GRAVITATIONAL TIME DELAY OF SIGNALS IN THE KERR METRIC

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ABSTRACT. The gravitational time delay of signals in a gravitational field of a rotating massive body is considered both in a weak field approximation and in a strong field caused by a rotating black hole. The expressions describing the time of propagation of signals are obtained by integrating the light geodesics of the Kerr metric in a frame reference of a distant observer using the Boyer-Lindquist coordinates and assuming that the wave length of radiation is much less than the characteristic scale of the field. The existence of the asymmetry in the time delay is shown depending on the mutual orientation of a photon propagation direction and of the rotation axis. As a result of this asymmetry, the effects of relative time delay are predicted and calculated for the signals focused by a rotating gravitational lens.

The Kerr metric describes in a number of cases the gravitational field of a rotating massive body including a rotating black hole. The investigation of the gravitational time delay of signals in this metric is of great importance both for the applications to observable effects of General Relativity, such as the Shapiro effect, and for the consideration of the course of physical processes in the environment of a massive rotating body, giving an answer to the question in what fashion does the frame dragging effect influence the time of propagation of signals. It appears that the gravitational time delay in a gravitational field of a rotating body depends strongly on the mutual orientation of the photon propagation direction and the rotation axis. As a result, the gravitational retardation of signals appears in many cases to be the gravitational acceleration.

The results presented here are obtained by integrating the light geodesics of the Kerr metric in two cases :
1) in a weak field approximation when the distance of the nearest approach of a photon to the deflecting body is much larger than the radius of the event horizon :

$$r_+ = GMc^{-2}\,(1+(1-a^2)^{1/2})$$

where M is the mass and a is the dimensionless specific angular momentum of a gravitating body, G is the gravitational constant and c is the speed

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of light ;
2) in a strong field in the vicinity of a rotating black hole provided that the signal is propagating at the distance close to the gravitational capture radius. The Boyer-Lindquist coordinates (Boyer and Lindquist, 1967) are used coinciding at infinity with the ordinary spherical coordinates.

The time of propagation of a signal in the equatorial plane of a rotating body is described in the weak field approximation by the expression

$$t_{\perp} = (R^2-d^2)^{1/2} + 2\ln((R+(R^2-d^2)^{1/2})/d) + ((R-d)/(R+d))^{1/2} + (15\Pi-8)/(4d) - 4a\,\mathrm{sgn}(\rho)/d \quad GMc^{-3} \tag{1}$$

Here $RGMc^{-2}$ is the distance from the source of radiation to the deflecting body, d is the dimensionless distance of the photon nearest approach and ρ is the photon impact parameter connected with its orbital angular momentum L and the energy E by the relation $\rho = L/(Ec)$.

The terms which do not depend on the rotation, describe the gravitational time delay measured in Shapiro experiments. The rotational correction has been recently obtained independently (Kostyukovich and Mitjanock, 1979 ; Dymnikova, 1982) using different techniques. For Shapiro type experiments it should be characterized by a quantity of the order of 10^{-10} second. It is sufficiently small, but taking into account the rotation influence on the gravitational time delay, it is physically very important, because the additional gravitational time delay due to rotation can have different signs. It appears to be positive only for the signals propagating in the direction opposite to the rotation. For a signal propagating in parallel to the axis, the propagation time has the following form :

$$t_{\parallel} = ((R^2-d^2)^{1/2} + 2\ln((R+(R^2-d^2)^{1/2})/d) + ((R-d)/(R+d))^{1/2} + (15\Pi-8)/(4d) + (120\Pi - 23 - 4a^2)/(4d^2)) \quad GMc^{-3} \tag{2}$$

The additional gravitational time delay due to rotation appears in this case to be negative. The quadratic behaviour of the corresponding term is in some sense similar to the transversal Doppler effect of the special theory of relativity.

In the case of a photon falling into a rotating black hole along a trajectory spiralling inward towards the conical surface $\theta = \theta_o$ =Const, the infall time is given by

$$t_i = (r + \ln(r^2-2r) + \ln((r^2-2r)(1+a^2/(r^2-2r))) - (1-a^2)^{-1/2}\ln((r-1+(1-a^2)^{1/2})/(r-1-(1-a^2)^{1/2}))) \quad GMc^{-3} \tag{3}$$

This time is less than the infall time in the case of a non-rotating black hole which is described by the first two terms of the expression (3).

The asymmetry in the gravitational time delay depending on the orien-

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tation of a photon orbital angular momentum about the rotation axis, results in the effect of the relative time delay of signals propagating along the different optical ways from a given point of a source of radiation into the same point of observation. This can be important in such situations when the rotating massive body plays the role of a gravitational lens.

