Proceedings of the Edinburgh Mathematical Society (1984) 27, 103–104 ©

BOOK REVIEWS

James, G. and Kerber, A., *The representation theory of the symmetric group* (Encyclopedia of Mathematics and its Applications, Vol. 16, Addison-Wesley, Reading, Mass., 1981), pp. 510, £37.80.

In many respects this book does for the symmetric group and its representations what the Encyclopaedia Britannica does for knowledge in general. It has however the drawback of any encyclopedia that it requires some stamina to read from page one to the end and, owing partly to the customary logical progression of a mathematical text and partly to the abounding complexity of the combinatorial concepts and their specialised notations, it is unlike most encyclopedias in being a text into which one cannot easily dip. Nevertheless, having made this mild criticism, one can only admire the comprehensiveness of the text and the care which has been exercised to cover all aspects of the symmetric group.

After a brief foreword by P. M. Cohn there is a short, largely historical, introduction by G. de B. Robinson. In the subsequent preface and elsewhere friends and former students of D. E. Rutherford will be pleased to see that fitting recognition is given to his book *Substitutional Analysis*. Finally after six pages listing symbols the main work begins. Granted the intrinsic difficulty of the topic the prerequisites for reading the text are not excessive, a moderate knowledge of the ordinary and modular representations of a finite group being all that is essentially required. The text is fairly lucid but the pace is relentless and the specialised notation comes thick and fast.

Without going to inordinate length it is impossible to give more than an indication of the wealth of material in the text; nevertheless it is probably worthwhile to give some hints as to the gems to be found. The first chapter begins with the difinitions of the symmetric and alternating groups leading to the construction of Young subgroups, tableaux and the so-called horizontal and vertical sub-group. The second chapter begins with intertwining numbers for representations and there is determined theoretically a complete set of ordinary irreducible representations of the symmetric group, proving, in the passing, that any field is a splitting field for the symmetric group, Young's Rules appear in several versions and from Nakayama's notion of a hook a precise formula for the degrees of the ordinary irreducible representations is derived and the recursion formula of Murnaghan and Nakayama is obtained. The symmetric group is shown to be characterised by its character table and the rule of Little and Richardson is proved. In the case of any finite group there is a well-known formula for the primitive central idempotents of the group algebra over the complex numbers: for the symmetric group there is even a formula for the primitive non-central idempotents. Chapter four discusses wreath products, a formula for the conjugacy classes of the wreath product of a finite group with a symmetric group is obtained and a complete set of irreducible representations of such groups over algebraically closed fields is derived. A long chapter is devoted to the applications of representations theory to combinatorics. Pólya's theory of enumeration is discussed and used to count the number of non-isomorphic groups with certain prescribed properties. Early, overlooked, results of J. H. Redfield are given due place. At this stage in the text, Schur functions are mentioned but only to be passed over; since they have played a key role in previous accounts it is surprising that an account as encyclopedic as this does not afford them more than a fleeting comment. Various operations, such as plethysms, on the representations of the symmetric group are discussed at some length. The next two chapters are concerned with the representations themselves, both modularly and with regard to an aribitrary field. The modular treatment goes as far as proving Nakayama's conjecture that two ordinary irreducible representations belong to the same p-block if an only if their p-cores are equal. There is also a long analysis of actual techniques for finding decomposition numbers which, for workers in the area, will be very informative. The approach independent of the field characteristic leads naturally to Specht modules and to the role of the hook lengths. The final chapter discusses representations of the general linear groups and explores the similarity of the behaviour of the Weyl modules for these groups with the behaviour of the Specht modules for the symmetric groups. The text concludes with over 100 pages of tabulated data, 10 pages of notes and over 700 references.

It is to be hoped that the above will give some impression of the contents of this remarkable book which will be indispensable to workers in the area, to whom it offers a virtually all-embracing, concise, sometimes too concise, account of a difficult and entrancing topic. For other algebraists it will be extremely useful as a reference although the warning must be reiterated that it is a text into which one can easily dip. From the mathematical community thanks are due to the authors for a very valuable synthesis of a somewhat indigestible topic and to the Publisher for a very high standard of exacting typography.

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