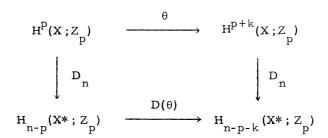
AN APPROXIMATION TO {X, Y}

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1. Notation and Terminology. All homology and cohomology groups are reduced. $\mbox{$\hat{\phi}$}(\hat{p})$ is the class of finite abelian groups whose orders are prime to p. $\mbox{$\hat{\phi}$}(\hat{p})$ as the class of abelian groups whose orders are products of primes less than a. If G is a finitely generated abelian group, G is the quotient of G by the subgroup of G made up of all elements whose orders are prime to p. If X and Y are finite C-W complexes and $\mbox{$\hat{\phi}$}$ a class of abelian groups, $\mbox{$\hat{\phi}$}$ stem $\mbox{$\hat{\phi}$}$ stem $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ is the quotient of $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ are finite C-W complexes and $\mbox{$\hat{\phi}$}$ a class of abelian groups, $\mbox{$\hat{\phi}$}$ stem $\mbox{$\hat{\phi}$}$ stem $\mbox{$\hat{\phi}$}$ or $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ or $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ are finite C-W complexes and $\mbox{$\hat{\phi}$}$ a class of abelian groups, $\mbox{$\hat{\phi}$}$ stem $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ are finite C-W complexes and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ are finite C-W complexes and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ are finite C-W complexes and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ are finite C-W complexes and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ are finite C-W complexes and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ are finite C-W complexes and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat{\phi}$}$ are finite C-W complexes and $\mbox{$\hat{\phi}$}$ and $\mbox{$\hat$

If θ is a stable cohomology operation of type (k, Z_p, Z_p) , $D(\theta)$ is the stable homology operation of type (k, Z_p, Z_p) which for any finite C-W complex X and n-dual X* of X makes the following diagram commutative:



where the vertical maps are the duality isomorphisms. Note that "n-dual" is used in the sense of [1]. D is well defined, and is an isomorphism from the group of stable cohomology operations of type (k, Z_p, Z_p) to the group of stable homology operations of type (k, Z_p, Z_p) (see [2]).

 δ is the Z_p cohomology Bockstein and β the Z_p homology Bockstein. Let $h:Z\to Z_p$ be the canonical projection; $h_{\#}\colon H_i(Y;Z)\to H_i(Y;Z_p) \quad \text{is the induced homomorphism and}$

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 $\begin{array}{l} h_{*}: \ H^{j}(X\,;\, H_{i}(Y\,;\, Z))_{p} \ \xrightarrow{} \ H^{j}(X\,;\, H_{i}(Y\,;\, Z_{p})) \ \ \text{is the homomorphism induced} \\ \text{by } \ h_{\#}\,. \end{array}$

2. Statement of Results.

THEOREM C. X and Y are finite C-W complexes, p a prime, p \neq 2, s = $\varphi(p)$ stem $\{X, Y\}$. Let a(q) = 2q(p-1)-1. Then the spectral sequence with

$$E_{1}^{\mathbf{r}, o} = \sum_{t} H^{t}(X; H_{t-r}(Y; Z))_{p}$$

$$E_{1}^{\mathbf{r}, q} = \sum_{t} H^{t}(X; Z_{p}) \otimes H_{t-a(q)-r}(Y; Z_{p}) \qquad q > 0$$

$$E_{1}^{\mathbf{r}, q} = 0 \qquad q < 0$$

$$d_{1}^{r, q} = H^{t}(X; Z_{p}) \otimes H_{t-a(q)-r}(Y; Z_{p})$$

$$((q+1)p^{1}\delta - q\delta p^{1}) \otimes 1 + \delta \otimes D(p^{1}) + (-1)^{t}p^{1} \otimes \beta$$

$$+ (-1)^{t} \otimes ((q+1)D(p^{1})\beta - q\beta D(p^{1}))$$

COROLLARY 1. Let $s = stem \{X, Y\}$.

$$\{X,Y\} \stackrel{\sim}{=} \sum_{t} H^{t}(X; H_{t}(Y;Z)) \mod \emptyset < \frac{s+4}{2}$$
.

COROLLARY 2. Let $s = stem \{X, Y\}$.

Suppose

$$A: H^{2i}(X; Z_p) = H_{2i}(Y; Z_p) = 0$$
 for all i;

or

$$B: H^{2i+1}(X; Z_p) = H_{2i+1}(Y; Z_p) = 0$$
 for all i;

 $\underline{\text{then if}} \quad p \, \geq \, \frac{1 + \sqrt{2s + 5}}{2} \ ,$

$$\{X,Y\}_p \stackrel{\sim}{=} \sum_t H^t(X;H_t(Y;Z))_p$$
.

