INSTRUMENTS AND METHODS

THE DIFFICULTIES OF MEASURING THE WATER SATURATION AND POROSITY OF SNOW

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ABSTRACT. Liquid saturation and porosity control most of the important material properties of wet snow, hence accurate measurements of these two parameters is of the utmost importance for both field research and glaciological applications. For example, the movement of liquid water through snow is highly sensitive to the volume of water present and accurate measurements of the water saturation are necessary in order to infer the temporal and spatial variations in the flow field. Nevertheless, most of the instruments in use are not capable of making accurate determinations of saturation.

An error analysis shows that only direct measurements of the liquid volume can provide accurate values of water saturation, hence the melting calorimeter is inherently inaccurate. While centrifuges extract some of the liquid for direct measurement, there is always some residual liquid left depending on the grain size and structural parameters of the ice matrix. Therefore, some uncertainty exists over the interpretation of the data obtained from centrifuges. High-frequency capacitance probes can be used either *in situ* or on the surface and are very sensitive to the volume of liquid present. Capacitance probes are by far the best of the available devices. Remote-sensing techniques, like the active microwave system, require more development for use in operational forecasting schemes and as research tools.

Résumé. Les difficultés de la mesure de la teneur en eau et de la porosité de la neige. La teneur en liquide et la porosité commandent la plupart des principales propriétés physiques de la neige mouillée; il en résulte que des mesures précises de ces deux paramètres sont de la plus haute importance pour la recherche sur le terrain comme pour les applications glaciologiques. Par exemple, le mouvement de l'eau liquide à travers la neige est extrêmement sensible au volume d'eau présent et des mesures précises de la teneur en eau sont nécessaires pour en déduire les variations dans le temps et dans l'espace du flux d'eau. Cependant, la plupart des appareils de mesure utilisés ne sont pas capables de donner des mesures précises de la teneur en eau.

Une discussion des erreurs montre que seules des mesures directes de volume liquide peuvent donner des valeurs précises de la teneur en eau, puisque le calorimètre de fusion est congénitalement imprécis. Lorsqu'on extrait l'eau liquide par centrifugation pour une mesure directe, il reste toujours une certaine quantité de liquide dans la neige variable selon la dimension des grains et les paramètres structurels de la matrice de glace. Par conséquent, il y a toujours une certaine incertitude sur l'interprétation des résultats obtenus par centrifugation. Les sondes à capacité à haute fréquence peuvent être utilisées soit in situ ou en surface et sont très sensibles au volume d'eau liquide présent. Les sondes à capacité sont, de loin, les meilleures parmi les appareils disponibles. Les techniques de télédétection, comme les systèmes à micro-ondes actives, demandent à être perfectionnées avant d'être utilisables en prévision opérationnelle et comme outils de recherche.

ZUSAMMENFASSUNG. Die Schwierigkeiten bei der Messung der Wassersättigung und der Schneeporosität. Flüssigkeitsgehalt und Porosität sind ausschlaggebend für die meisten wichtigen Materialeigenschaften nassen Schnees; genaue Messungen dieser beiden Parameter sind daher äusserst wichtig für die glaziologische Forschung wie für deren Anwendung. So ist zum Beispiel die Bewegung flüssigen Wassers durch Schnee stark abhängig vom vorhandenen Wasservolumen und genaue Messungen der Wassersättigung sind notwendig, um auf die zeitlichen und räumlichen Schwankungen des Fliessfeldes schliessen zu können. Trotzdem lassen sich mit den meisten gebräuchlichen Instrumenten keine genauen Bestimmungen des Sättigungsgrades vornehmen.

