BOOK REVIEWS

HALMOS, PAUL R., *Naive Set Theory* (Van Nostrand, Princeton, 1960), 26s. 6d.

This is an admirable account of those parts of set theory needed by the prospective pure mathematician. Although the title may suggest the contrary the treatment is abstract and axiomatic; but the axioms are regarded as a source from which facts are to be drawn as quickly as a logically clear exposition to the beginner will allow. To achieve this in such a small book (it consists of 25 short sections) some proofs are merely hinted at and many are left entirely as exercises. Thus many sections take on a predominantly descriptive character. Proofs are presented informally in the language and notation of ordinary mathematics and in this sense the account is naive.

In the first few sections the author introduces axioms for the basic set-constructing processes and deduces their consequences. Here, as in most of the later sections, he supplies a wealth of informal comment and explanation with which to ease the beginner's difficulties. For example he takes care on several occasions to allay possible suspicion of vacuous conditions; at another point he remarks that Burali-Forti was one man and not two. He describes a formal language for specifying subsets but soon replaces it in favour of ordinary usage and he makes no use of the special term " class ".

In the sections dealing with relations, families, mappings and order, he gives a straightforward account of practically all the standard terms and notations associated with these concepts. With the aid of the axiom of infinity he constructs the natural numbers as transitive sets, proves the recursion theorem and is then able to give inductive definitions of addition and multiplication followed by a brief sketch of arithmetic. The transfinite version of the recursion theorem is reached by way of the axiom of choice, Zorn's lemma and the well-ordering theorem. The properties of ordinal numbers and those of cardinal numbers are treated independently of each other and the subject of their inter-relation is left until the final section.

T. W. PARNABY

MCSHANE, E. J., AND BOTTS, T., *Real Analysis* (van Nostrand, 1959), 272 pp., 49s. 6d.

This book is an excellent introduction to certain parts of real analysis and functional analysis. Nevertheless the real analysis, as usually understood, does not begin until Chapter IV and, when I was reading through the book, this chapter came as a real pleasure to read after the close concentration required in the study of generalised convergence (in R. L. Moore's sense) followed by applications to continuous functions in a Hausdorff space in Chapter III. If the book is to appeal to beginning graduate students (and the chapters on *real* real analysis are excellent for this purpose) then surely it is a psychological error to begin with such abstract topics. By contrast, the measure theory in Chapter IV is restricted to subsets of Euclidean spaces, though a large number of important theorems in measure theory and functional analysis seem to have been included. One might mention the Hahn-Banach theorem, Hahn and Lebesgue decompositions, Parseval's formula, the Radon-Nikodym theorem and the Riez-Fischer theorem.

Chapter I, on fields, is not difficult, but it seems to include too much material of doubtful relevance to the rest of the book. For instance, what use can Theorem 1.10

(characterising a subfield) be to a real analyst? If the theory of ordered fields is to be done at all, surely the existence of one such field should be proved (assuming, if you like, the properties of the rationals).

As to Chapters II and III, they include very important ideas and add greatly to the usefulness of the book. However, for the less mature reader, it would seem better to define convergence and continuity for real spaces only at first, consigning the general theory to a later stage—perhaps between the present Chapters V and VI.

Lastly, one should mention the excellent collection of exercises, including hints for the more difficult ones. The book is certainly valuable to any graduate student, more particularly if he is given guidance about the order in which it should be read.

A. M. MACBEATH

HYSLOP, J. M., *Real Variable* (University Mathematical Texts Series, Oliver and Boyd, 1960), viii+136 pp., 8s. 6d.

This book has chapters on numbers, bounds, limits, continuity and differentiability, exponential and logarithmic functions, Taylor's expansion, the evaluation of limits, upper and lower limits, and circular functions. It is intended for the use of Honours students, either at the beginning of their formal study of analysis or for private reading at an earlier stage. No attempt is made to give all that is necessary for an Honours course; for example, the real numbers are not defined by means of Dedekind sections, but there is a careful statement of the properties which they are assumed to have.

To avoid overlaps with other University Texts, the present book includes a minimum of convergence theory and does not discuss integration. These are severe handicaps in a subject which holds formidable difficulties for the average student without the introduction of artificial ones. If the beginner is to be attracted to a further study of analysis, an elegant presentation is essential, and the reviewer cannot accept the author's claim that this has been achieved.

In a book of this size, there are naturally omissions which will not meet with universal approval. The possibility that a sequence may oscillate is not mentioned in the chapter on limits. The significance of the theorem on the limit of a function of a function (Theorem 23) will surely be lost in the absence of a counterexample in which the result does not hold because the inner function takes its limiting value in every neighbourhood of the limit point. Often an extra sentence would make an explanation easier to understand; thus in several proofs the author establishes a contradiction but fails to say explicitly what the contradiction is.

On the other hand several proofs could be shortened or simplified, notably those of Theorems 5 and 17, and the book contains many unimportant theorems, such as $\overline{\lim}\{f(x)-g(x)\}\leqslant\overline{\lim}\,f(x)-\overline{\lim}\,g(x)$. If these theorems were omitted or given as examples, it would be easier for the reader to see which results were important.

The circular functions are defined by series expansions, and are not identified with the functions encountered in trigonometry. The behaviour of $f(x)$ as x tends to a is dealt with by considering the behaviour of $f(a+y^{-1})$ as y tends to infinity, so that one group of theorems may be deduced from another. In a textbook for beginners, the reviewer feels that it is unwise to use statements such as " $\overline{\text{bd}}\,f(x) = +\infty$ " or " every monotonic function of x has a limit as x tends to infinity ".

The printing is of the high standard of other books in the series, except for the large vertical bars. There are singularly few misprints, the most serious of which are the superfluous commas in the statement of Theorem 18.

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