HALOS AND DISK STABILITY

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ABSTRACT. The need for halos on stability grounds is not at all compelling. Compared to a bulge, a halo is not very efficient in stabilising a disk. Because the effect of a modest halo is small, it is difficult to infer its presence or absence from stability arguments.

1. INTRODUCTION

Historically the instabilities of disks have added much fuel to the argument for the existence of massive haloes around galaxies. We happen to live in that part of the Galaxy where the circular motion is much larger than the random motions of stars. Until fairly recently, we were ignorant about the velocity dispersion elsewhere in the Galaxy, and it was therefore quite reasonable to assume that the rest of the Galaxy, as well as other disk galaxies, would be mainly rotationally supported. Toomre's pioneering stability analysis (Toomre 1964) seemed to provide a reason for the small velocity dispersion: it was just enough to make the solar neighbourhood stable against axisymmetric instabilities. There was another, less worthy reason for wanting to think about cool disks: the dynamics could be worked out fairly simply if epicycles were to be good approximations to stellar orbits.

The first inkling that all was not well with this concept of a disk galaxy came with the advent of numerical simulations (Miller, Quirk, and Prendergast 1970; Hohl 1970). Disks with just enough velocity dispersion to make them stable against axisymmetric instabilities, evolved quite rapidly, chiefly through bar-making instabilities. These instabilities warmed up the disks to the extent that in the ensuing equilibrium the pressure from random motions became as important as rotation. The demonstration by Ostriker and Peebles (1973) that the random part of the kinetic energy should exceed the rotational part by about a factor of three in order to avoid the bar-making instabilities, seemed to rule out the idea that the Galaxy (where the local value of that energy ratio is two orders of magnitude less) could be a self-gravitating disk. One solution of the stablity problem is to assume that the disk is surrounded

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by a rigid halo. In view of the virial mass discrepancies in clusters of galaxies, the idea of unseen matter surrounding galaxies is very appealing. It becomes even more appealing if the halos are needed to keep the galactic disks cool.

In this review I will attempt to convince you that the stability problems are associated with the inner parts of disk galaxies. They can be overcome by hot centers or small bulges. Halos will do it as well, if they provide a significant part to the equilibrium force field in the central regions. A halo with a scale length larger than the disk, and a comparable mass within a Holmberg radius, will not contribute significantly to the stability.

2. NUMERICAL EXPERIMENTS WITH DISKS

The problem faced by numerical simulations is the lack of knowledge about the orbital eccentricities of the stars within the disk. There is only a lower bound for the radial velocity dispersion, which is needed to insure axisymmetric stability (Toomre 1964). The earlier experiments all started out with just enough random velocities to satisfy this criterion.

The actual value of the radial dispersion measured in the units of the minimum Toomre denoted by Q. One can write Q as

$$1/Q = 2\pi G\mu(r)*(0.5345)/(\kappa r e) \qquad (1)$$

if we express the radial velocity dispersion as (κre), where e is the mean eccentricity, κ the epicyclic frequency, and r the radius. In the denominator, μ is the surface density, and G the gravitational constant. The value of Q at the sun, which at first seemed to be tantalisingly close to 1.0, now appears to be in the range 1.2 - 2.0 (Toomre 1973). The corresponding eccentricity is around 0.16, which is a reasonably small number.

The Q = 1.0+ disks were quite lively in the non-axisymmetric sense, with the result that Q quickly rose to values in the range of 2 - 6, the highest values obtaining in the outer parts. Attempts to cool these hot disks were only moderately succesful (Hohl 1971). The value of Q could be reduced to about two, but then the bar reappeared and the subsequent stirring kept the value of Q in the range 2 - 3. Actually the measured velocity dispersions were annular averages, and as such they included any systematic motions due to the bar and spiral arms. Thus it is quite conceivable that the true dispersion may yet be a factor of 1.4 lower, implying final Q´s in the range 1.4 - 2.1, which are not that discrepant with the solar value.

The cooling experiments probably deserve to be re-examined, in view

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of the fact that the bar is a negative energy creature, and hence can be provoked by indiscriminate cooling.

