ON THE RAMSEY NUMBERS OF TREE GRAPHS VERSUS CERTAIN GENERALISED WHEEL GRAPHS

ZHI YEE CHNG

This thesis presents a series of Ramsey results on tree graphs versus generalised wheel graphs, with the focus on the generalised wheel graphs $W_{s,6}$ and $W_{s,7}$, and the wheel graph W_8.

The thesis comprises seven chapters. In Chapter 1, we give a brief historical introduction to Ramsey theory and Ramsey's theorem, as well as some brief introduction to the contents of the thesis. Then in Chapter 2, we introduce notation and definitions that will be consistently used throughout the thesis, including some basic knowledge of graph theory which is particularly useful in our discussion.

In Chapter 3, we present Ramsey numbers for tree graphs T_n of order n versus the generalised wheel graphs $W_{s,6}$ and $W_{s,7}$. We determine the Ramsey number $R(T_n, W_{2,6})$ for $n \geq 5$. Then we generalise these results to find $R(T_n, W_{s,6})$ for $s \geq 2$. After that, we also determine the Ramsey number $R(T_n, W_{s,7})$ for $n \geq 5$ and $s \geq 1$. In the last section of Chapter 3, we discuss results on the Ramsey numbers for tree graphs versus generalised wheel graphs, $R(T_n, W_{s,m})$, and propose a conjecture.

Chapters 4 and 5 present the Ramsey numbers T_n for tree graphs of order n versus the wheel graph of order 9, W_8. In Chapter 4, we focus on the tree graphs with maximum degree of at least $n - 3$. In Chapter 5, we provide results for the tree graphs with maximum degree of $n - 4$ and $n - 5$.

In Chapter 6, we present the Ramsey numbers $R(T_n, W_8)$ for the tree graphs with maximum degree of at most $n - 6$, where n is sufficiently large.

Chapter 7 concludes the thesis with suggestions for possible future work.

Part of this research has been published in [1, 2].

References

[1] Z. Y. Chng, T. Britz, T. S. Tan and K. B. Wong, 'The Ramsey numbers for trees of large maximum degree versus the wheel graph W_8', *Bull. Malays. Math. Sci. Soc.* **47** (2024), Article no. 134.
[2] Z. Y. Chng, T. S. Tan and K. B. Wong, 'On the Ramsey numbers for the tree graphs versus certain generalised wheel graphs,' *Discrete Math.* **344** (2021), Article no. 112440.

ZHI YEE CHNG, School of Mathematics and Statistics,
University of New South Wales, Sydney, New South Wales 2052, Australia
e-mail: zhi_yee.chng@unsw.edu.au