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DISCUSSION NOTE

# Further Reflections Based on the Black–Scholes (B-S) Model

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## Abstract

The Black–Scholes (B-S) model is considered by the academic environment one of the greatest achievements of financial economics. Yet it brings with it some conundrums. The model is often used in a manner that contradicts one of its assumptions, and its predictions are not supported by market reality. Here we address, from the perspective of philosophy of science, an additional issue related to this model: the distinction between its explanatory and predictive capabilities.

## 1. Introduction

The Black–Scholes (B-S) model occupies a prominent place in the financial economics landscape (Black and Scholes 1973). The academic establishment considers it one of the most remarkable achievements of the last 50 years: a wonderful display of mathematical showmanship combined with practical relevance. It earned Scholes a Nobel Prize in 1997 (Black died in 1995). Additionally, this model figures prominently in all financial economics textbooks and in most business, economics, and finance curricula.

For our discussion, we will distinguish between (i) the B-S equation and (ii) the B-S pricing formulas. We reserve the term *model* to refer to both the equation and the formulas together. In the original derivation of the B-S model, the equation precedes the formulas; in other words, the equation is a requirement to derive the formulas. The equation and the pricing formulas are obviously intimately related, but the relevance of this distinction will be apparent in what follows.

The B-S equation is a partial differential equation whose solution, depending on the boundary conditions used to solve it, allows one to arrive at the B-S pricing formulas, one for calls and another for puts. Although the B-S equation is not a stochastic differential equation, its derivation requires a good command of stochastic calculus, a discipline that is considered advanced mathematics.

An option is a contract that gives the right, but not the obligation, to buy (call) or sell (put) an asset whose price we denote as $S$, at some future time ($T$), for a previously agreed price ($K$). The B-S model deals with European options, that is, options that can only be exercised at expiration ($t = T$) but not before. Note that American options can be exercised at any time between $t = 0$ and $T$. The B-S pricing formulas are mathematical expressions to calculate the fair prices of these options, denoted as $C$ (calls) and $P$ (puts). The derivation of the B-S equation assumes that $S$, the asset price, evolves from $t = 0$ (today) to $T$, according to a stochastic process known as *geometric Brownian motion* (GBM), which depends, among other factors, on a parameter, $\sigma$, which stands for volatility and is assumed to be constant. Finally, the B-S formulas depend on $T$, $K$, $\sigma$, the value of the asset today ($S_0$), and $R$ (the risk-free interest rate).

Yet despite the model's distinguished pedigree, a nagging problem persists with the B-S formulas: they often predict prices that do not agree with market reality. In fact, the most salient feature of the B-S pricing formulas is that they do not work for their intended purpose, that is, estimating the price of European calls and puts. The empirical evidence is overwhelming (e.g., Asay 1976; Kumar and Agrawal 2017; Redroban and Cifuentes 2021). Furthermore, there is substantial evidence that traders do not use the B-S formulas for pricing such options —instead, they rely on a number of different approaches based on more realistic assumptions and heuristics. This fact has been well known since the late 1970s (Haug and Taleb 2011). However, in spite of this empirical failure, the ubiquitousness of the B-S model in the financial economics arena has proven to be persistent, which, in turn, raises interesting considerations. Some of them go beyond the scope of financial economics and fall within the realm of philosophy of science.

In what follows, we take as a starting point the observations made by Weatherall (2018) in relation to the volatility smile (a peculiar feature of the B-S model) and offer an additional observation that complements Weatherall's insights; specifically, we elaborate on an aspect of the B-S model from the viewpoint of philosophy of science: the distinction between explanatory and predictive models.

## 2. The volatility smile phenomenon

Weatherall (2018) identified a peculiarity associated with the B-S model: the fact that one of the assumptions behind its derivation—that the volatility of the reference asset is constant—is in contradiction with the empirical evidence. Nevertheless, despite this shortcoming, the B-S model is frequently used in an "inverted" fashion (Weatherall's term)—that is, to estimate the volatility, assuming that all the other variables involved in the formulas are known. The peculiarity results from the fact that the model is employed to explore how the volatility changes as a function of the other variables, which is in contradiction with one of the hypotheses on which the model is based. In this case, the price of the option ($C$ or $P$), which can be observed from market data, is taken as an input for the B-S formulas. Weatherall advances three explanations for why the model is still used, which depend on the specific attitude one has taken toward the model. He concludes that even a broken model, in the right context, can be used to illuminate certain aspects of the problem under study.

Incidentally, the usefulness of other financial economics models that are based on unrealistic assumptions has also attracted the attention of academics in the philosophy of science field. The most salient case is perhaps the Modigliani–Miller theorems (e.g., Hindriks 2008, 2013; Vergara-Fernández and de Bruin 2021). However, to the best of our knowledge, an analogy similar to the peculiarity described by Weatherall—that is, exploring how something changes based on a formula that assumes it does not change—has not been found in other financial applications.

## 3. Explanatory models versus predictive models

Weatherall focuses on the fact that the B-S pricing model formulas are not used to price calls and puts but rather to estimate implied volatilities. Furthermore, it is important to note that the B-S formulas are also used to design hedging strategies, a very different application of the B-S model. Hedging is the use of options to reduce the risk in an investment portfolio when there is a change in the value of one of the key parameters, for instance, an interest rate. Traders and practitioners use the Greeks to structure these hedge arrangements. The Greeks are partial derivatives that express how the price of an option changes when the value of one of the reference variables changes. The Greeks, notwithstanding the fact that they are expressions derived from formulas that are inaccurate for pricing, are widely employed for hedging purposes.