Eine Fehleranalyse zeigt, dass nur direkte Messungen des Flüssigkeitsvolumens genaue Werte der Wassersättigung liefern; das Schmelz-Kalorimeter ist deshalb von Hause aus ungenau. Wenn Zentrifugen auch einen Teil der Flüssigkeit zur direkten Messung verfügbar machen, so bleibt doch stets je nach Korngrösse und Struktur des Eisgefüges ein gewisser Rest zurück. Deshalb herrscht bei der Interpretation von Zentrifugendaten eine gewisse Unsicherheit. Hochfrequente Kapizitanzsonden können entweder *in situ* oder an der Oberfläche verwendet werden; sie besitzen hohe Empfindlichkeit für vorhandene Wassermengen. Kapazitanzsonden sind bei weitem die besten verfügbaren Geräte. Fernerkundungsverfahren, etwa mit aktiven Mikrowellensystemen, bedürfen noch der Fortentwicklung zur Nutzung für operationelle Vorhersagen und als Forschungsmittel. SYMBOLS

 c_0 mass of solution introduced

- E(x) relative error in parameter x
 - F liquid-water content, liquid volume divided by total volume
 - g acceleration due to gravity
 - g' acceleration due to centrifuging
 - k intrinsic permeability of snow
 - L equivalent length of centrifuged sample
 - L' length of centrifuged sample
 - m_0 molal concentration of sodium hydroxide
 - $m_{\rm s}$ mass of snow sample
 - r mean radius of centrifuge
 - $S_{\rm m}$ mobile water, $S_{\rm w}-S_{\rm wi}$
 - S_{w} water saturation, liquid volume divided by pore volume
- S_{wi} irreducible water saturation

 S^* effective water saturation, $(S_w - S_{wi})/(1 - S_{wi})$

- t equivalent time for a sample drained by gravity
- t' time of centrifuging
- T temperature
- *u* flux of water, flow through a unit area per unit time
- $v_{a}, v_{i}, v_{s}, v_{w}$ volumes of air, ice, sample, and liquid
 - $\alpha 5.47 \times 10^{6} \text{ m}^{-1} \text{ s}^{-1}$
 - β molal temperature depression constant of the dissolved substance
 - ρ_i, ρ_s, ρ_w densities of ice, snow, and water
 - ϕ porosity, pore volume divided by total volume
 - ω angular velocity of centrifuge

I. INTRODUCTION

There are many reasons for wanting to know the liquid-water saturation and porosity of snow. Every investigation of the snow cover uses information such as the snow density, state of metamorphism, liquid-water storage capacity, and/or liquid-water transmission rate. Furthermore, virtually all of the important material properties of snow are related to its density and history of liquid saturation. Two of the most important pieces of information about snow, either on the scale of kilometers or millimeters, are water saturation and porosity. These two pieces of information are necessary for even a crude approximation of such varied properties as snow strength and water flow rates. Accordingly, much attention has been given to developing devices for determining the liquid-water content and density of snow. While some of these devices work rather well (see Table I), others can be shown to be unacceptable conceptually. Because of the widespread use of devices which fall into the latter category, all of the popular devices are analyzed here in order to identify their limitations.

Table I. Typical calculation errors when $E(S_{wi}) = E(v_s) = E(m_s) = 0$

Method	Measured values	$E(\pmb{\phi})$	$E(S_{\mathbf{w}})$	E(u)
i	$v_{\mathbf{a}}, v_{\mathbf{s}}, m_{\mathbf{s}}$	$10.2E(v_{a})$	52.4 $E(v_{a})$	$376E(v_{a})$
ii	v_i, v_s, m_s	$E(v_i)$	$5. \mathbf{I} E(v_i)$	$36.6E(v_i)$
iii	$v_{\rm W}, v_{\rm S}, m_{\rm S}$	0.16 $E(v_{\rm w})$	$0.84E(v_w)$	$6E(v_{w})$
iv	$v_{\rm w}, v_{\rm i}, v_{\rm s}$	$E(v_i)$	$E(v_{\mathbf{w}}) + E(v_{\mathbf{i}})$	$13.6E(v_i) + 5.6E(v_w)$

Snow hydrologists have been concerned principally with the "free-water content F" of snow, i.e. the fraction of the total volume occupied by the liquid phase. For most purposes it is more meaningful to separate the free-water content into its two component parameters, the liquid-water saturation S_w and porosity ϕ . These three are related by

$$F = S_{\mathbf{w}}\phi. \tag{1}$$

191

Since it is necessary to determine both S_w and ϕ for virtually every wet-snow problem, F can be calculated from known values of S_w and ϕ if desired.