3. HALOS AND BULGES

If we start from the premise that disks are rotationally supported, we can already see that the central parts will have to contain stars in fairly eccentric orbits just to satisfy the axisymmetric stability criterion. Equation (1) can be rearranged slightly,

$$1/Q = [2\pi G\mu(r)*r/v^2]*[0.535*\Omega^2/\kappa^2]/e \qquad (2)$$

by bringing in the circular velocity v, the azimuthal frequency Ω, and recalling that $v = \Omega*r$. The ratio of the squares of the frequencies is a slowly changing function, which goes from 0.25 at the center, to 0.5 at the maximum of the rotation curve, and approaches 1.0 at large radii. This variation is much slower than that of the left-most bracket, and from now on we will pay most of our attention to the latter.

The denominator is the square of the rotation curve, produced by the combination of surface density and radius appearing in the numerator. Figure 1 shows the two quantities for an exponential disk. The ordinate is ln(r), because the relation between these two functions is independent of the radial scale. The relation is also linear, and hence they are related by a convolution on the ln(r) scale.

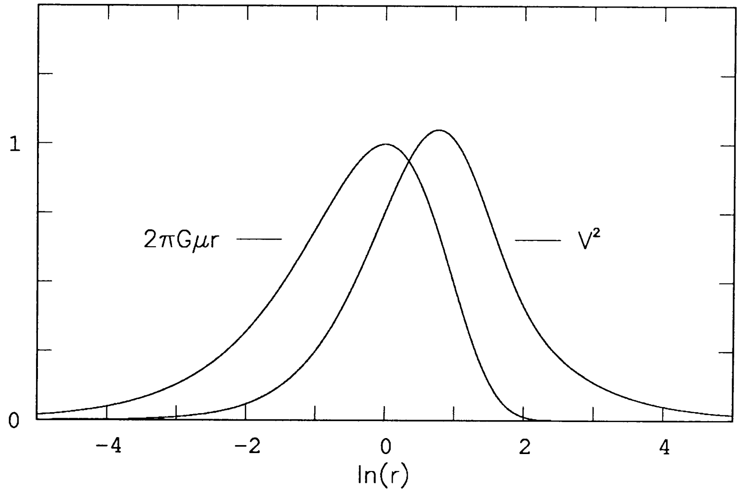

Figure 1. The radius-weighted surface density and the square of the circular velocity for an exponential disk of unit scale length.

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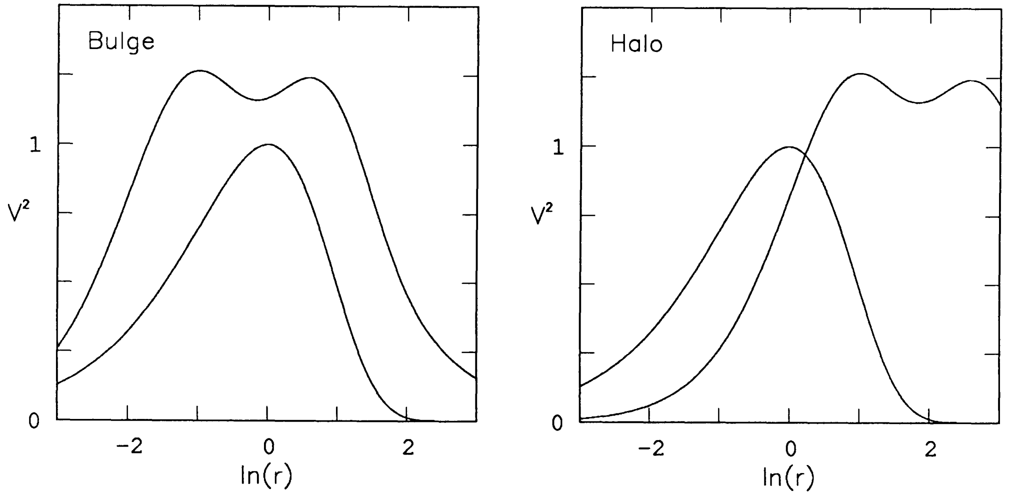
The similarity in shapes and the relative displacement of the two curves is typical, provided the surface densities are as wide and smooth as the exponential.

