Notation

Common notation	*Meaning*
$\operatorname{Re} z$, $\operatorname{Im} z$	real, imaginary parts of $z \in \mathbb{C}$
$\lfloor x \rfloor$, $\lceil x \rceil$	largest (smallest) integer $\leq x$ ($\geq x$)
$\langle \alpha, u \rangle$	Hermitian form
$(u \mid v) = u \cdot v$	inner-product
M^t	transpose of matrix M
$M^\dagger$	matrix-adjoint $= \overline{M}^t$
$\|S\|$	cardinality of a set, order of a group
$\overline{z}$	complex conjugation
$\gcd(a, b)$	greatest common divisor
$\mathcal{B}_n$	the braid group on n strands (1.1.9)
$\mathbb{C}$	the complex numbers
$\mathbb{C}^\times$	the multiplicative group of nonzero $z \in \mathbb{C}$
CFT	conformal field theory
δ_{ij}	Kronecker delta: 1 if $i = j$, otherwise 0
$\delta(x)$	Dirac delta distribution
$\eta(\tau)$	Dedekind eta function (2.2.6b)
$\mathbb{H}$	the upper half-plane (0.1.1)
$\overline{\mathbb{H}}$	the upper half-plane with cusps (0.1.3)
I_n	$n \times n$ identity matrix
j, $J = j - 744$	Hauptmoduls for $\mathrm{SL}_2(\mathbb{Z})$ (0.1.8)
Λ	Leech lattice (Section 1.2.1)
$\mathbb{M}$	Monster finite simple group
$\mathbb{N}$	the nonnegative integers $\{0, 1, 2, \ldots\}$
$\mathbb{P}^1(\mathbb{C})$	Riemann sphere $\mathbb{C} \cup \{\infty\}$
q	$e^{2\pi i \tau}$, $\tau \in \mathbb{H}$
$\mathbb{Q}$	the rational numbers
$\mathbb{R}$	the real numbers
RCFT	rational conformal field theory
Σ	Riemann surface, complex curve
$\mathrm{SL}_n(R)$	$n \times n$ $\det = 1$ matrices, entries in R
τ	point in $\mathbb{H}$
θ_3	Jacobi theta function (2.2.6a)
T_g	McKay–Thompson series (0.3.3)

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$V^\natural$	Moonshine module (Section 7.2.1)
VOA	vertex operator algebra (Definition 5.1.3)
ξ_n	root of unity $\exp(2\pi \mathrm{i}/n)$
$\mathbb{Z}$	the integers

Section 1.1

e	the identity in a group
$G \cong H$	groups G and H are isomorphic
$\mathbb{Z}_n$	ring and additive group $\mathbb{Z}/n\mathbb{Z}$
$\mathbb{F}_q$	finite field with q elements
$N \lhd G$	N normal subgroup of G
$H < G$	H subgroup of G
$\mathcal{F}_n$	the free group on n generators
$\langle g_1, \ldots, g_n \rangle$	the group generated by elements g_i
$\mathcal{D}_n$	dihedral group (1.1.1)
$N \times H$	direct product
$N \rtimes H$	semi-direct product
$\mathcal{S}_n$	symmetric group
$Z(G)$	centre of G
$\mathcal{A}_n$	alternating group
$M_{11}, \ldots, M_{24}$	Mathieu sporadics
$\mathrm{GL}_n(\mathbb{K})$	invertible matrices over field $\mathbb{K}$
K_g	conjugacy class $\{hgh^{-1}\}$
$\mathbb{C}G$	group algebra
$\mathcal{P}_n$	pure braid group
$\mathbb{C}[w, w^{-1}] = \mathbb{C}[w^{\pm}]$	Laurent polynomials
$[G, G]$	commutator subgroup $\langle ghg^{-1}h^{-1} \rangle$