If either A or B holds for all p such that

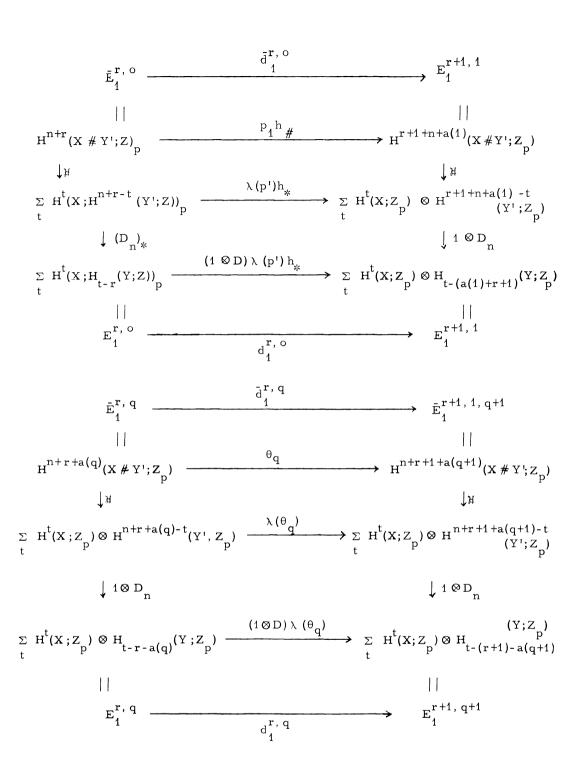
$$\frac{1+\sqrt{2s+5}}{2} \le p \le \frac{s+3}{2}$$
 then

$$\{X,Y\} \stackrel{\sim}{=} \sum_{t} H^{t}(X;H_{t}(Y;Z)) \mod c < \frac{1+\sqrt{2s+5}}{2}$$
.

COROLLARY 3. {CP(n), CP(m)} is isomorphic mod \diamondsuit < $\frac{1+\sqrt{4n+1}}{2}$ to the free abelian group on min (m, n) generators.

3. Proof of Theorem C. Y is a finite C-W complex so, for some n, it has an n-dual Y'. $\varphi(\hat{p})$ stem $\{X,Y\} = \varphi(\hat{p})$ stem $\{X \# Y', S^n\}$. Theorem B of [4] is a special case of Theorem C and is valid when the image space is a sphere. Thus we have a spectral sequence with $\bar{E}_1^{r,o} = H^{r+n}(X \# Y';Z)_p$; $\bar{E}_1^{r,q} = H^{r+n+a(q)}(X \# Y';Z_p)$, for q > o; $\bar{E}_1^{r,q} = 0$ for q < 0; $\bar{d}_1^{r,o} = p'h_{\#}$; $\bar{d}_1^{r,q} = (q+1)p'\delta - q\delta p'$ and this spectral sequence converges to $\{X \# Y', S^{r+n}\}_p$ when r > s - 2p(p-1) + 2.

Now we will describe an isomorphism from this spectral sequence to the one described in the statement of Theorem C. Let $\mbox{\ensuremath{\mathtt{H}}}$ be the appropriate Kunneth isomorphism, $\mbox{\ensuremath{\mathtt{\lambda}}}$ the diagonal map in the Steenrod Algebra and $\mbox{\ensuremath{\mathtt{\theta}}}$ the primary operation (q+1) p' $\mbox{\ensuremath{\mathtt{\delta}}}$ - $q\mbox{\ensuremath{\mathtt{\delta}}}$ p'. Then the following two diagrams are commutative and the vertical maps are isomorphisms.



So the spectral sequences \bar{E} and E both converge to $\{X \# Y', S^{r+n}\}_p$ when $r \ge s - 2p(p-1) + 2$. But $\{X \# Y', S^{r+n}\}_p \approx \{X, S^rY\}_p$ (cf. [1]) and the proof of Theorem C is complete.

In each of the three corollaries the hypothesis assures us that $E_1^{o,\,q}=0 \ \text{except when} \ q=0 \ \text{and that} \ d_1^{o,\,o}=0. \ \text{Consequently}$ $E_{\infty}^{o,\,o}=E_1^{o,\,o} \ \text{and} \ E_{\infty}^{o,\,q}=0 \ (q \ \ \ ^{\dagger} \ 0). \ \text{The conclusions then follow}$ easily.

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