The reciprocal Q has another interpretation: it is the maximum local response of the disk at zero temporal frequency. At a fixed radius and eccentricity the response is proportional to the surface density, and inversely proportional to the square of κ, which is a measure of the restoring force against radial displacement. Thus the surface density in the numerator promotes instability, while the frequency dependent terms in the denominator are stabilising. The two curves in Figure 1 play similar roles in the stability of the disk.

Axisymmetric stability requires a minimum eccentricity of 0.16 at the peak of the rotation curve in Figure 1. We can see from the ratio of the two curves that this value is more than adequate for stability at larger radii, but as we move towards the center the eccentricity has to increase to quite large values. While the quantitative aspect of this analysis becomes suspect for large eccentricities, the conclusion that the center must be hot remains.

If we want to keep the center of the disk cool, we must raise the rotational velocities there. This can be accomplished by introducing external masses in the form of a rigid bulge or halo. For example, by adding another rotation curve of the same shape, but shifted two units to the left, we obtain the curves shown in Figure 2a.



Figures 2a and 2b. The effect of adding a bulge (left) or a halo component (right) to the rotation curve shown in Figure 1.

The additional component could be produced by an exponential disk whose mass and scale length is reduced by exp(2) = 7.39, which we would

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call a bulge. This is to be contrasted with what we obtain if we add a similar velocity component, but now shifted to the right by two units. Figure 2b shows that such a halo-like component helps in the outer parts where help is not needed, but does nothing to the center. It is similar to the halos inferred from flat rotation curves. The mass of this component is 7.39 times that of the disk, or 55 times that of the bulge. It would have to be much heavier still to have any effect on the central part of the disk.

Figure 3 shows the contribution of the bulge to the rotation curve in the more conventional way. The halo version has the same shape, but a different radial scaling.

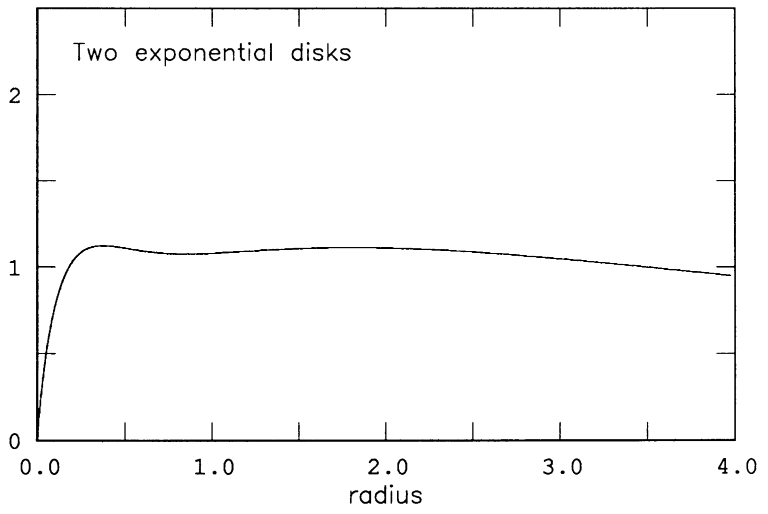

Figure 3. The rotation curve produced by a sum of two exponential disks with scale lengths and masses in the ratio of exp(-2) to one.

Real bulges and halos are not flat. However the same modifications to the rotation curves can be produced by spherical distributions, with very similar projected surface densities, provided the amplitudes are increased by about 50% . The mass ratio of halo to bulge remains the same.

The importance of Figure 1 lies in the relative displacement of the two curves: it shows that the outer regions benefit from the stabilising effect of the equilibrium mass distribution, to the detriment of the center.

