DYNAMICAL INSTABILITIES IN SPHERICAL STELLAR SYSTEMS

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Abstract. Equlibrium spherical stellar systems exhibiting instabilities on a dynamical timescale were first studied by Henon (1973), using a spherically symmetric N-body code. We have re-examined Henon's models using an improved code which includes non-radial forces to quadrupole order. In addition to the radial instability reported by Henon, two new non-radial instabilities are also observed. In one, found in models with highly circular orbits, the mass distribution exhibits quadrupole-mode oscillations. In the other, seen in models with highly radial orbits, the system spontaneously breaks spherical symmetry and settles into a tri-axial ellipsoid. These instabilities, which are driven by fluctuations of the mean field, offer some analogies to the well-known dynamical instabilities of a cold disk of stars. While our models are rather artificial, they indicate that dynamical instabilities may be more common in spherical systems than had been thought.

Henon's generalized polytropic models are equlibrium spherical stellar systems based on the distribution function

$$f(r,v) \propto \max(E_1-E,0)^{n-3/2}\ J^{2m},$$

where E_1, n, and m are constants, $E = \phi(r)+\frac{1}{2}v^2$, and $J = rv_t$. Here $n \geq \frac{1}{2}$; if $n = \frac{1}{2}$ then all stars have the same E. The stellar density of these models vanishes at

$$r_1 = (4m+6)/(3m-n+5)$$

in units where G = 1, total mass $M_1 = 1$, and binding energy $T+U = -\frac{1}{4}$. The velocity anisotropy is independent of radius. Instead of m, we use the parameter

$$\kappa = \langle v_t^2\rangle/(\langle v_v^2\rangle+\langle v_r^2\rangle) = (2m+2)/(2m+3).$$

The cases κ = 0, 2/3, and 1 correspont to radial, isotropic, and circular orbits respectively.

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We numerically solve Poisson's equation for a given choice of n and κ, obtaining the potential Φ and radius r as functions of the enclosed mass M. To produce an N-body realization of a generalized polytrope, particle radii are selected by applying r(M) to a series of uniformly distributed random numbers, while the radial and transverse velocity components are selected by a von Neumann rejection procedure.

Most of our results were obtained with a "reduced" N-body code (White 1983) in which the mean force field is expressed as a sum of monopole, dipole and quadrupole terms. Individual 2-body interactions are not included, so this code is collisionless. Our key results have been checked with a direct-summation Aarseth code. In general, the two codes yield similar results, although the more expensive direct-summation experiments also show signs of 2-body relaxation.

The first non-radial instability occurs in models with orbits circular enough to initially confine most of the mass to a spherical shell. Figure 1 shows a snapshot of such a model; particles with high angular momentum have been plotted as circles to bring out the quadrupole form of the instability. The net ellipticity of the model, derived from the quadrupole moment, oscillates with a period of approximately half the mean orbital time. In the limit $\kappa = 1$, particles are confined to the surface of a sphere; since all have the same orbital period, any initial perturbation will recur at regular intervals. The self-gravity of a perturbation attracts unperturbed particles, so its amplitude grows with time.

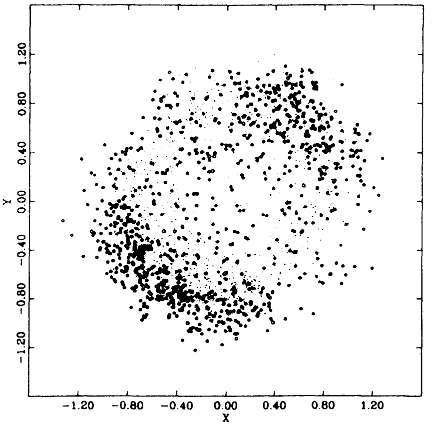

Figure 1. Snapshot of an N-body model of the polytrope $n = \frac{1}{2}$, $\kappa = 11/12$ after 13 time units. Particles with $J \geq 0.8$ are plotted as circles.

The second non-radial instability is seen in models with highly radial orbits. In these cases the system does not oscillate, but quickly

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settles into a highly tri-axial ellipsoid, as shown in figure 2. No further evolution has been seen in the longest experiments to date.

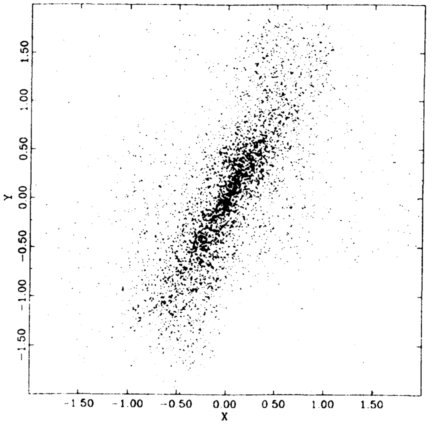

Figure 2. Snapshot of an N-body model of the polytrope $n = 1$, $\kappa = 0$ after 10 time units.

To map out the domain of these instabilities, we have run a grid of 120 experiments in the (n,κ) plane, spanning the range from $n = 5/10$, $\kappa = 2/12$ to $n = 16/10$, $\kappa = 11/12$. We confirm Henon's results for the radial instability, which appears to be confined to models with small values of n and κ. The non-radial instabilities occur over a much wider range of parameters; there is little indication that either instability becomes weaker with increasing n up to $n = 16/10$.

Antonov (1962) and Lynden-Bell and Sanitt (1969) have shown that no instabilities exist if the distribution f is a function of the energy E only and $df/dE < 0$. The latter condition alone is sufficient for stability if only radial modes are allowed (Gillan et. al. 1976). Our results show that the condition $\partial f/\partial E < 0$ is not sufficient for stability against non-radial perturbations. The unstable models presented here may shed some new light on the general stability problem for stellar systems.

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