

## RADIOCARBON TO CALENDAR DATE CONVERSION: CALENDRIAL BAND WIDTHS AS A FUNCTION OF RADIOCARBON PRECISION

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**ABSTRACT.** Accurate high-precision  $^{14}\text{C}$  dating (*i.e.*,  $\pm 20$  yr precision or less on the  $^{14}\text{C}$  date) provides the narrowest calendrical band width and, hence, the best age range determination possible. However, because of the structure in the  $^{14}\text{C}$  calibration curve, the calendar age range for a given  $^{14}\text{C}$  precision is not constant throughout the calibration range. In this study, we quantify the calendar band widths for a range of  $^{14}\text{C}$  precisions throughout the calibration range. We show that an estimate of the likely calendar band width in years can be obtained from the expression: Band width (yr) =  $2.12 \times ^{14}\text{C}$  precision ( $1 \sigma$ ) + 54.6. We also show that calendar band widths are widest around 4000 BP at the start of the Bronze Age, and become narrow through the later Bronze Age and Iron Age and back into the Neolithic.

### INTRODUCTION

$^{14}\text{C}$  dates are reported as BP values with an associated precision (Stuiver & Polach 1977). Typically, the quoted error represents  $\pm$  one standard deviation ( $1 \sigma$ ), as determined by the total number of accumulated counts for that sample. Most laboratories also include an error multiplier that reflects the uncertainty associated with repeated dating of identical sample material. The  $^{14}\text{C}$  date and total associated error are then converted to a calendar date (Stuiver 1989), using the internationally agreed  $^{14}\text{C}$  calibration curve determined by Stuiver and Pearson (1986) and Pearson and Stuiver (1986) using a computer program developed by Stuiver and Reimer (1986). Probabilistic calibration techniques have also been developed by van der Plicht and Mook (1989) and Stuiver and Reimer (1987). Calibration introduces two additional sources of uncertainty in the final calendar date. First, because the calibration curve, itself, is a set of  $^{14}\text{C}$  measurements made at decadal/bidecadal resolution on dendrochronologically dated wood, each point on the curve has an associated  $^{14}\text{C}$  error and error multiplier. Second, because the curve is not smooth, but contains considerable structure, the calendrical band width for a  $^{14}\text{C}$  date of a given precision varies at different points on the calibration curve.

Stuiver and Pearson (1986), Pearson and Stuiver (1986) and Stuiver and Becker (1986) published high-precision  $^{14}\text{C}$  calibration data in the Calibration Issue of *RADIOCARBON* (Stuiver & Kra 1986). In their papers, they included tables of calibrated age ranges, for a set of  $^{14}\text{C}$  precisions, at dates in the 80–4020 BP range in bidecadal intervals. These tables provide a useful guide to the calendar band width that can be expected for a  $^{14}\text{C}$  date of a given precision, but are restricted in range (80–4020 BP), and do not readily inform the user of the variations in band width as a function of  $^{14}\text{C}$  age. Here, we quantify the calendar age ranges for a set of  $^{14}\text{C}$  precisions at all  $^{14}\text{C}$  dates within the calibration range. This will be of use to archaeologists, who should be able to obtain an estimate of laboratory  $^{14}\text{C}$  precision based on sample size and expected age, and thereby determine whether or not the probable calendrical band width is useful. We also show that calibration band widths are greatest for all precisions around 4000 BP, at the start of the Bronze Age, and decrease through the later Bronze Age and Iron Age and into the Neolithic. This is attributed to the greatest rate of change of  $\Delta^{14}\text{C}$  being near 4000 BP.

