The impression was gained that this extensive revision of a well established book was carried out by an author still very much absorbed in his subject.

J. FULTON

KOLMOGOROV, A. N., AND FOMIN, S. V., *Elements of the Theory of Functions and Functional Analysis*, vol. i : *Metric and Normed Spaces*, translated by LEO F. BORON (Graylock Press, Rochester, N.Y., 1957), 129 pp., 32s.

This is an excellent introduction to the ideas and methods of functional analysis. The original version was based on lectures given by the author in Moscow, and the translator has produced a very readable English text. Although nothing beyond elementary analysis is presupposed, the principal abstract concepts are illustrated by an ample variety of examples, and the theory is elegantly applied to problems of considerable practical interest. The approach is essentially " classical " : all limits are sequential, and no use is made of any principle of transfinite induction. This is a weakness from the point of view of the serious student of modern analysis, but it makes substantial parts of the subject easily accessible to many others.

There are four chapters. The first is concerned with some elementary facts of abstract set theory (with no mention of the Axiom of Choice). Chapter II, the longest, is devoted to metric spaces, with a brief reference to general topological spaces. The main themes here are completeness and compactness. The fixed-point theorem for contraction mappings in a complete metric space is thoroughly exploited in a discussion of iterative methods for solving equations, including differential and integral equations, for which several existence theorems are proved. The fundamental properties of compact sets in metric spaces are established (the word " compact " is used in the relative sense, which is now unusual), and there is a careful account of the relations between compactness and equicontinuity in spaces of continuous functions on compacta. The chapter ends with a useful discussion of rectifiable curves. Questions of category are not considered, and there is no mention of local compactness.

Chapter III, on normed vector spaces, is concerned largely with continuous linear functionals. Conjugate spaces are determined in some simple cases, and their are brief discussions of reflexivity and weak convergence. A proof of the Hahn-Banach extension theorem, and an account of weak* compactness, are restricted to separable spaces (of necessity, since transfinite induction is not available). Continuous linear operators on Banach spaces are also considered, and there is a proof of Banach's theorem on the continuity of inverse operators. The Banach-Steinhaus theorem is, surprisingly, omitted. An addendum to this chapter gives some of the main facts about " generalized functions " (distributions).

In Chapter IV, the idea of the spectrum of a linear operator is introduced, and the main theorems on completely continuous operators are proved, giving the Fredholm theory of integral equations.

There is a good index, and the translator has added a bibliography. The authors promise further volumes, in which they propose to discuss, among other things, Lebesgue integration and Hilbert-space theory.

J. D. WESTON

PONTRYAGIN, L. S., *Foundations of Combinatorial Topology* (Graylock Press, Rochester, N.Y., 1952), 99 pp., $3.00.

This is a translation of the first (1947) Russian edition of a book which the author says is " essentially a semester course in combinatorial topology which I have given several times at Moscow National University ".

There are three chapters. In Chapter I the Betti (homology) groups are defined for polyhedra ; in Chapter II the topological invariance of the groups is proved ;

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in Chapter III continuous mappings of polyhedra into polyhedra are studied by means of the induced homomorphisms of the homology groups.

The presentation is rigorous and straightforward, but very concise. No previous knowledge of topology as such is necessarily required. The book's one defect, acknowledged by the author in his preface, is the complete omission of illustrative examples; despite this, the reviewer warmly recommends it to a beginning research student in topology who wishes to obtain a basic source of information on homology groups in combinatorial topology. W. H. COCKCROFT

ALEKSANDROV, P. S., *Combinatorial Topology*, Vols. i and ii (Graylock Press, Rochester, N.Y., 1956, 57), 225 pp., 244 pp., $11.45.

These two volumes are a translation of Parts I, II and III of the first (1947) Russian edition of the author's *Kombinatornaya Topologiya*. A few English references and additions to the bibliography have been made by the translator. The work as a whole is intended for the student beginning topology, and is intended to give such a student a thorough grounding in the "classical" uses, in the subject, of homology (and cohomology) theory. It is written with great care; and, particularly with the beginner in mind, has a wealth of figures, illustrative examples, illuminating remarks and calculations.

Parts I and II constitute the first volume. Elementary properties of topological spaces are first well surveyed. There follows a proof of the Jordan curve theorem and of the fundamental theorem of the topology of surfaces. The volume closes with a full treatment of geometric complexes, a proof of Sperner's lemma with its corollaries, and an introduction to dimension theory.

The second volume contains Part III of the book. Here for the first time the homology and cohomology groups are introduced. They are defined first for locally finite abstract complexes and then for compacta. Various proofs of invariance are given. For polyhedra, for example, a direct proof (in the simplicial manner, by means of subdivisions) is given, in addition to the proof as a deduction from that for compacta. A variety of coefficient groups is employed throughout. The volume closes with a chapter on relative cycles and homology, and applications thereof to pseudomanifolds with boundary and to homology dimension theory.

As the author admits, the book does not begin to exhaust even the basic branches of modern combinatorial topology. It is nevertheless a substantial text, offering the beginner an excellent insight into the subject. W. H. COCKCROFT

GODEMENT, ROGER, *Topologie Algébrique et Théorie des Faisceaux*, Vol. i, Actualités scientifiques et industrielles 1252, Publications de l'Institut de Mathématique de l'Université de Strasbourg XIII (Hermann, Paris), 283 pp., 3600 francs.

This is the first book to appear on the general theory of sheaves and satisfies a long-felt want. The subject is developed from a general point of view, and contains no account of the applications to algebraic geometry and topology, although particular examples from these fields are used continually to illustrate the general theory. The opening chapters develop the required basis of homological algebra, semi-simplicial complexes and spectral sequences. The cohomology theory is defined by using a flabby resolution of the sheaf, a method which applies to an arbitrary topological space. The isomorphism with the Čech groups is shown when there is a paracompact family of supports. A second volume will deal with relations between sheaves and singular homology, duality in manifolds, fibre spaces, and cohomology operations. D. J. SIMMS