BULL. AUSTRAL. MATH. SOC. 10H08, 12B05, 32A30
VOL. 26 (1982), 317-319.

ON MEASURES OF POLYNOMIALS IN SEVERAL VARIABLES: CORRIGENDUM

GERALD MYERSON AND C.J. SMYTH

1.

In [2], it was asserted in Theorem 3 that the measure $M(x_0+\ldots+x_n)$ is asymptotically $c\sqrt{n}+O(1)$, where c was an explicit constant. The value of c given was incorrect, and should be $e^{-\frac{1}{2}\gamma}$ where γ is Euler's constant. This was pointed out by the first author. In fact

$$M(x_0+\ldots+x_n) = e^{-\frac{1}{2}\gamma}\sqrt{n}+O(\log n/\sqrt{n}) \ , \tag{1}$$

where we have tried to make amends by improving the error term.

2.

The mistake in the proof of Theorem 3 occurred in the third line, where, in the notation of [2], $\log_+\left|e^{i\theta_1}+\ldots+e^{i\theta_n}\right|$ was incorrectly split up as $f(C, S)+\chi_n(C, S)$. Here

$$C = C(n, \theta) = n^{-\frac{1}{2}}\left(\sqrt{2}\cos\theta_1+\ldots+\sqrt{2}\cos\theta_n\right) ,$$

$$S = S(n, \theta) = n^{-\frac{1}{2}}\left(\sqrt{2}\sin\theta_1+\ldots+\sqrt{2}\sin\theta_n\right) .$$

Using the corrected identity

$$\log_+\left|e^{i\theta_1}+\ldots+e^{i\theta_n}\right| = \tfrac{1}{2}\log n+\tfrac{1}{2}\max\left(-\log n, \log\left(\tfrac{1}{2}\left(C^2+S^2\right)\right)\right) \tag{2}$$

Received 2 June 1982.

and replacing the f of [2] by

$$f(x, y) = \max\left(-\log n, \log\left(\tfrac{1}{2}(x^2+y^2)\right)\right) ,$$

we have

(3) $\log M(x_0 + \ldots + x_n)$

$$= \frac{1}{(2\pi)^n} \int_0^{2\pi} \ldots \int_0^{2\pi} \log_+ |e^{i\theta_1} + \ldots + e^{i\theta_n}| d\theta_1 \ldots d\theta_n$$

$$= \tfrac{1}{2} \log n + \tfrac{1}{2} \int_{R^2} f(z) dQ_n(z) ,$$

where $z = (x, y)$ and $Q_n(z)$ is the distribution function of (C, S). Now

(4) $$\int f d\Phi = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \cdot \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2)} dx dy$$

$$= -\gamma + O(\log n/n) ,$$

on transforming to polar coordinates, and then using $\int_0^{\infty} \log u . e^{-u} du = -\gamma$, where $u = \frac{1}{2} r^2$.

To estimate $\int f d(Q_n - \Phi)$, use Theorem 20.1 of [1]. (Take $s = 4$. We have $P_0 \equiv \Phi$ and $P_1 \equiv 0$ since $\mu_\nu = 0$ for $|\nu| = 3$, $\chi_3(z) = 0$ from (6.21) and (7.6). Clearly $|f| \le \log 2n$, and one can check that $\int dP_2 = O(1)$, so that $\int f dP_2 = O(\log n)$.)

We obtain

(5) $$\int f d(Q_n - \Phi) = O(\log n/n) .$$

Combining (3), (4) and (5) gives (1), on exponentiation.

3.

Theorem 20.1 of [1] can be used to work out an asymptotic expansion for $\int f d(Q_n - \Phi)$. Also, it is not difficult to obtain more terms in the

https://doi.org/10.1017/S0004972700005773 Published online by Cambridge University Press

asymptotic expansion of $\int f d\Phi$ using (4), and hence one could obtain more terms in the expansion of $M\left(x_0 + \ldots + x_n\right)$. However, we have not worked out the details.

4.

Please note the following errata to [2]:

p. 50, line -10, insert "i with" before "non-zero";

p. 53, line 5, change cos to cot;

p. 54, line -2, change $\frac{(-1)^{j-1}}{j}$ to $\frac{(-1)^{j-1}}{j^2}$;

p. 59, line 13, insert "sup" before $|f(y_1)-f(y_2)|$;

p. 59, line -1, change $n^{\frac{1}{4}}$ to $n^{\frac{1}{2}}$;

p. 62, line 4, change log to $\log_+$.

References

[1] R.N. Bhattacharya and R. Ranga Rao, *Normal approximations and asymptotic expansions* (John Wiley & Sons, New York, London, Sydney, 1976).

[2] C.J. Smyth, "Measures of polynomials in several variables", *Bull. Austral. Math. Soc.* 23 (1981), 49-63.

Department of Mathematics,
State University of New York,
Buffalo,
New York 14214,
USA;

Department of Mathematics,
James Cook University of North Queensland,
Townsville,
Queensland 4811, Australia.