Neglecting the weight of the air phase,

$$\rho_{\rm s} = \rho_{\rm i}(\mathbf{I} - \phi) + \rho_{\rm w}\phi S_{\rm w},\tag{2}$$

relates the snow density ρ_s to the ice mass per unit sample volume plus the liquid mass per unit sample volume. Thus the snow density can also be calculated directly from known values of S_w and ϕ . The advantage of using S_w instead of F can best be illustrated by the flux-concentration relationship for water (Colbeck and Davidson, 1973),

$$u = \alpha k S^{\star 3}, \tag{3}$$

where

4

$$S^{\star} = (S_{\mathbf{w}} - S_{\mathbf{w}\mathbf{i}})/(\mathbf{1} - S_{\mathbf{w}\mathbf{i}}), \qquad (4)$$

 S_{wi} is the irreducible water content which cannot be removed by gravity drainage, α is a constant, the intrinsic permeability k is a function of only the porous matrix, and S^* is a function of only the mobile liquid fraction in the pore volume.

2. Calculation errors for S_w and ϕ

There are four fundamental equations; two describe the mass and volume balances and two are definitions (neglecting the air mass):

$$v_{\rm s} = v_{\rm i} + v_{\rm w} + v_{\rm a},\tag{5}$$

$$m_{\rm s} = \rho_{\rm w} v_{\rm w} + \rho_{\rm i} v_{\rm i}, \tag{6}$$

$$S_{\mathbf{w}} \equiv v_{\mathbf{w}}/(v_{\mathbf{a}} + v_{\mathbf{w}}),\tag{7}$$

$$\phi \equiv (v_{\mathbf{a}} + v_{\mathbf{w}})/v_{\mathbf{s}}.\tag{8}$$

These four equations contain seven variables hence three pieces of information must be supplied. Usually the sample mass and volume are determined independently, in which case either the air, water, or ice volume must be measured directly. If the sample mass or volume is not determined independently, then two other quantities would have to be measured. The errors inherent in four of the ten possible combinations of the volume and mass variables are analyzed here. First, the three possible cases which include v_s and m_s are considered. Then, the optimum combination for *in situ* sampling— v_s , v_1 and v_w —is analyzed:

(i) Measure v_{a} directly

This procedure would be attractive for a destructive sampling technique because phase changes during sample preparation would not matter except that the solubility of air in water would have to be considered. Unfortunately, in order to calculate S_w and ϕ , it is necessary to divide by the difference between two large numbers, a procedure which inevitably leads to large errors. Using the definition of relative error,

$$E(x) \equiv \left| \frac{\mathrm{d}x}{x} \right| \,, \tag{9}$$

and assuming no error in the measurements of m_s and v_s ,

$$E(\phi) = \frac{v_{\mathbf{a}}\rho_{\mathbf{w}}}{(\rho_{\mathbf{w}} - \rho_{\mathbf{i}})(v_{\mathbf{a}} + v_{\mathbf{w}})} E(v_{\mathbf{a}}), \tag{10}$$

and

$$E(S_{\mathbf{w}}) = \frac{v_{\mathbf{a}}}{\rho_{\mathbf{w}} - \rho_{\mathbf{i}}} \left(\frac{\rho_{\mathbf{i}}}{v_{\mathbf{w}}} - \frac{\rho_{\mathbf{w}}}{v_{\mathbf{a}} + v_{\mathbf{w}}} \right) E(v_{\mathbf{a}}). \tag{11}$$