In the simplest case when the source of radiation, the observer and the point like non-rotating gravitational lens are located along the same line, the arising images are simultaneous, i.e. the times of propagation of signals between the emission and observation points along different optical paths are identical. The rotation of the lens results in non-simultaneity of the images in the considered case. The magnitude of the relative time delay between the photons arriving at the same point along the different paths is the highest possible in the equatorial plane while it vanishes for signals propagating in parallel to the axis. The azimuthal deflection of rays in the equatorial plane is described in the approximation d<<1 by the relation :

$$\Delta\phi = 4/d+(15\,\Pi-16)/(4d^2)-4a\,\mathrm{sgn}(\rho)/d^2 \qquad (4)$$

The rays with the negative values of the impact parameter are deflected stronger than the rays with the positive ρ . Let us note that combining the expressions $\Delta\phi$ for two rays propagating with impact parameters of the same magnitude and different signs, one could in principle determine the value of the angular momentum of the deflecting body.

The condition of the alignment of the source of radiation, the lens and the observer has the form :

$$d/b + d/f = 4/d-4a\,\mathrm{sgn}(\rho)/d^2+(15\Pi -32)/(4d^2) \qquad (5)$$

where b is the distance from a rotating gravitational lens to the emission point and f is the distance to the observation point, all three being situated along the same line in the equatorial plane of the rotating lens (see Figure).
It follows that the rays emitted from some given point with the positive

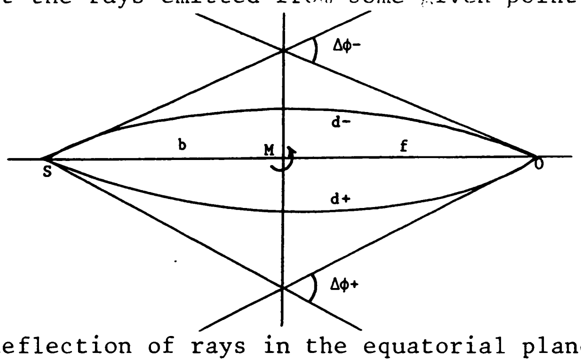

Figure : The deflection of rays in the equatorial plane of a rotating gravitational lens M. S is the radiation source and O is the observer ; d and $\Delta\phi$ are the distance of the nearest approach and the azimuthal angles of the photons.

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It follows that the rays emitted from some given point with the positive and negative values of the photon impact parameter arrive at the same observation point only in such a case when they are subject to the condition.

$$d_- = d_+ + a \tag{6}$$

As a result, the relative time delay arises, in connection with the difference in the optical paths between the rays. It is determined by the formulae (3) and (6) and given by

$$t = t_- - t_+ = 8a/d\ (GMc^{-3}) \tag{7}$$

For the case of a compact relativistic object (neutron star or black hole) as a gravitational lens, this relative time delay is characterized by the quantity :

$$\Delta t = 2\times10^{-6}s(M/3M_\theta)(a/1)(100/d) = 200s(M/5\times10^8 M_\theta)(a/1)(100/d) \tag{8}$$

One can imagine another situation, when a rotating black hole situated in the picture plane of a distant source of radiation, would play the role of gravitational lens. In this case, the black hole will look literally like the black hole within the image of the source. Its shining boundary coincides with the curve restricting the gravitational capture cross-section which is formed by the photons deflected by the hole at the angles :

$$\Delta\phi = \pm 3k\Pi, \quad \text{where } k=1,2,3,\ldots.. \tag{9}$$

Due to the rotation of a hole, the gravitational capture cross-section is very asymmetrical and depends upon the infall angle of the photons. The asymmetry of the cross-section should result in the relative time delay between the photons forming the boundary of the hole within the image of the source of radiation. It will be maximum for the photons propagating in the equatorial plane on the opposite sides of the black hole.

