Nonlinear Evolution of Acoustic Waves in Dust Interacting With Dark Matter in Newtonian Cosmology: Biasing, Voids, and the Kadomtsev-Petviashvili Equation

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This paper shows that the Kadomtsev-Petviashvili equation of type I

$$[A_\tau + \frac{3}{4}\Gamma(A^2)_\xi - \frac{\Gamma}{2}A_{\xi\xi\xi}]_\xi = -\frac{\Gamma^2}{2}A_{\eta\eta} \qquad \text{(KP-I)}$$

governs the evolution of certain weakly nonlinear fluctuations on a static or very slowly varying cosmological background. The model consists of a mixture of nonrelativistic, collisionless (hereafter called "dark") matter at nonzero temperature together with a smaller amount of zero-temperature rotation-free dust. The two components are coupled only by gravitational forces.

Consider the case of a static background. For wavelengths much longer than the Jeans length scale, the mixture supports a weakly (Landau-) damped, weakly dispersive acoustic wave, if the concentration of ordinary matter is smaller. (Pure collisionless matter would not support such a wave.) The linear dispersion relation is

$$D(k,s) = k^2 - \alpha\frac{k^2}{s^2} - (1-\alpha)\left[1 - \frac{s}{k}\frac{\sqrt{\pi}}{\sqrt{2}}W\left[\frac{i}{\sqrt{2}}\frac{s}{k}\right]\right],$$

where W is the plasma dispersion function and $s = i\omega$. In order to take into account long-term effects of nonlinearities, a two-timing method is employed, which consists of defining new coordinates slow coordinates ξ, τ, η, expanding the functions of interest in powers of the amplitude parameter ϵ, and collecting the two leading orders of all equations. KP-I arises in second order as a condition on the first-order amplitude.

Solutions of the Korteweg-deVries (KdV) equation are also solutions of KP-I. However, KdV-solitons are not stable solutions of KP-I. KP-I has stable <u>lump solutions</u> which decay algebraically in both spatial directions and interact like solitons. **The density contrast at the center of a lump is negative.**

PREDICTIONS:

- <u>nonlinearities</u> effective over timescale $[4\pi G\rho_b(t_0)]^{-1/2}\epsilon^{-5/2}$
- <u>biasing</u>: density fluctuations of $O(\epsilon^{-2})$ larger than dark matter fluctuations.
- <u>peculiar velocities</u> should decrease with mass like $M^{-2/3}$
- <u>cosmic voids</u>: lumps are persistent solutions with <u>negative</u> density contrast of ordinary matter (small, positive density contrast of dark matter).

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