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ERRATUM

Negative energy standing wave instability in the presence of flow – ERRATUM

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The concept of negative energy (NE) waves is very useful in studying stability. For long time only the NE wave instability of propagating waves was studied. Some time ago, Ruderman (2018) (Paper I below) studied for the fist time this type of instability for standing waves. He used a model problem of stability of a tangential magnetohydrodynamic (MHD) discontinuity in an incompressible plasma. The main conclusion that he made is that the NE wave instability of standing waves occurs when the flow velocity exceeds a critical velocity. This condition is the same as the condition of the NE wave instability for propagating waves. While this conclusion is perfectly correct, the expression for the instability increment obtained by Paper I is wrong. We aim to correct the error made in that paper and derive the correct expression for the instability increment.

We start from a brief reminder of the problem set-up and its solution given in by Paper I. The equilibrium state is an MHD tangential discontinuity in an incompressible plasma. The equation of this discontinuity is $z = 0$ in Cartesian coordinates x, y, z. The plasma below the discontinuity is immovable and viscous, while the plasma above the discontinuity is moving with the velocity U in the positive x-direction. It was assumed that the Reynolds number is large, so dissipation in the viscous plasma only takes place in a narrow dissipative layer near the discontinuity. The perturbed discontinuity is defined by $z = \zeta(t, x)$. In the linear approximation the evolution of $\zeta(t, x)$ is described by the equation derived by Ruderman & Goossens (1995). It was assumed that the magnetic field lines are frozen in a dense plasma at $x = 0$ and $x = L$, so $\zeta(t, 0) = \zeta(t, L) = 0$. The wave propagating in the direction opposite to the direction of flow becomes an NE wave when $U > U_c$, where

$$U_c^2 = \frac{\rho_1 V_1^2 + \rho_2 V_2^2}{\rho_2} = \frac{\rho_1 V_{KH}^2}{\rho_1 + \rho_2}, \qquad V_{1,2} = \frac{B_1^2}{\mu_0 \rho_{1,2}}, \tag{1}$$

where ρ is the density, B the magnetic field, μ_0 magnetic permeability of free space, the indices 1 and 2 refer to the equilibrium quantities below and above the discontinuity and V_{KH}^2 is the Kelvin-Helmholtz threshold velocity.

The NE wave instability increment is proportional to the coefficient of kinematic viscosity ν. It was assumed in Paper I that the instability growth time is much higher than the oscillation period of a standing wave. In accordance with this the 'slow' time $T = \epsilon t$, $\epsilon \ll 1$, and the scaled coefficient of kinematic viscosity $\bar{\nu} = \epsilon^{-1}\nu$ were introduced. Then the solution to the problem was looked for in the form of expansion $\zeta = \zeta_1 + \epsilon\zeta_2 + \ldots$ In the first-order approximation the expression for ζ_1 was obtained. It reads

$$\zeta_1 = A(T)[\cos(\omega t - k_+ x) - \cos(\omega t - k_+ x)], \tag{2}$$

where $A(T)$ is a function to be determined and the frequency ω is given by equation (3.12) in Paper I. The expression for ω is incorrect. The correct expression is

$$\omega = \frac{kn\rho_2(U^2 - U_c^2)}{\sqrt{\rho_1\rho_2(V_{KH}^2 - U_c^2)}}, \quad k = \frac{\pi}{L}, \quad n = 1, 2, \ldots \tag{3}$$

The wavenumbers $k_\pm$ are defined by

$$k_\pm = \omega\frac{\rho_2 U \mp \sqrt{\rho_1\rho_2(V_{KH}^2 - U^2)}}{\rho_2(U^2 - U_c^2)}. \tag{4}$$

This quantities are related by $k_- - k_+ = 2kn$.