## METHOD

Stuiver and Reimer (1986) compiled a data set, ATM20, for use in their calibration computer program. This file consists of calendar dates derived by dendrochronology and the associated  $^{14}\text{C}$  dates and errors at decadal/bidecadal intervals. In this study, we used ATM20 to determine the calendrical band widths of  $^{14}\text{C}$  dates at a series of precisions. The method involved fitting a cubic spline curve to the data set and interpolating at intervals of one year. Figure 1 shows the cubic spline interpolation superimposed on a linear interpolation (Stuiver & Reimer 1986) of a section of the original data. Checks at a range of  $^{14}\text{C}$  dates showed that both techniques yielded calendrical band widths that were within a few years of each other. We chose the spline because we agree with Suess and Linick (1990) that unknown parts of a function in nature can best be approximated by a spline. Having obtained the spline we then stepped through  $^{14}\text{C}$  age (1 step every 5  $^{14}\text{C}$  yr) from 8100–150 BP, and recorded the calendar ages at the intercepts of the values

$$\pm ((^{14}\text{C precision})^2 + (\text{calibration curve } \sigma)^2)^{1/2}$$

with the calibration curve. The difference between the calendar ages at the intercepts gives the calendrical band width for a given  $^{14}\text{C}$  date and associated precision. The band widths, as a function of  $^{14}\text{C}$  age (BP), were plotted for four  $^{14}\text{C}$  precisions in Figure 2 (A–D).

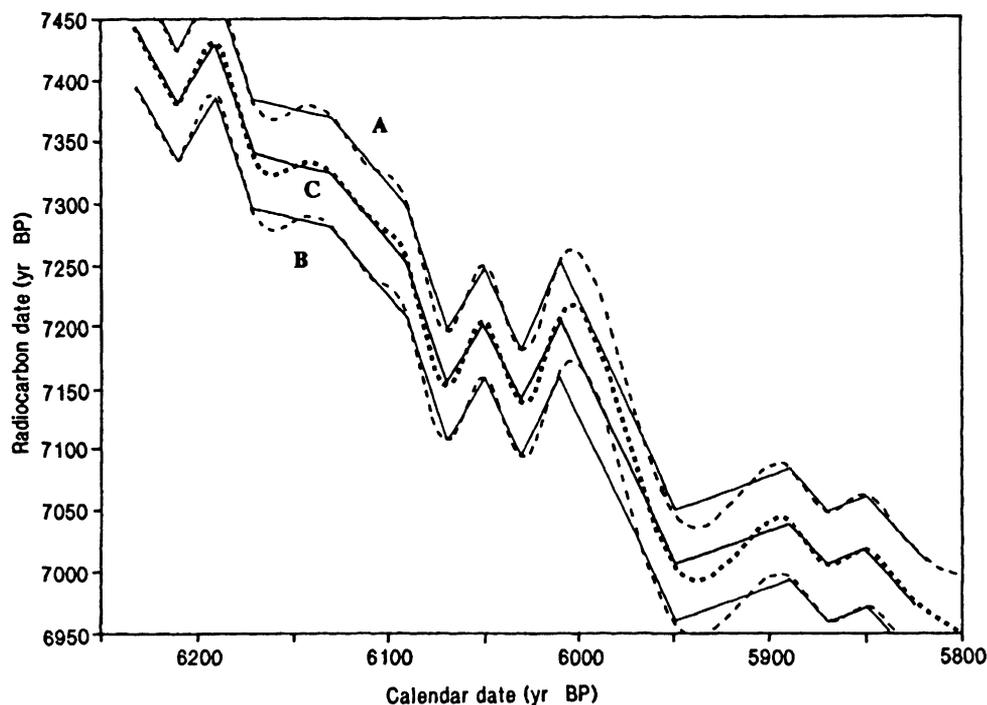


Fig. 1. Section of the  $^{14}\text{C}$  calibration curve. — = linear interpolations between data points, ---- = cubic spline interpolations; C = interpolated measurements; A and B =  $\pm 1 \sigma$  errors on the measurements