The above discussion of stability is based on the axisymmetric criterion Q. There does not appear to be any simple stability criterion for the non-axisymmetric case, where new complications such as shear and resonances arise. Nevertheless the simple notion of keeping the disk stiff by means of external masses, is still correct. The stabilising effect of freezing a fraction of the disk, with the frozen part acting the role of the halo, has been illustrated by Toomre (1981). If the eccentricity in equation (2) is replaced by the reciprocal of the azimuthal wave number, the right side becomes proportional 1/X. X is an important parameter for the swing amplifier: large values of X will turn it off (Toomre 1981), which is in line with the above reasoning.

Of the many numerical experiments examining the stabilising effect of external masses, the recent study of the stability of the BSS model of our Galaxy (Bahcall, Schmidt, and Soneira 1982) by Sellwood (1985) is perhaps the most relevant. Much of the credit for the stability of this model goes to the combination of velocity dispersion and a central bulge component. The effect of the halo is minimal: removing it makes the disk lop-sided. The possibility exists that the lop-sidedness may have resulted from keeping the central component fixed during the simulation. The BSS model undoubtedly has some slack, which could be utilised to make it even more robust.

4. DISKS WITHOUT BULGES

The above discussion suggests that pure disk systems need massive halos if they are to remain cool. Lacking these, they must be quite hot, at least in the central parts. Surprisingly little work has been done on disks with hot centers, the notable exception being that by Athanassoula and Sellwood (1986). One reason for this is the lack of nice and simple equilibrium models, for there is little observational or theoretic basis for choosing the velocity structure.

The lack of specifications for a real disk galaxy is a big handicap in a talk of this nature. My own choice would be a stable differentially rotating disk which became cooler with increasing radius, and resembled the solar vicinity close to the peak of the rotation curve. It would be nice if I could produce such an example, for then I could quantify such phrases as "hot center", and provide the incentive for an observer to go out and prove me wrong.

The best I can offer at the moment is a model I will call JW , an abbreviation for "Jacobi waterbag". My purpose is to give you an idea of what the notion of a hot center may imply. JW is the coolest member of a family obtained by adapting Lynden-Bell´s violent relaxation argument to disks (Lynden-Bell 1969). The phase space distribution is a function of Jacobi´s integral, and has a constant value. It is truncated so that the disk is finite, and the mean and circular velocities become equal at the edge. The claim of stability rests on a 500-body numerical simulation.

Figure 4a summarises the kinematical data, and 4b shows the run of Q. The stellar disk rotates uniformly in the mean. The central velocity dispersion is about 66% of the maximum circular velocity. The velocity dispersion of the planetary nebulae and OH masers at the center of our Galaxy is 150 km/s, which is a similar fraction of the circular velocity

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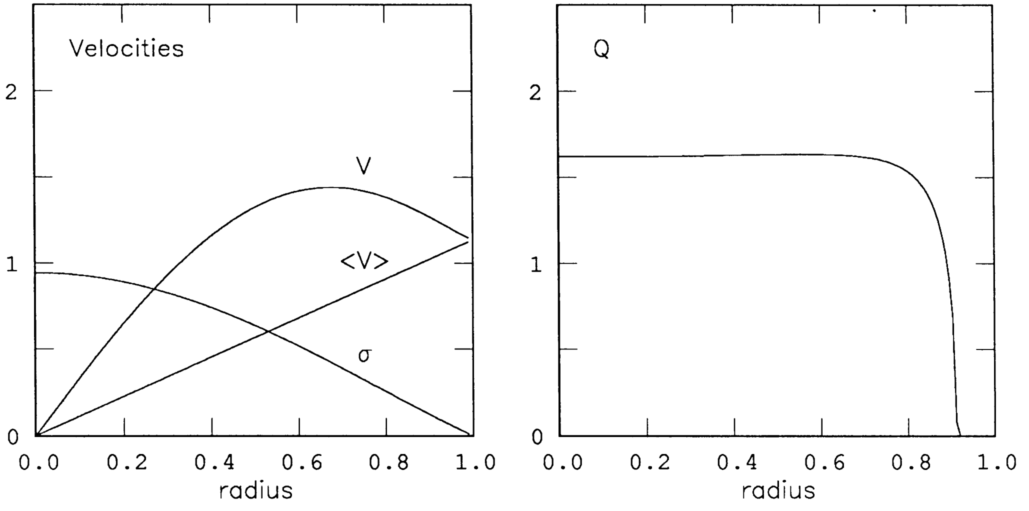
of 220 km/s at the sun. The ratio of random to rotational kinetic energy is 3.6 - 20% higher than that implied by the Ostriker-Peebles criterion. The vanishing of Q reflects the vanishing of the epicyclic frequency, caused by a rapid decrease of the surface density near the edge. This is a blemish, for it means that the outermost circular orbits are unstable.