In the second-order approximation the equation for ζ_2 was derived (see equation (3.12) in Paper I. It is convenient to transform this equation to

$$(\rho_1 + \rho_2)\frac{\partial^2\zeta_2}{\partial t^2} + 2\rho_2 U\frac{\partial^2\zeta_2}{\partial t\partial x} + \rho_2(U^2 - U_c^2)\frac{\partial^2\zeta_2}{\partial x^2} = -\,\mathrm{i}\,\mathrm{e}^{\mathrm{i}\omega t}\left(a_+\,\mathrm{e}^{-\mathrm{i}k_+ x} + a_-\,\mathrm{e}^{-\mathrm{i}k_- x}\right) + \text{c.c.}, \tag{5}$$

where c.c. indicates complex conjugate and

$$a_\pm = k_\pm\sqrt{\rho_1\rho_2(V_{KH}^2 - U^2)}\frac{dA}{dT} \pm 2\bar{\nu}\omega\rho_1(k_\pm^2 + k_y^2)A. \tag{6}$$

We note that $\zeta_2 = 0$ at $x = 0, L$. The condition of existence of solution to equation (5) bounded with respect to time results in the equation determining $A(T)$. This equation was incorrectly derived in Paper I. This resulted in a wrong expression for the instability increment γ. We now derive the correct expression for this quantity. We look for the solution to equation (5) satisfying the zero boundary conditions in the form $\zeta_2 = \zeta_p + \zeta_h$, where ζ_p is a particular solution to equation (5) and ζ_h is the general solution to its homogeneous counterpart. It can be shown that the solution bounded with respect to time to the homogeneous counterpart satisfying the zero boundary conditions and arbitrary initial conditions always exists. We do not give this proof here. We look for ζ_p in the form $\zeta_p = f(x)\,\mathrm{e}^{\mathrm{i}\omega t} + \text{c.c.}$ Substituting this expression in equation (5) yields

$$\rho_2(U^2 - U_c^2)f'' + 2\,\mathrm{i}\rho_2\omega U f' - (\rho_1 + \rho_2)\omega^2 f = -\,\mathrm{i}\left(a_+\,\mathrm{e}^{-\mathrm{i}k_+ x} + a_-\,\mathrm{e}^{-\mathrm{i}k_- x}\right), \tag{7}$$

where the prime indicates the derivative with respect to x. We look for the solution to this equation in the form

$$f(x) = x\left(H_+\,\mathrm{e}^{-\mathrm{i}k_+ x} + H_-\,\mathrm{e}^{-\mathrm{i}k_- x}\right). \tag{8}$$

Substituting this expression in equation (7) and collecting terms proportional to e^{-ik_+x} and e^{-ik_-x} we obtain

$$H_\pm = \mp\frac{a_\pm}{2\omega}\sqrt{\rho_1\rho_2(V_{KH}^2 - U^2)}. \tag{9}$$

The function $f(x)$ must satisfy the conditions $f(0) = f(L) = 0$. Obviously $f(0) = 0$. Using equations (3), (4), (6), (9) and the relation $k_- - k_+ = 2kn$ we obtain from the condition $f(L) = 0$ is $H_+ + H_- = 0$. Using equations (6) and (9) we obtain from this relation

$$\sqrt{\rho_1\rho_2(V_{KH}^2 - U^2)}\frac{dA}{dT} = \frac{\bar{\nu}\rho_1\rho_2(U^2 - U_c^2)(k_+^2 + k_-^2)}{\sqrt{\rho_1\rho_2(V_{KH}^2 - U^2)}}A. \tag{10}$$

It follows from this equation that $A = A_0\,e^{\gamma T}$. Using equations (3) and (4) we obtain that the expression for γ is given by

$$\gamma = 2\bar{\nu}\frac{U^2 - U_c^2}{V_{KH}^2 - U^2}\left(k^2n^2\frac{\rho_2U^2 + \rho_1(V_{KH}^2 - U^2)}{\rho_1(V_{KH}^2 - U^2)} + k_y^2\right). \tag{11}$$

Supplementary data

There are no supplementary data and movies.

Declaration of interests

The authors report no conflict of interest.

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