The time of propagation of signals in the equatorial plane of a rotating black hole close to the capture region is described by the expressions presented in the Appendix. Using them, one can show that the time delay between the photons of the shining halo, surrounding the image of the hole within the source of radiation, is described in the case a = 1 by the following approximate formula

$$\Delta t = 3\,|\Delta\phi|\ GMc^{-3} \tag{10}$$

If the luminosity of the source of radiation would change rapidly enough, one should expect that the relative time delay (10) will lead to the effect of "sunbeam in a mirror" running along the shining boundary of a hole with the speed depending on the distances between the observer and the hole, and between the observer and the source of radiation, as well as on the hole angular momentum. For the case of the extreme black hole with a=1 the "sunbeam" speed is given by

$$v_S = (c/6)(R_S/R_H). \tag{11}$$

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APPENDIX

The time of propagation of a photon in the equatorial plane of a rotating black hole from the point R_1 to the point R_2 close by the capture radius r_c (see, e.g., Bardeen 1973 ; Dymnikova, 1982) is described by the following formula where $d-r_c = \delta \ll 1$:

$$t=(((R_1^2+2R_1d)^{1/2}+2\ln(((R_1^2+2R_1d)^{1/2}+R_1+d)/d)-d\sqrt{3}-2\ln(2+\sqrt{3})+$$
$$+A_t(\sqrt{3})^{-1}(-\ln\delta+\ln(3d/(1+\sqrt{3}/2))+B_t^{\pm}(r_\pm/(2d+r_\pm))^{1/2}\ln(d((3r_\pm(2d+r_\pm))^{1/2}+$$
$$+2r_\pm+d)/((r_\pm(2d+r_\pm))^{1/2}+r_\pm+d)(d-r_\pm)))\,GMc^{-3} \qquad (A1)$$

+ preceding term with R_1 being replaced by R_2.

Here the coefficients A_t and B_t are determined by

$$A_t=(d^4-3d^3+a^2d^2+a^2d)/((d-3)(d^2-2d+a^2)) \qquad (A2)$$
$$B_t^{\pm}=((d-3)(d^2-2d+a^2))^{-1}(-2d^2+6d-a^2d-3a^2\pm(6d-2d^2-a^2d^2-3a^2)(1-a^2)^{-1/2})$$

and r_- is the smaller root of the equation $r^2-2r+a^2=0$ (the inner event horizon).

The influence of the rotation becomes the strongest in the case of extremely fast rotating black hole with a=1. In this case the photons with the impact parameters $-7\leqslant\rho\leqslant 2$ (Bardeen, 1973) are captured at the points $r_c=\rho-1/2$. Actually the black hole can not have a larger than 0.998. The azimuthal deflection of a ray near the event horizon is expressed in terms of δ and $\varepsilon=1-a\ll 1$:

$$\Delta\phi=(2\sqrt{3}(2\delta+\sqrt{\varepsilon/6}))^{-1}(8/3(-\ln\delta+\ln(3/(1+\sqrt{3}/2))(2\delta+\sqrt{\varepsilon/6})+\sum_{+,-}(1+(2\pm\sqrt{3})$$
$$(\sqrt{2\varepsilon/3}/\delta)(1+(1\pm4\sqrt{3})\delta/(2\sqrt{2\varepsilon/3}) \qquad (A3)$$

for the rays propagating in the direction of the rotation and :

$$\Delta\phi=(8/3\sqrt{3})\ln\delta-(8/3\sqrt{3})\ln 12+(8/3\sqrt{3}-10/9)\ln(1+\sqrt{3}/2)$$

for the photons going in the opposite direction and deflected at the points $d=4+\delta$. The time of propagation of a signal in the direction of the rotation is, in this case, given by

$$t=GMc^{-3}((R_1+R_2+2\ln(R_1R_2)-4\ln(1+\sqrt{3}/2)+2(1-\sqrt{3})+ +(3(2+\sqrt{\varepsilon}/6))^{-1}(-(13/3)\ln\delta(4d+\sqrt{2}\varepsilon/3)+ + \sum_{+,-} \ln(1+(2\pm\sqrt{3})(\sqrt{2\varepsilon/3}/\delta))(1+(1\pm4\sqrt{3})\delta/(2\sqrt{2\varepsilon/3}))), \tag{A4}$$

while the time of propagation of the signals going opposite to the rotation has the form

$$t=GMc^{-3}(R_1+R_2+2\ln(R_1R_2) + 8(1-\sqrt{3}) - 4\ln4 - -56/(3\sqrt{3})\ (\ln\delta-\ln12 + (1+1/\sqrt{3})\ln(1+\sqrt{3}/2))) \tag{A5}$$

The photons deflected by the hole at the angles (A3) are forming the shining halo surrounding the black hole.

DISCUSSION

Grishchuk : what are the perspectives of testing your effect experimentally ? For the Sun it is too small, for the black holes, we don't know masses with sufficient precision, so we cannot separate Newtonian and relativistic effects.

Dymnikova : the aim of my work was to estimate whether it is necessary to take these effects into consideration. It turns out that now they are negligible, but they will be measurable by perspective micro arc second measuring devices (POINTS etc.).