For a typical case where $v_i = 500 \times 10^{-6} \text{ m}^3$, $v_w = 75 \times 10^{-6} \text{ m}^3$, $m_s = 0.533 \text{ kg}$, $v_s = 10^{-3} \text{ m}^3$ and $v_a = 425 \times 10^{-6} \text{ m}^3$,

$$E(\phi) = 10.2E(v_{\rm a}),$$
 (12)

and

$$E(S_{\rm w}) = 52.4E(v_{\rm a}). \tag{13}$$

These large values render this approach useless unless extremely precise measurements of v_a are possible. It is important to note that $E(S_w)$ increases rapidly as S_w approaches S_{wi} .

(ii) Measure v_i directly

While this method is widely used because of its simplicity, large errors are associated with calculating S_w from the measured quantities. Again assuming no error in the measurements of m_s and v_s ,

$$E(\phi) = \frac{v_{\mathrm{i}}}{v_{\mathrm{s}} - v_{\mathrm{i}}} E(v_{\mathrm{i}}), \qquad (14)$$

and

$$E(S_{\mathbf{w}}) = \left(\frac{v_{\mathbf{i}}}{v_{\mathbf{s}} - v_{\mathbf{i}}} - \frac{\rho_{\mathbf{i}}v_{\mathbf{i}}}{m_{\mathbf{s}} - v_{\mathbf{i}}\rho_{\mathbf{i}}}\right) E(v_{\mathbf{i}}).$$
(15)

Note that the liquid saturation only enters this equation through the denominator, $m_s - v_i \rho_i$. As the liquid saturation decreases, $m_s - v_i \rho_i$ decreases and a large error occurs in the calculation.

For the sample case,

$$E(\phi) = E(v_{\rm i}),\tag{16}$$

and

$$E(S_{\mathbf{w}}) = 5.\mathbf{I}E(v_{\mathbf{i}}),\tag{17}$$

but $E(S_w)$ is much higher at lower values of saturation. As shown on Figure 1, $E(S_w)/E(v_i)$ approaches negative infinity as S_w approaches zero.

While this method may be useful for calculating the snow porosity, it will inevitably lead to large errors in the calculated value of water saturation unless very precise measurements of v_i are possible. As shown later, if this method were used to calculate or infer variations in the flow field of liquid water, a great deal of uncertainty would be associated with the results.

(iii) Measure v_w directly

This is the principle of many common devices and, since the error analysis shows that this method is most likely to produce consistent and accurate results, it is analyzed in more detail. We find

$$E(\phi) \leq \frac{v_1}{v_s\phi} E(v_s) + \frac{m_s}{\rho_1 v_s\phi} E(m_s) + \frac{\rho_w v_w}{\rho_1 v_s\phi} E(v_w), \tag{18}$$

192



Fig. 1. The ratios of error in calculating S_w to the error in measuring v_1 and v_w are shown for methods (ii) and (iii) respectively (assuming $E(m_8) = E(v_8) = 0$, $v_8 = 10^{-3} m^3$ and $v_1 = 0.5 \times 10^{-3} m^3$). For method (ii) the ratio is very large for all values of S_w and approaches negative infinity as S_w vanishes. For method (iii), however, the ratio reaches an upper limit of unity as S_w vanishes and is rather insensitive to the value of S_w over the common range of saturations.

