

## The measurement of velocity

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In the past forty years in many fields of physics, geophysics, and oceanography, signals, usually in the form of waves, have been observed using arrays of receivers or detectors. The practical problems of estimating the velocity and direction of one signal observed with such an array in the presence of noise which is not correlated between receivers have been solved by many people and the statistical theory has been covered by Hannan [3].

This thesis firstly extends this theory to the situation where the noise is correlated between receivers. It is shown that the problem can only be solved if the structure of the noise correlation is largely known beforehand and then the resulting estimators are related to the adaptive methods [5].

These estimators are then extended to the situation where more than one signal is present. This problem is complicated by the fact that usually in practice the signal spectra are not known. Due to this the maximum likelihood estimator is not very useful and other estimators were derived. These were analysed in terms of their asymptotic properties and by simulations. Also in this context several *ad hoc* estimators [2], [4], commonly used in practice were examined.

The above results apply to an array with an arbitrary geometry and usually involve considerable computations. If the design of the array is restricted it is sometimes possible to simplify processing considerably. A circular array first suggested by Aki [1] is examined in detail in this respect with a view to providing quick initial estimates of velocity.

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Finally an example of applying the methods to real seismic data is given.

### References

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