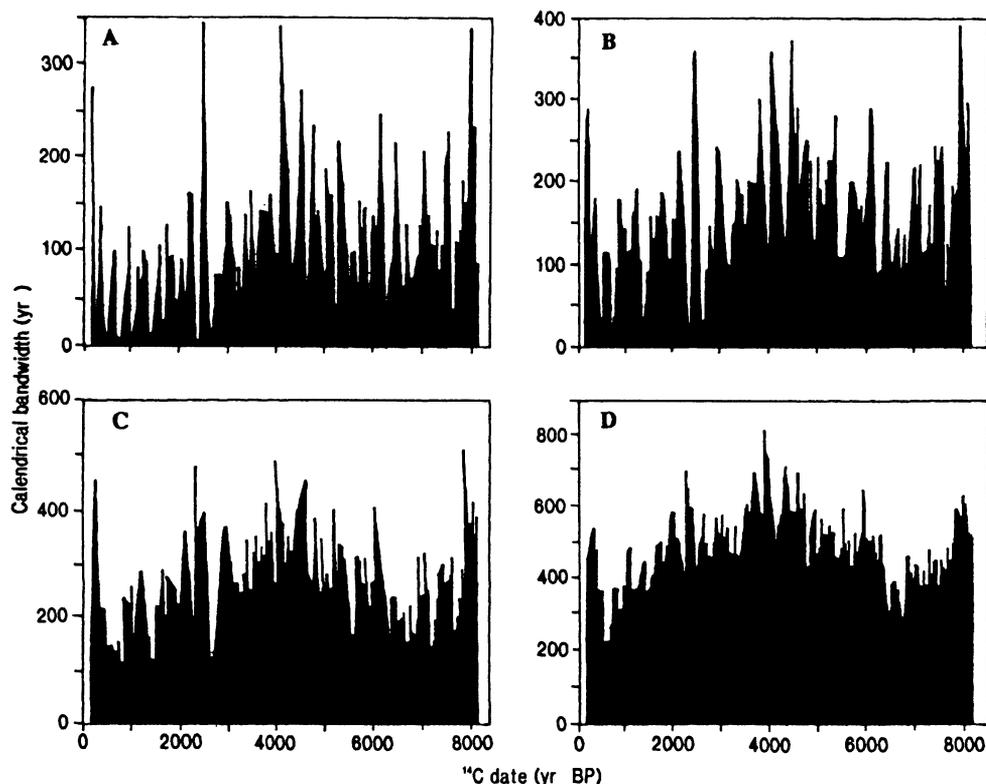


Fig. 2. Calendrical band widths in years as a function of  $^{14}\text{C}$  age, in increments of 5 yr from 8100–150 BP. A.  $^{14}\text{C}$  precision of  $\pm 20$  yr; B.  $^{14}\text{C}$  precision of  $\pm 50$  yr; C.  $^{14}\text{C}$  precision of  $\pm 100$  yr; D.  $^{14}\text{C}$  precision of  $\pm 200$  yr

## RESULTS

Figures 2(A–D) allow users of  $^{14}\text{C}$  dates to ascertain what level of  $^{14}\text{C}$  precision is required to obtain a given calendar band width at any time during the past nine millennia. The figures also show that even at the highest precisions (Fig. 2A), short intervals (*e.g.*, 2400–2500 BP and 7900–8000 BP) exist when the advantages of high-precision dating are minimal. The period, 4000–5300 BP (*i.e.*, the Neolithic to the beginning of the Bronze Age) gives the largest calendrical band widths. As the dating precision decreases from “high” to “routine” (*i.e.*,  $\pm 50$  yr) the calendrical band widths widen (Fig. 2B), but the trends in band width remain similar, with the 4000–5300 BP period giving the largest uncertainties in calendar dates. The mean calendrical band widths for precisions of  $\pm 20$  and  $\pm 50$  yr, found by averaging all band widths over 8100–150 BP, is 101 yr and 158 yr, respectively.

