?

If the galactic disk had a discrete mode of vertical oscillation, it could vibrate like a drum under its own gravity. Any forces which perturb the disk would excite the bending mode; once a warp had been set up it would persist for many rotation periods. Unfortunately an <u>isolated</u> self-gravitating disk has no discrete bending modes (Hunter and Toomre 1969) if the density falls smoothly to zero at the edge; any initial warp is lost within a couple of revolutions. The galactic rotation curve suggests that over half its mass may not lie in the disk, but form a massive unseen halo. This paper discusses how an axisymmetric (but not spherical) heavy halo affects the warping modes of a galactic disk.

The disk (of surface density falling exponentially with radius within a sharp edge) is approximated as a system of gravitating rings, subject to the halo potential. The halo density $\rho_{\rm H}$ depends on spherical radius R and polar angle θ as

$$\rho_{\rm H} = \frac{\rho_{\rm o}}{R^2 + R_{\rm o}^2} \left[1 - \epsilon(R) P_2(\cos\theta) \right]; \qquad (R < R_{\rm t})$$

 ε > 0 in an oblate halo. Modes are sought with the form

$$z(r,\phi,t) + h(r)e^{i(\omega t-\phi)}$$

in cylindrical polar coordinates (r,ϕ,z) ; material at any given radius 499

H. van Woerden et al. (eds.), The Milky Way Galaxy, 499–502. © 1985 by the IAU. lies above the mean plane on one side of the disk and below that plane on the other, while the warped shape precesses at an angular rate ω .

When the halo is <u>spherical</u> or <u>oblate</u> with $\varepsilon(R)$ increasing no faster than linearly with R, the shape of the most slowly precessing warped mode is found to depend on where the sharp disk edge is taken to lie. The precession rate tends to zero as the disk extends further out. As the edge is made smooth rather than sharp, the frequencies of different modes come together to form a continuum. This is exactly what Hunter and Toomre found for the isolated disk; a smooth-edged disk has no discrete warping modes, but supports a continuum of dispersive waves which carry away the energy of any initially imposed warp.

If the ellipticity ε rises more rapidly (for example, quadratically) with radius, and the disk is 2 or 3 times smaller than the halo radius R_t , a qualitatively different result is found. The disk has at least one discrete mode, which is not sensitive to the edge; its shape and precession rate stay constant as the disk is extended. But as the disk radius approaches R_t , the mode behaviour reverts to that in a uniformly oblate halo; discrete modes are found only if the disk has a sharp edge. Disks in <u>prolate</u> halos appear to have at least one discrete warping mode even if the disk edge is smooth, so that the system can sustain a long-lived warp.

These results can be understood using the WKB (short-wavelength) theory of bending waves. If the precession rate Ω_p of a free particle in the halo <u>decreases</u> outward at the disk edge, then a discrete warping mode exists independently of the details of the edge; otherwise, a smooth-edged disk has a continuum of bending waves. If the halo is uniformly prolate, the (positive) precession rate falls at all radii; if it becomes rapidly more oblate, the (negative) precession rate may reach a minimum before rising towards zero at large radii. In these cases the Galaxy has an inner region where Ω_p is falling, and a smooth-edged disk can have discrete warping modes. By contrast, a uniformly oblate halo yields no region of falling precession rate, and only a sharp-edged disk can keep a long-lived warp.

So, if our Galactic disk is in a normal mode of warping, the massive unseen halo is either a) <u>increasingly oblate at large radii</u>, or b) <u>prolate</u>. The warped outer disk then crosses the plane of the inner disk at a constant azimuthal angle at all radii (the warp has no sense of spirality). The figure precession (which can in principle be measured from kinematic data) is in the same direction as the circular motion if the halo is prolate, and retrograde if the halo is oblate. The precession is much slower than the disk rotation if the halo is not far from round.

A full account of this work has been submitted for publication in the Astrophysical Journal.

REFERENCE

Hunter, C., and Toomre, A.: 1969, Astrophys. J. 155, 747

DISCUSSION

J.P. Ostriker: I would be interested in any comparison you might wish to make between your proposed explanation and that of R.H. Sanders. I believe that he found that a warp would persist for a long time (but not for ever) in regions where the disk-to-halo mass ratio was small.

Sparke: Sanders considered the warp at distances of 30-35 kpc, where the disk has become quite light. The Galactic warp is observed out to 18-20 kpc radius (with the Sun at 8-10 kpc), and would require a lot of heavy halo out there. Toomre argued at Besançon (IAU Symposium 100, page 177) that the required halo mass would be implausibly large. Of course all these arguments are about factors 2 or 3.

H. van Woerden: Would our halo be made prolate or flattened by tidal interaction with the Andromeda Nebula?

Sparke: I suspect not the halo, but I am not sure. I do not know why haloes should have the shape they have.

<u>D. Lynden-Bell</u>: A galaxy with an extensive heavy halo is more susceptible to tides than a galaxy without one (cf. my review paper, in Section III.1 in this volume). The tide distorts the halo, which distorts the visible parts of the galaxy. For a galaxy whose halo extends a factor of 10 beyond the visible disk, this enhancement of an imposed tide is by a factor of about 3.

<u>R. Güsten</u>: We know from the recent CO surveys that in the inner Galaxy there is a distinct ripple in the position of the mean CO-layer relative to the galactic plane. Is this an expected (second-order?) response to the galactic warp phenomenon?

<u>Sparke</u>: Probably not. I think it is more likely that the gas layer in the inner Galaxy has been perturbed by star formation. (In the outer Galaxy, the mean plane of the CO follows that of the neutral hydrogen.)

<u>D. Lynden-Bell</u>: There is an important observational criterion by which one may distinguish steady modes from transient, propagating ones. A steady mode must have its nodal line along a diameter, whereas in a propagating disturbance the nodal line will have a spiral shape. Is there an observational answer to this?

F.J. Kerr: Yes, the warp in our Galaxy does have a radial form in the observations. It has always been a puzzle, in fact, that the phase (or azimuth) of the warp is roughly constant with radius, while the structure lies in a field of differential rotation.

Lynden-Bell: Is that true in all galaxies, or just in ours?

Kerr: I think it is true in all galaxies.

C.A. Norman (Chairman): Can we have some discussion of that point?

SILENCE!

Norman: No one willing? What about Renzo?

T.S. Jaakkola: This is a question to the theorists: if the rotation of galaxies is indeed the relic of a primordial vortex, as the round-thecorner explanation is usually given, how can it still be so extremely virile <u>even within massive haloes</u>, as seen in the rapid rotation of spiral galaxies out to their very outskirts??

SILENCE!



Bernard Burke (left), Frank Kerr (centre), Lloyd Higgs and Bill Shuter have a beer during Wednesday excursion. Burke (1957, Astron. J. **62**, 90) and Kerr (1957, Astron. J. **62**, 93) were the first to discuss the warp in the Galactic hydrogen layer. GSS

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