discouraged research student, who will take heart from the story of how the mathematical world was made to sit up and take notice by the young van der Waerden, Mann, and Linnik.

It should be recorded that, since Chapter II was first written in 1945, it has been partly superseded by the work of van der Corput, Martin Kneser, and others.

A. M. MACBEATH

APOSTOL, T. M., *Mathematical Analysis* (Addison-Wesley, Reading, Mass., 1957), 553 pp., 76s.

This book aims to fulfil the long-felt want of a textbook which will deal rigorously with the part of the subject now known as "advanced calculus". The author proves very carefully, with a proper statement of conditions, theorems like Green's theorem, which are unsatisfactorily dealt with in most textbooks.

Inevitably, the book suffers slightly from the complication which so rigorous treatment must at first involve. Some of this is a result of the intrinsic difficulty of the material, but there are places at which simplification would be possible. For instance, the statement of the Mean Value Theorem of the Differential Calculus is needlessly complicated by allowing the function to have, at the endpoints, two jump discontinuities which cancel one another out.

The following features are particularly welcome:

(i) use of vector notation and the treatment of functions mapping one Euclidean space into another,

(ii) the use of set theory and simple topological terms and ideas,

(iii) a satisfactory treatment of Gauss's, Stokes's and Green's theorems.

The book also includes chapters on Riemann-Stieltjes integration, on Fourier analysis, and on Cauchy's theorem and calculus of residues. It seems a pity that the Stieltjes integral $\int f(x)dg(x)$ is treated without first dealing with the special case $\int f(x)dx$. One also regrets the absence of a definition of real numbers, either using Dedekind section or Cauchy sequences. And one would have liked to see a treatment from first principles of the exponential and trigonometric functions.

There would appear to be a misprint in the statement of the inverse function theorem on p. 144. Surely condition (ii) ought to be not $X = f^{-1}(Y)$, but $X \subset f^{-1}(Y)$. Incidentally the proof given of the inverse function theorem is unusual and rather interesting.

By and large, the book succeeds in its aim. Teachers of analysis should not be without it, though for students it is at times a little severe. A. M. MACBEATH

SPRINGER, G., *Introduction to Riemann Surfaces* (Addison-Wesley, Reading, Mass., 1958), pp. viii+305, 76s.

This is a modern presentation of the classical theory of Riemann surfaces. The author assumes that his reader has a knowledge of elementary complex variable theory and a little algebra and real variable theory, but gives a sufficient introduction to topology and Hilbert space for his purpose. The material is clearly and carefully explained and frequently illustrated by figures. The book is not intended to be an account of modern work in the subject but would be a useful introductory text for advanced undergraduate reading. E. M. WRIGHT

HAYMAN, W. K., *Multivalent Functions* (Cambridge University Press, 1958), 168 pp., 27s. 6d.

The object of this tract is to study the growth of univalent and multivalent functions $f(z)$ which are regular in the unit circle, and, in particular, to obtain bounds for the absolute values and coefficients of such functions.



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In Chapter 1 univalent functions are considered and various classical inequalities are obtained. The class $\mathfrak{S}$ of univalent functions $f(z)$ with Taylor expansions

$$f(z) = z + a_2 z^2 + a_3 z^3 + \dots \qquad (\,|\,z\,| < 1)$$

is studied, and it is shown that $|\,a_2\,| \leqslant 2$, not only for $f(z) \in \mathfrak{S}$, but also for a rather wider class of functions. It is also shown that the Bieberbach conjecture $|\,a_n\,| \leqslant n$ holds for certain subclasses of $\mathfrak{S}$ such as Rogosinski's typically real functions.

In Chapter 2 the Cartwright-Spencer theorem on areally mean $p$-valent functions is proved. A regular function $f$ is said to be areally mean $p$-valent in $|\,z\,| < 1$ if the average number of roots of the equation $f(z) = w$ is not greater than $p$, as $w$ ranges over any disc $|\,w\,| < R$, and the theorem states that

$$\sup_{|z|=r} |\,f(z)\,| < A(p)\mu_p(1-r)^{-2p} \qquad (0 < r < 1),$$

where $\mu_p$ is max $|\,a_q\,|$ for $1 \leqslant q \leqslant p$. This is used to obtain upper bounds for $|\,a_n\,|$ in Chapter 3.

Chapter 4 is devoted to Steiner and circular (Pólya) symmetrisation and their applications to functions regular in the unit circle.

In Chapter 5 circumferentially mean $p$-valent functions, which form a subclass of the class of areally mean $p$-valent functions, are studied. Most of the theorems are due to the author himself, and they include the theorem which states, in the particular case when $p = 1$, that, for a fixed function $f \in \mathfrak{S}$, $|\,a_n\,| \leqslant n$ for all $n > n_0(f)$; thus, for a fixed function, Bieberbach's conjecture holds for all sufficiently large $n$.

The final chapter gives an account of the deep and difficult theory of K. Löwner, from which it is deduced, in particuar, that $|\,a_3\,| \leqslant 3$.

The author's style is succinct and clear and the book is beautifully printed. Most of the material contained in it has not appeared in book form before and some of it is quite new. The tract forms a most valuable addition to the library of any mathematician interested in the theory of functions of a complex variable.

R. A. RANKIN

GRENANDER, U., AND SZEGÖ, G., *Toeplitz Forms and their Applications* (University of California Press, 1958), 246 pp., \$6.00.

It should perhaps be remarked at the outset, for those to whom Toeplitz matrices mean the matrices of regular summability methods, that this book is concerned with quadratic forms and specialisations of them. The finite Hermitian forms $T_n = \Sigma\, c_{\nu-\mu} u_\mu \bar{u}_\nu (\mu, \nu = 0, 1, \dots, n)$, $c_{-\nu} = \bar{c}_\nu$, are called the Toeplitz forms associated with a given function when the $c_n$ are either the coefficients in a power series expansion of a harmonic function, or the complex Fourier coefficients of a real-valued function Lebesgue-integrable over $[-\pi, \pi]$, or the Fourier-Stieltjes coefficients of a distribution function. Toeplitz's original L-forms were associated with Laurent series, and various writers have established connections between these L-forms and the work of Carathéodory on the Fourier coefficients of a harmonic function. Szegö has done much work on Toeplitz forms associated with a Lebesgue-integrable function, in particular on the distribution of their eigenvalues, and some of this work is incorporated into the text. In the place of the Toeplitz matrix $(c_{\nu,\mu}) = (c_{\nu-\mu})$ there is an analogue for functions, using a kernel $K(s, t) = K(s-t)$.

The preliminary chapter provides, with the minimum of proofs, a useful source of reference material for the later chapters. Chapters 2 and 3 deal with the algebraic and limit properties of orthogonal polynomials, and chapters 4, 5, 6 with the trigonometric moment problem, eigenvalues of Toeplitz forms, and generalisations