Figure 2(C, D) shows that, as the dating precision decreases, the graphs representing calendrical band width as a function of  $^{14}\text{C}$  precision take on a sinusoidal form, with an amplitude of *ca.* 250 yr and a period of 12,000 yr (assumed by extrapolation). The peak of the curve is at *ca.* 4000 BP which is the point of greatest rate of change of  $\Delta^{14}\text{C}$  in the radiocarbon record (Fig. 3). Thus, as the precision decreases, the band width reflects more closely the rate of change of  $\Delta^{14}\text{C}$ . To illustrate this graphically, we fitted a 4th-order Legendre polynomial to the  $\Delta^{14}\text{C}$  data derived from the ATM20 data set. We differentiated the resultant polynomial, plotted the calendrical band widths

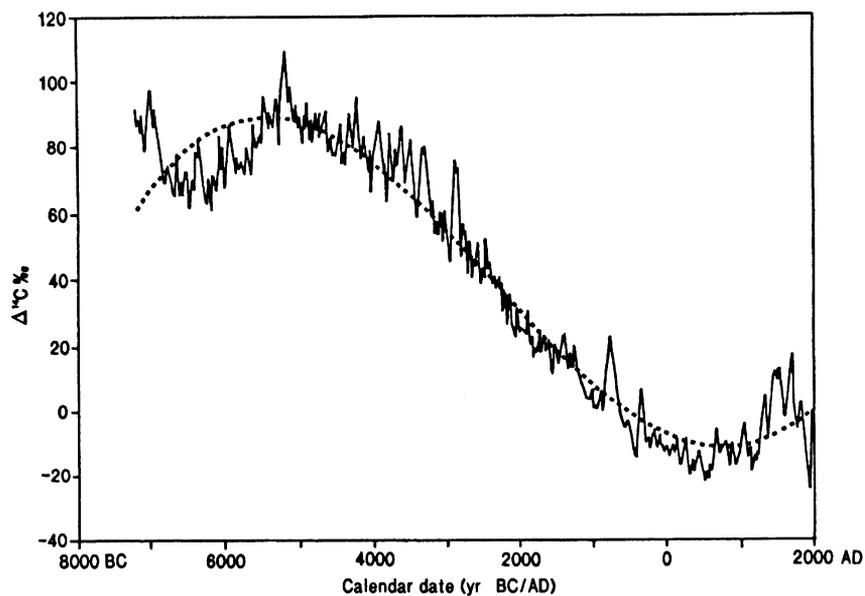


Fig. 3.  $\Delta^{14}\text{C}$  (‰) as a function of calendar year. Solid line is a 4th-order polynomial fit to the data.

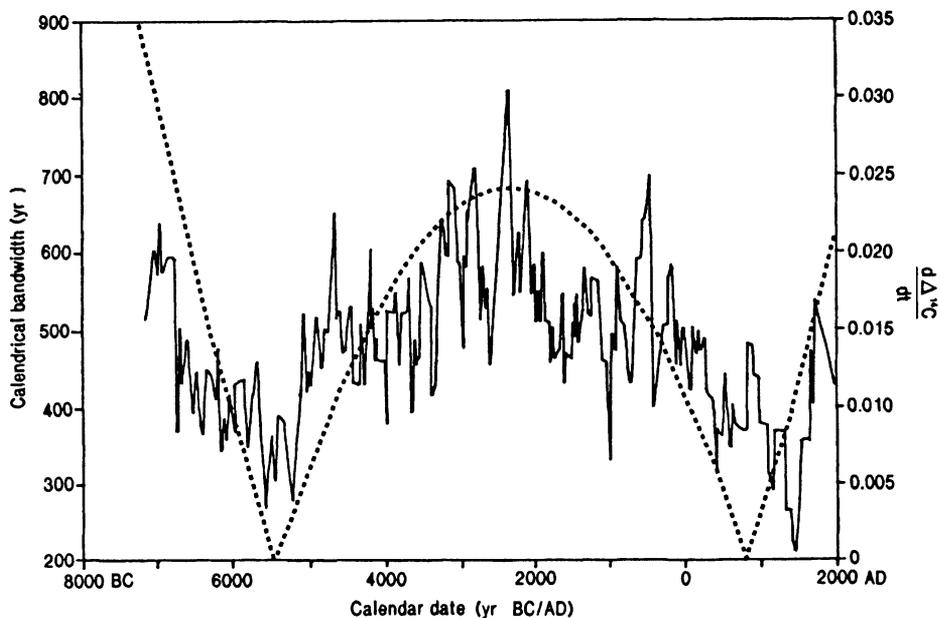


Fig. 4. Calendar band width as a function of calendar age for a  $^{14}\text{C}$  precision of  $\pm 200$  yr with the differentiated 4th-order polynomial shown in Figure 3 superimposed. This illustrates the good correspondence between the rate of change of  $\Delta^{14}\text{C}$  and calendrical band width at lower precisions.