Section 1.2

L	a lattice (Section 1.2.1)
L^*	the dual of a lattice (Section 1.2.1)
$I I_{m,n}$	indefinite even self-dual lattice
$\lvert L \rvert$	determinant of lattice
$L_1 \oplus L_2$	orthogonal direct sum of lattices
S^n	the n-sphere
C^∞	smooth; all partials are continuous
$C^\infty(U)$	C^∞-functions $f : U \to \mathbb{R}$
$T_p(M)$	tangent space at $p \in M$
TM	tangent bundle
$\mathrm{Vect}(M)$	vector fields
$T_p^*(M)$	differential 1-forms
T^*M	cotangent bundle
$\pi_1(M, v) = \pi_1(M)$	fundamental group

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$\mathbb{P}^n(\mathbb{R})$, $\mathbb{P}^n(\mathbb{C})$	projective n-space
$\mathfrak{C}_n$	configuration space (1.2.6)
Section 1.3	
$\mathcal{H}$	complex separable Hilbert space
$\ell^2(\infty)$	Hilbert space of square-summable sequences
$\mathcal{C}^\infty_{cs}(\mathbb{R}^n)$	smooth functions with compact support
$\mathcal{S}(\mathbb{R}^n)$	Schwartz space
$L^2(X)$	Hilbert space of square-integrable functions
$\mathrm{d}\mu(x)$	Lebesgue measure
T^*	adjoint of operator T
$\int_X \mathcal{H}(x)\mathrm{d}\mu(x)$	direct integral of Hilbert spaces
$\mathcal{L}(\mathcal{H})$	bounded operators on $\mathcal{H}$
S'	commutant of set S
$M_n(\mathbb{C})$	$n \times n$ matrices over $\mathbb{C}$
type I_n, II_1, II_∞, III_λ	families of factors
$M \rtimes G$	crossed-product (1.3.4)
Section 1.4	
$\mathfrak{g}$	a Lie algebra
$[xy]$	bracket (multiplication) in Lie algebra
$\mathfrak{Heis}$	Heisenberg algebra (1.4.3)
$\mathrm{SO}^+_{3,1}(\mathbb{R})$	Lorentz group
$\widetilde{G}$	universal cover of G
$\mathfrak{gl}_n$	Lie algebra of $n \times n$ matrices
$[\mathfrak{g}\mathfrak{h}]$	span $[xy]$, $x \in \mathfrak{g}$, $y \in \mathfrak{h}$
$Z(\mathfrak{g})$	centre of $\mathfrak{g}$
$\mathrm{ad}\, x$	adjoint operator $(\mathrm{ad}\, x)(y) = [xy]$
$\kappa(x\|y)$	Killing form on $\mathfrak{g}$
$\mathfrak{g}(A)$	Lie algebra associated with Cartan matrix A
A_r, B_r, C_r, D_r	Lie algebras $\mathfrak{sl}_{r+1}(\mathbb{C})$, $\mathfrak{so}_{2r+1}(\mathbb{C})$, $\mathfrak{sp}_{2r}(\mathbb{C})$, $\mathfrak{so}_{2r}(\mathbb{C})$
E_6, E_7, E_8, F_4, G_2	exceptional simple Lie algebras
$\mathfrak{Witt}$	Witt algebra (1.4.9)
$\mathrm{Diff}(M)$, $\mathrm{Diff}^+(M)$	(orientation-preserving) diffeomorphism group
Section 1.5	
$L(\lambda)$	irreducible module with highest weight λ
$P_+(\mathfrak{g})$	dominant integral weights
$M(\lambda)$	Verma module with highest weight λ
$\mathfrak{h}$	a Cartan subalgebra
$\alpha \in \Phi$	roots
$\mathfrak{g}_\alpha$	root-space (1.5.5b)
$(\alpha\|\beta)$	Killing form on $\mathfrak{h}^*$
r_α	Weyl reflection (1.5.5c)
W	Weyl group

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$\alpha_i \in \Delta$	simple roots in a base
ω_i	fundamental weights
$\beta, \mu \in \Omega(\rho)$	weights of representation ρ
V_β	weight-spaces (1.5.6b) in module V
$U(\mathfrak{g})$	universal enveloping algebra
$\mathrm{ch}_V(z)$	character (1.5.9a) of module V
ch_λ	character of $L(\lambda)$
ρ	representation; also, the Weyl vector $\sum_i \omega_i$
$\mathrm{ch}_\lambda^\gamma$	γ-twisted character
z, h	elements in $\mathfrak{h}$
$\gamma \in \mathrm{Aut}(\mathfrak{g})$	automorphisms of $\mathfrak{g}$
$\mathcal{B}(\mathcal{H})$	bounded operators with bounded inverse
$\widehat{G}$	unitary dual of Lie group G