The Greeks are useful because they successfully establish a direct causal relationship between the reference variables (e.g., $K$, $T$) and the price of the option. They are used to estimate, assuming one knows the correct value of an option, how much that value will change if there is a small variation in the value of the reference variable. Additionally, the Greeks allow potential investors to compare the merits of different option positions in terms of their sensitivity. This seemingly strange phenomenon—the success of the Greeks despite the failure of the pricing formulas—requires a brief mathematical aside.

In broad terms, the success of the Greeks lies in the fact that approximating the derivative of a function (the Greeks) is easier than approximating the function itself (the price) simply because the derivative operator "smooths out" variations. More precisely, consider a single-variable function $F(x)$. Assuming that $F$ satisfies some continuity conditions, $F$ can always be approximated, with whatever degree of accuracy we want, using a polynomial expansion, that is, a Taylor series. Suppose that we have approximated $F$ with a polynomial of degree $N$; clearly, its derivative will be a polynomial of degree $N - 1$. Note also that a polynomial of degree $N$ has $N$ roots, whereas a polynomial of degree $N - 1$ has $N - 1$ roots. From this result, it follows that the derivative of $F$ will be smoother than $F$. The following example provides a more intuitive explanation: Suppose $F(x) = x^2$; its derivative is $2x$. A graphic representation of $F(x)$ is a parabola that opens upward and has a minimum at $x = 0$; on the other hand, $2x$ is just a straight line.

The usefulness of the Greeks, despite the failure of the B-S formulas to estimate prices, brings us to a topic that has already grabbed the attention of philosophy of science in a much broader context: the difference between explanatory models and predictive models (e.g., Hempel and Oppenheim 1948; Helmer and Rescher 1959; Breiman 2001; Shmueli 2010; Sainani 2014).

Thus, it is probably fair to say that the B-S model is a failure from a predictive viewpoint because the B-S formulas result in prices that do not match market reality, but the B-S model is a good explanatory model: the Greeks offer a reliable account of several causal relationships that influence options prices.[1] Moreover, the Greeks have passed the empirical test and are widely used by market participants. We believe that recognizing this dichotomy is important.

## 4. Implied volatility and market stress

Weatherall (2018) mentions, correctly, that traders often use the implied volatility as a vehicle to convey information. Strictly speaking, this type of use, however legitimate, falls outside the explanatory-versus-predictive discussion. It is simply another dimension of the usefulness of the B-S model. In the same vein, it is important to mention that the implied volatility is also used to assess the level of uncertainty in financial markets (Londono and Wilson 2018).

The Standard & Poor's (S&P) 500 is an index that reflects the overall price level of the U.S. stock market—by far the most important equities market in the world in terms of size and transaction volume. The volatility index (VIX), the volatility implied by the S&P 500 options calculated with the B-S formulas, is a metric that captures the uncertainty (or stress) in the market. It is important to note that the concept of volatility, or the standard deviation of returns, was first introduced by Markowitz (1952) in his seminal paper as a useful risk metric. Since then, it has been widely adopted by regulators and investors alike as a means of assessing uncertainty. In fact, market participants normally refer to the VIX as the *fear gauge*. The VIX is closely monitored by central bankers and financial regulators throughout the world. In a sense, it is taken to indicate the well-being of the capital markets. When the VIX reaches very high levels (more than 50, based on historical evidence), regulators might take actions such as opening liquidity facilities via the so-called central bank window and/or offering credit lines that would not be available under normal market conditions. This occurred, for example, in November 2008, during the subprime crisis, and in March 2020, during the COVID crisis (Weinberg 2015; Federal Reserve Bank of New York 2021). In both cases, regulators took actions—ultimately based on the B-S formulas and some additional information—that had profound effects on the functioning of the financial markets.

Additionally, when a regulator or any other market participant uses the VIX in combination with other information to guide a decision, they are implicitly assuming that the B-S formulas are right and that the market is functioning in a distorted way—something that needs to be corrected. This interpretation is more in line with what Weatherall calls *compatibilism*, that is, the view that the B-S model is not broken. Clearly, using the B-S model in this manner (to intervene when the market is not working the way it should) implies that the regulators are using the B-S formulas not in a classical positive economics fashion but as a tool within the realm of normative economics.

[1] Milton Friedman, keenly aware that some assumptions behind many economic models had invited criticism, stated a pragmatic view regarding this matter. He claimed that whether the assumptions behind a model were realistic or not was irrelevant as long as the model resulted in reliable inferences (Friedman 1953). A full discussion of this assertion from a philosophical angle is beyond the scope of this commentary. The topic has been treated in detail elsewhere (e.g., Wible 1982; De Scheemaekere 2009; Pfleiderer 2020). Anyhow, at least in reference to the Greeks, Friedman's view, however loaded or controversial, seems to have some validity.

## 5. Conclusions

The B-S model plays an outsized role in the financial economics arena: it is widely praised as an important intellectual achievement, yet it is mostly used in a manner contrary to the initial intention. Moreover, the option prices predicted by the B-S formulas—that is, the model-predicting capacity—fail the empirical evidence test. This puzzling situation, fascinating in itself, offers an opportunity to reflect on several topics that are beyond the sphere of finance and fall under the umbrella of philosophy of science.

Weatherall (2018) focused on one feature of the B-S model (the volatility smile) and advanced some perceptive observations. The purpose of this commentary has been to expand on another aspect of the B-S model that we believe is interesting, namely, its predictive versus its explanatory capabilities. It would be fair to say that the B-S model is a failure from the predictive viewpoint but a success from an explanatory viewpoint. Furthermore, strictly from a teaching perspective, given that this model is covered in most financial and economics courses, we think it would be wise to emphasize this distinction.

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