Figures 4a and 4b. Left: the rotation curve (V), mean velocity (<V>), and velocity dispersion (σ) of the JW model. Right: the axisymmetric stability parameter Q.

The JW model could conceivably be the first step in building a more realistic disk galaxy. Starting from somewhere close to the peak of the rotation curve, one would try adding a tapered ring, which should remain cool enough to match the solar neighbourhood.

The JW model is one example of a stable disk. Undoubtedly there are many more, and nicer examples.

5. CONCLUSIONS

The need for halos on stability grounds is <u>not</u> very compelling. The fact that we have not come up with a decent halo-less Galactic model is <u>not</u> an argument for the existence of halos. Small halos cannot be ruled out, and might even be useful. At the other extreme, very massive halos can be ruled out since they would make the disk dynamically dead. That would eliminate such things as spiral structure, tidally induced spiral structure, bars, lop-sidedness, and dynamical evolution.

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DISCUSSION

DEKEL: How would you explain long-lived warps without massive halos?

KALNAJS: I probably would not. But the sort of halo needed to disturb the outer fringes of a galaxy will not contribute significantly to the force field in the central parts, and therefore is only of marginal interest to disk stability.

J. BAHCALL: Could you comment on Sellwood's numerical simulations of the Galaxy and how they affected your discussions?

KALNAJS: Sorry. I failed to mention the work by Sellwood, who simulated the BSS model for the Galaxy and got reasonable results. The model certainly doesn't have any of the obvious instabilities. At the moment, the rotation curve near the center is a bit high, but a little bit of tweaking will probably result in a fairly respectable dynamical model for our Galaxy. It is certainly a promising start.

OSTRIKER: You gave our T/W criterion much more credit than we ever intended. I have been surprised by how well this rule of thumb has worked for a variety of different stellar systems.

Most models of the Galaxy have moderate halos, i.e., within several scale lengths a third to a half of the mass is in a hot component. This amount is quite adequate for removing gross instabilities, as Sellwood and others have found. You can throw away that component if you don't want the disks to be stable, and in any case you probably don't want to stabilize galaxies against all m = 2 modes. We suggested that galaxies <u>are</u> in fact moderately unstable and <u>do</u> produce bars. M31 has a bar, our Galaxy has a bar, and it may be that most ordinary spirals have a significant m = 2 component because they are not entirely stable.

KALNAJS: I agree with everything you say. At the time you did your work, chaps like me were thinking in terms of epicycles and rotationally supported disks, and your T/W criterion had a shattering impact on us, from which we have just about recovered.

TREMAINE: As a result of a great deal of work in recent years, we now have for a lot of disk galaxies good rotation curves, photometry, and a few measurements of velocity dispersions. It wasn't completely clear from your talk whether or not we should worry about the stability of those disks.

KALNAJS: They certainly should be stable on a timescale of a rotation or two. Halos can help, but are probably not necessary. You can have them, but not too much.

TREMAINE: The rotation curves say that you <u>have</u> to have them.

KALNAJS: Yes, but the mass required to explain the large rotation velocities in the outer regions of galaxies does not help with the

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stability of disks. It is the inner parts you have to stabilize some way if you want to have a cool disk. Otherwise the disk has to be very hot in the center.

FABER: I'd like to ask about hot disks. The fact that edge-on spirals have constant linear scale height as a function of distance from the center must imply that at least the z dispersion σ_z is increasing toward the center. Can you give us a rough estimate of σ_z and the scale height in the inner part of the disk of our Galaxy? If someone measures a velocity dispersion of 100 km s^{-1} for stars in Baade's window, does that necessarily mean that they are measuring the spheroid or could they be measuring a hot disk in the middle of the Galaxy?

