Fifth Meeting, Friday, 12*th March* 1897.

C. G. Knott, Esq., D.Sc., F.R.S.E., in the Chair.

On a Proof of the Fundamental Combination Theorem.

By J. B. Clark, M.A.

The following proof of the fundamental Combination Theorem does not appear in any of the current text-books on Algebra. It has the twofold advantage of being exceedingly simple and of being quite independent of the fundamental Permutation Theorem.

Let the n different things be represented by n letters

$$a, \quad b, \quad c, \quad d, \quad . \quad . \quad . \quad .$$

We can form a 1-combination in n ways.

We can form a 2-combination by taking any one of the n 1-combinations and along with it any one of the remaining $(n-1)$ letters : this gives $n(n-1)$ combinations, but each combination is formed twice, *e.g.*, ab arises when b is taken with a, and also when a is taken with b.

Hence
$$_nC_2 = n \, . \, \frac{n-1}{2}.$$

We can form a 3-combination by taking any one of the $\frac{n(n-1)}{2}$ 2-combinations, and along with it any one of the remaining $(n-2)$ letters : this gives $n \, . \, \frac{n-1}{2} \, . \, (n-2)$ combinations, but each combination is counted thrice, *e.g.*, abc arises from bc with a, from ca with b, and from ab with c.

Hence
$$_nC_3 = n \, . \, \frac{n-1}{2} \, . \, \frac{n-2}{3}.$$

In general we can form an r-combination in

$$n \, . \, \frac{n-1}{2} \, . \, \frac{n-2}{3} \, . \, . \, . \, \frac{n-\overline{r-2}}{r-1} \, . \, n-\overline{r-1}$$

ways, and since each combination is counted r times, we have

$$_nC_r = \frac{n(n-1)(n-2) \, . \, . \, . \, . \, (n-r+1)}{r!}$$

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