Section 1.6

$\mathrm{Hom}(A, B)$	set of arrows = morphisms
Vect	category of vector spaces
Riem	category of Riemann surfaces
Braid	category of braids
a_{UVW}	associativity constraint
c_{UV}	commutativity constraint
Ribbon	category of ribbons
$\mathbf{Ribbon}_S$	category of ribbons labelled from S

Section 1.7

$\mathbb{K}, \mathbb{L}$	fields
$\mathbb{K}[\alpha_1, \ldots, \alpha_n]$	field of polynomials in (algebraic) α_i
$[\mathbb{L} : \mathbb{K}]$	degree of field extension $\mathbb{K} \subset \mathbb{L}$
$\mathrm{Gal}(\mathbb{L}/\mathbb{K})$	Galois group
$\mathbb{Q}[\xi_n]$	cyclotomic field

Section 2.1

$\mathcal{K}(S)$	field of meromorphic functions
$\mathfrak{p}(z)$	Weierstrass function (2.1.6a)
$\theta_{r,s}(\tau, z)$	theta functions (2.1.7a)
$\mathcal{H}^k(S), \mathcal{M}^k(S)$	holomorphic/meromorphic k-differentials
$\mathfrak{M}_{g,n}$	moduli space for genus g, n punctures
$\Gamma_{g,n}$	mapping class group for genus g, n punctures
$\mathrm{Jac}(\Sigma)$	Jacobian variety
$\overline{\mathfrak{M}_{g,n}}$	Deligne–Mumford compactification
$\widehat{\mathfrak{M}}_{g,n}$	enhanced moduli space
$\widehat{\Gamma}_{g,n}$	enhanced mapping class group

Section 2.2

$\zeta(s)$	Riemann zeta function (2.2.3c)
$\Gamma(N)$	the principal congruence subgroup (2.2.4a)

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$\Gamma_0(N)$	a congruence subgroup (2.2.4b)
Γ_θ	(2.2.5)
$\Theta_{\mathbf{t}+L}$	lattice theta function (2.2.11a), (2.3.7)
Section 2.3	
$\Gamma(s)$	Gamma function
$\theta_1, \theta_2, \theta_4$	Jacobi theta functions
Section 2.4	
$\widehat{\mathbb{Z}}_p$	p-adic integers
$\lim_{\leftarrow}$	projective limit
$\mathrm{Mp}_2(\mathbb{R})$	metaplectic group (2.4.9)
Section 2.5	
A-D-E	the A_n, D_n, E_6, E_7, E_8 meta-pattern
PF2,PF2$^-$	condition on graphs
Section 3.1	
$\mathfrak{Witt}$	Witt algebra (1.4.9)
ℓ_n	standard basis for $\mathfrak{Witt}$
$\mathfrak{Vir}$	Virasoro algebra (3.1.5)
L_n, C	standard basis of $\mathfrak{Vir}$
L_0	Hamilton operator, gives grading on $\mathfrak{Vir}$-modules
c, h	central charge, conformal weight
$M(c,h)$	Verma module
$V(c,h)$	irreducible module
$c_m, h_{m;rs}$	c, h for discrete series (3.1.6)
$\mathrm{ch}_{c,h}(\tau)$	the character of $V(c,h)$
$\mathrm{Diff}^+(S^1)$	diffeomorphism group of S^1
Section 3.2	
$\overline{\mathfrak{g}}$	finite-dimensional semi-simple Lie algebra
$\mathcal{L}_{poly}\overline{\mathfrak{g}}$	polynomial loop algebra $S^1 \to \overline{\mathfrak{g}}$
$\overline{\mathfrak{g}}^{(1)} = X_r{}^{(1)}$	nontwisted affine algebra
$a_i, a_i^\vee$	labels, co-labels (Figure 3.2)
$\mathfrak{h}, \overline{\mathfrak{h}}$	Cartan subalgebras of $\mathfrak{g}$ and $\overline{\mathfrak{g}}$
$\overline{\mathfrak{g}}^{(N)}$, $N > 1$	twisted affine algebra
$M(\lambda) = M(\overline{\lambda}, k, u)$	Verma module with highest weight λ
$L(\lambda)$	irreducible module with highest weight λ
k	level
δ	imaginary root
P_+^k	integrable level k highest weights (3.2.8)
χ_λ	character of $L(\lambda)$
c_λ, h_λ	central charge, conformal weight (3.2.9)
$\lambda, \mu \in \Omega(V)$	weights