KALNAJS: There is very little I can tell you about z motions; in this business one ignores them. But if you do have a large velocity dispersion in the plane, the disk will not remain thin. It will buckle to such a height that the z velocity dispersion becomes 1/3 of that in the plane. So that implies a minimum thickness for the disk. Presumably you could have any additional amount of σ_z for other reasons.

SANDERS: Sellwood and I did a simulation of a model galaxy which exactly resembled NGC 3198, discussed earlier by Sancisi. The model had a maximum disk, the right amount of halo to give a flat rotation curve and an initial Q of 1.5. It was violently unstable; it made a strong bar within a few dynamical times. This is a practically bulgeless galaxy, so if you want to stabilize the disk, it must have a hot center.

GUNN: I would like to describe a set of simulations that Jens Villumsen has just finished. This is a galaxy which at the end of the calculations is cooked up to resemble our own, except that it doesn't have a bulge. The simulations are designed to look at the effect of the infall of cold matter. Apart from the fact that they are three-dimensional, they are very much like the models that Carlberg and Sellwood have made. The models evolve with essentially a constant value of Q over the whole disk, and are <u>never</u> violently unstable. At the end, the models have essentially a constant value of Q and a constant scale height over the whole disk. The disk is thus quite hot at the center, with the velocity dispersion rising like 1/r. I think that is more or less in accord with the observations that Ken Freeman showed us. And certainly this is what one expects σ_z to do, since the scale heights of galactic disks are constant with radius.

VAN DER KRUIT: I would like to draw attention to a measurement by Freeman and myself of the stellar velocity dispersion in NGC 7184. The dispersions are determined from the measured asymmetric drift between about one and two scale lengths from the center (the system has only a minor bulge), and are increasing toward the center. The extrapolated central value is over 100 km s^{-1}, hot enough to be significant in contributing towards stability (van der Kruit and Freeman, <u>Ap.J.</u>, in press).

KORMENDY: I have measured stellar velocity dispersions in the disks of two S0 galaxies, NGC 1553 and NGC 936 (Ap.J., 286, 116 and 132, 1984). Both disks are very hot in their central parts. In fact, in NGC 1553, the line-of-sight velocity dispersion rises from < 100 km s^{-1} at $r > 4$ kpc to 179 ± 4 km s^{-1} at $r = 1$ kpc in the disk. This is the sort of behavior inferred from the constant scale heights, but the magnitude of the dispersion at small radii is remarkably large for a disk. The density increases inward, too, and so $Q \simeq 2.5$ varies little with radius. Unlike NGC 1553, NGC 936 is barred. It has an even larger value of Q, rising from ~4 at r = 8 kpc to ~7 at r = 2.5 kpc. Not surprisingly, neither galaxy has any spiral structure. Since the inner parts of both disks locally satisfy the Ostriker-Peebles criterion, it is tempting to think that the dispersion is large enough to contribute to global stability. However, the fact that NGC 936 is barred shows that even if this argument is correct, it is not safe to use the present value of the velocity dispersion to decide whether a disk was stable in the past. The disk could have been cold, made a bar, and then used it to heat itself up after the fact.

SANCISI: There is least one case in which we can rule out the possibility that the halo is dominant in the inner parts of a galaxy. This is NGC 2403, a normal spiral which I showed in my talk. If we try to make a model with an insignificant disk, we find that $M/L \simeq 0.3$ for the disk. The maximum disk gives $M/L = 1$. Now unless you are willing to believe that $M/L < 1$, you are forced to conclude that inside 2 or 3 scale lengths the dark matter cannot be dominant.

RUBIN: There is not very much difference in what we are all saying. A variety of approaches suggest $M_{dark}/M_{lum} \sim 1$ within the optical parts of galaxies. No one claims that there are enormous amounts of dark matter at these small radii. I have suggested that dark matter contributes, but not that it is dominant there.

KALNAJS: I am not questioning that dark halos exist. I am only saying that for stability arguments they are not necessary.

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