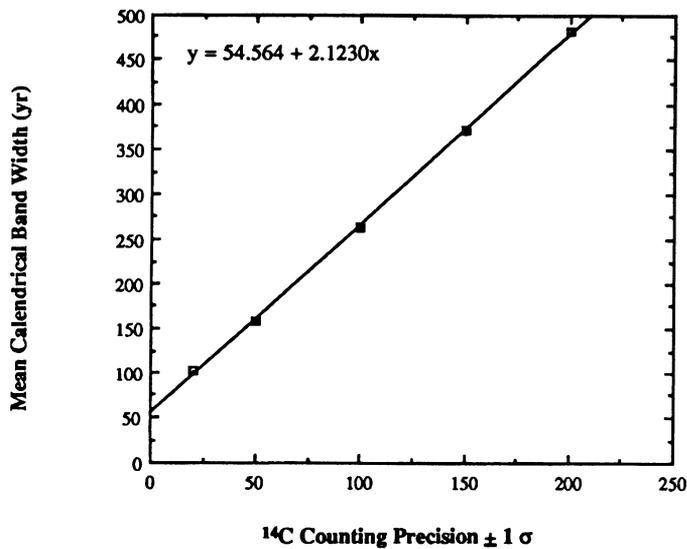


Fig. 5. Mean calendrical band width as a function of <sup>14</sup>C precision

as a function of calendar year, and superimposed the differentiated curve to show the good correspondence between peaks and troughs (Fig. 4). This illustrates that subtle changes in slope of the calibration curve are mimicked in the calendrical band widths at the lower precisions.

As the <sup>14</sup>C dating precision decreases, the calendrical band width increases. Figure 5 shows the mean calendrical band width over 7950 <sup>14</sup>C yr vs. the <sup>14</sup>C precision of ± 20, ± 50, ± 100, ± 150 and ± 200 yr. A linear regression fit to these data gives

$$\text{Mean band width (yr)} = 2.12 \times \text{<sup>14</sup>C precision (1 } \sigma) + 54.56. \tag{1}$$

### CONCLUSIONS

The band width of calendar dates vs. <sup>14</sup>C age (for dates with <sup>14</sup>C 1 σ > 100 yr) can be approximated by a sinusoid with an amplitude of ca. 250 yr and a period of ca. 12,000 yr with a peak at 4000 BP. For low-precision dates, this result illustrates the response of calibrated age range to long-term variations of atmospheric <sup>14</sup>C, and specifically, to the long-term rate of change of Δ<sup>14</sup>C.

Although evidence for a similar cyclical variation exists in the band widths obtained at higher precisions (*i.e.*, 1 σ < 50 yr), the calendar age range is dominated by a series of isolated periods with very large band widths, even for precisions as low as ± 20 yr. These correspond to periods of rapid fluctuations or slow change in the atmospheric <sup>14</sup>C content.

A linear relationship exists between <sup>14</sup>C dating precision and the mean calendrical band width, found by averaging all band widths over 8100–150 BP

$$\text{Band width (yr)} = 2.12 \times \text{<sup>14</sup>C precision (1 } \sigma) + 54.6. \tag{2}$$

Thus, for high-precision dates (*i.e.*, ± 20 yr), the calendar age range is typically 100 yr, whereas for routine dates (± 50 yr), a 160-yr band width can be expected. Of course, these dates will be modulated by the variability noted above and can only be used as a guide.

## ACKNOWLEDGMENTS

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