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$h^\vee$	dual Coxeter number
KZ	Knizhnik–Zamolodchikov (Section 3.2.4)
$\mathcal{L}G$	loop group (Section 3.2.6)
Section 3.3	
$\mathfrak{g}^e$, $\mathfrak{h}^e$	$\mathfrak{g}$, $\mathfrak{h}$ extended by derivations
$\mathfrak{g}_\tau$	toroidal algebra associated with 2-cocycle τ
$\mathfrak{g}_{\Sigma,P}$	Krichever–Novikov algebra
Section 3.4	
χ_λ^α	α-twisted character
$\mathfrak{m}$	Monster Lie algebra (Sections 3.3.2, 7.2.2)
Section 4.1	
L	Lagrangian
H	Hamiltonian
p^i	momentum components
c	speed of light
$\mathcal{L}$	Lagrangian density
Section 4.2	
$\psi(\mathbf{x}, t)$	wave-function
$\hbar$	Planck's constant
$\mathcal{H}$	state-space (Hilbert space)
Ω	Fock space
$\widehat{a}, \widehat{a}^\dagger$	annihilation, creation operators
$\widehat{H}$	Hamiltonian operator
$\|0\rangle$	vacuum
$\|\star\rangle$	state
$\|\text{in}\rangle$, $\|\text{out}\rangle$	incoming, outgoing states
$\langle\text{out}\|\text{in}\rangle$	transition amplitude
$\varphi(x)$, $\phi(x)$, $\psi(x)$	quantum fields
P^μ	energy–momentum operators
QED	quantum electrodynamics
Section 4.3	
OPE	operator product expansion
$\mathfrak{B}^{(g,n)}$	space of chiral blocks
$\mathcal{V}$	chiral algebra (VOA)
$T(z)$	stress–energy tensor
WZW	Wess–Zumino–Witten model
$M \in \Phi(\mathcal{V})$	irreducible $\mathcal{V}$-module
$\chi_M(\tau)$	graded dimension (4.3.8a)
$\mathcal{Z}(\tau, \overline{\tau})$	1-loop partition function (4.3.8b)
$\mathcal{V}^h$	h-twisted sector

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$\mathcal{V}^G$	fixed-point subalgebra for group G
$\mathcal{Z}_{(g,h)}(\tau)$	(4.3.14b)
$\chi_{(h,\rho)}(\tau)$	(4.3.15c)

Section 4.4

C_m	disjoint union of m circles
$\mathbf{C}_{g,k}$	connected component in Segal's Hom(C_m, C_n)
$\mathbf{Vect}_f$	category of finite-dimensional spaces

Section 5.1

$W[z^{\pm 1}]$	Laurent polynomials in z, coefficients in W
$W[[z^{\pm 1}]]$	formal power series in z
$\delta(z)$	multiplicative Dirac delta (5.1.2)
$\mathrm{Res}_z(f)$	residue (5.1.3a)
$\mathcal{V}$	a VOA
$Y(a, z)$	vertex operator
$\mathcal{V}_n$	space of conformal weight n vectors
$u_{(n)}$	mode of $u \in \mathcal{V}$
ω	conformal vector
$\mathbf{1}$	vacuum vector in $\mathcal{V}$
$\chi_{\mathcal{V}}(\tau)$	graded dimension (5.1.10b)
$(\star\|\star)$	invariant bilinear form (5.1.11)

