## THE BIOLOGICAL TREATMENT OF WASTEWATER: MATHEMATICAL MODELS

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## 1. Introduction

The activated sludge process is one of the major aerobic processes used in the biological treatment of wastewater. A significant drawback of this process is the production of excess sludge, the disposal of which can account for 50-60% of the running costs of a plant. Thus, there is a growing interest in methods that reduce the volume and mass of excess sludge produced as part of biological wastewater treatment processes. In practice a target value is often set for the sludge content inside the bioreactor. If the sludge content is higher than the target value, the process is stopped and the reactor is cleaned. This is undesirable as it increases running costs.

In Chapter 2 we investigate a simple model for the activated sludge process in which the influent contains a mixture of soluble and biodegradable particulate substrate. Within the bioreactor the biodegradable particulate substrate is hydrolysed to form soluble substrate. The soluble organics are used for energy and growth by the biomass. Biomass decay produces soluble substrate in addition to inert material. We use steadystate analysis to investigate how the amount of sludge formed depends upon the residence time and the use of a settling unit. We show that when the steady-state sludge content is plotted as a function of the residence time, there are five generic response diagrams, depending upon the value of the effective recycle parameter. Four of them are desirable because the sludge content is below the target value if the residence time is higher than some critical value that is not 'too large' in practice.

In Chapter 3 we investigate how the volume and mass of excess sludge produced by the activated sludge process can be reduced by coupling the bioreactor used in the process to a sludge disintegration unit.

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In Chapter 4 a seemingly minor modification is made to the model in Chapter 2. Instead of biomass decay producing soluble substrate and inert material, it produces slowly biodegradable substrate and nonbiodegradable particulates. We use steady-state analysis to investigate how the amount of sludge formed depends upon the residence time and the use of a settling unit. We show that when the steady-state sludge content is plotted as a function of the residence time, there are three generic response diagrams, depending upon the value of the effective recycle parameter. In particular, the seemingly minor change has had a pronounced effect on the behaviour of the model. Unlike the model investigated in Chapter 2, we find that in practice excessive sludge formation is inevitable. The results of the successive stages of this research have been published in [1-3].

In Chapter 5 we investigate how increasing the decay rate affected the behaviour of models one and two. We find that this is not an effective way to reduce sludge formation.

In the final chapter we summarise the results of the thesis and include recommendations for future research.

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