Section 5.2

$o(u)$	the 'zero-mode' $u_{(n-1)}$, for $u \in \mathcal{V}_n$
$\mathcal{PV}_n$	conformal primaries (5.2.3)
$\mathrm{Aut}(\mathcal{V})$	automorphism group of $\mathcal{V}$
$\mathcal{V}(\mathbb{C}^n)$	Heisenberg VOA
$\mathcal{V}(\mathfrak{g}, k)$	affine algebra VOA
$\mathcal{V}(L)$	lattice VOA
$\mathcal{V}^G$	G-fixed points in $\mathcal{V}$

Section 5.3

M	V-module
$Y_M(u, z)$	vertex operator for M
M_α	conformal weight α space
$M^\star$	contragredient (= dual) module
$\Phi(\mathcal{V})$	irreducible $\mathcal{V}$-modules
$M \boxtimes N$	fusion
$\mathcal{N}_{MN}^P$	fusion multiplicities (5.3.3)
$h(M)$	conformal weight of M
$M_{n\times n}$	the algebra of $n \times n$ matrices
$A(\mathcal{V})$	Zhu's algebra
$\chi_M(\tau, v)$	character of M (5.3.13)
$\chi_M^J(\tau, u, v)$	Jacobi character (5.3.14)

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$L[n]$	a second Virasoro action on $\mathcal{V}$
$\mathcal{V}[n]$	homogeneous spaces for $L[0]$
$\mathrm{Inn}(\mathcal{V})$	inner-automorphisms of $\mathcal{V}$
$\mathcal{Z}(M, h; \tau)$	h-twisted graded trace (5.3.23)
Section 5.4	
X	smooth complex variety
$\mathcal{MSV}_X$	chiral de Rham complex
$\mathcal{MSV}_X(X)$	global sections
$\mathcal{T}(X)$	Tamanoi's invariant
Section 6.1	
$\mathcal{N}_{ab}^{c}$	fusion multiplicities
$S = (S_{ab})$	modular matrix
$\mathcal{N}_{a_1,\dots,a_n}^{(g,m+n)\ b_1,\dots,b_m}$	Verlinde dimensions (6.1.2)
$T = (T_{ab})$	diagonal matrix in modular data
$\mathcal{Z} = (\mathcal{Z}_{ab})$	modular invariant
$\mathcal{Y}(w, z)$	intertwining operator
$\mathcal{V}\binom{c}{a\ b}$	space of intertwining operators of type $\binom{c}{a\ b}$
Section 6.2	
$\mathrm{Gr}(m, n)$	Grassmannian
$L(x)$	Roger's dilogarithm
RVOA	rational vertex operator algebra
$U_q(\mathfrak{g})$	quantum group
$R(q)$	family of solutions to (6.2.8)
$C_G(g)$	centraliser of g in G
$N \subset M$	subfactor
$[M : N]$	Jones index
${}_M X_N$	$M - N$ bimodule
Section 6.3	
F_n	Fermat curve $x^n + y^n + z^n = 0$
$\mathrm{Jac}(F_n)$	Jacobian of Fermat curve
$\Gamma_{\mathbb{Q}}$	absolute Galois group of $\mathbb{Q}$
$\overline{\mathbb{Q}}$	algebraic closure of $\mathbb{Q}$
χ^{cyclo}	cyclotomic character
$\widehat{G}$	profinite completion of G
Section 7.1	
$\mathbb{B}$	Baby Monster
Fi'_{24}	a Fischer group
$\mathbb{M} \wr \mathbb{Z}_2$	Bimonster
Γ_g	fixing group of T_g
$o(g)$	order of g

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$\Gamma_0(p)+$	(7.1.5)
$k\|N^\infty$	any prime dividing k divides N

Section 7.3

$A^{f\natural}$	vertex operator superalgebra for Co_1
$V\mathbb{B}^\natural$	Baby Monster Moonshine module
$N_{(g,h)}(\tau)$	Norton series
$\beta(g)$	Brauer character
$B(g,h;\tau)$	Modular Moonshine series
$\|\star\rangle\rangle$	boundary states in CFT

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