and

$$E(S_{\mathbf{w}}) \leqslant \left(\phi^{-1} - \frac{m_{\mathbf{s}}}{\rho_{\mathbf{i}} v_{\mathbf{s}} \phi}\right) E(v_{\mathbf{w}}) + \phi^{-1} E(v_{\mathbf{s}}) + \left(\frac{m_{\mathbf{s}}}{\rho_{\mathbf{i}} v_{\mathbf{s}} \phi}\right) E(m_{\mathbf{s}}), \tag{19}$$

where the inequality sign is used to show that we only know the upper limit of the error. Unlike the two previous methods, as the volume of water approaches zero, the error in S_w approaches a small upper limit instead of becoming infinite (see Fig. 1). For the sample case,

$$E(\phi) \leq 0.164 E(v_{\rm w}) + 1.0 E(v_{\rm s}) + 1.16 E(m_{\rm s}), \tag{20}$$

and

$$E(S_{\rm w}) \leq 0.84 E(v_{\rm w}) + 2.0 E(v_{\rm s}) + 1.16 E(m_{\rm s}).$$
⁽²¹⁾

From these considerations it is clear that the direct measurement of the liquid is inherently a more accurate method of determining both S_w and ϕ (see Table I). The use of methods (i) or (ii) could only be justified if it could be shown that the error in measuring the volume of water directly was much greater than the error in measuring the volumes of ice or air. However, as shown later, the dielectric constant is very sensitive to the liquid volume, hence an accurate determination of S_w is possible.

For method (iii) the errors in calculating ϕ or S_w are more sensitive to the errors of measurement of m_s and v_s than of v_w . As shown by Equations (18) and (19), $E(\phi)$ and $E(S_w)$ increase inversely with the quantity $\rho_1 v_s \phi$. For a given situation, the errors in S_w and ϕ decrease as sample size increases, thus demonstrating the importance of avoiding small samples.

(iv) Measure v_w , v_i and v_s directly

The use of a snow sampling kit to measure m_s and v_s is inherently undesirable because it is frequently necessary to use two samples to get one calculated value of S_w or ϕ . Direct measurements of v_w , v_i and v_s are most desirable. In principle these measurements are possible, for example, by an electromagnetic instrument which senses the volumes of solid and liquid in a volume of sample which is predetermined by the nature of the instrument. The errors in calculating S_w and ϕ are given by

$$E(\phi) \leqslant \frac{v_{\mathbf{i}}}{v_{\mathbf{s}} - v_{\mathbf{i}}} \left(E(v_{\mathbf{i}}) + E(v_{\mathbf{s}}) \right), \tag{22}$$

and

$$E(S_{\mathbf{w}}) \leqslant E(v_{\mathbf{w}}) + \frac{v_{\mathbf{s}}}{v_{\mathbf{s}} - v_{\mathbf{i}}} E(v_{\mathbf{s}}) + \frac{v_{\mathbf{i}}}{v_{\mathbf{s}} - v_{\mathbf{i}}} E(v_{\mathbf{i}}).$$
(23)

For the sample case these errors are

$$E(\phi) \leqslant E(v_1) + E(v_s), \tag{24}$$

and

$$E(S_{\mathbf{w}}) \leqslant E(v_{\mathbf{w}}) + 2E(v_{\mathbf{s}}) + E(v_{\mathbf{i}}).$$
⁽²⁵⁾

If the predetermined volume being sensed v_s is known accurately, this method should work well for a remote-sensing system which can accurately distinguish between the liquid and solid phases.

3. CALCULATION ERRORS FOR u

The liquid-water content is most often measured to infer or calculate variations in the flow field of water. Because of the sensitivity of water flux to water saturation as shown by

$$u = \alpha k S^{\star 3}, \qquad (3)$$

any error in the determination of S^* will be magnified in the calculation of u. The related error in flux is bounded by

$$E(u) \leq 3E(S^{\star}) + E(k). \tag{26}$$

Accordingly, errors inherent in the device used to determine S_w must be avoided as much as possible. The error in calculating flux could be significantly reduced by taking a large number of samples in order to determine S_w more accurately. This procedure is difficult to follow in practice, however, since destructive sampling procedures are inherently undesirable and *in situ* instrumentation is rarely used in multiple arrays at each level in a snow-pack. The question of lateral variability must also be addressed if multiple samples are to be taken from a single snow layer.

At worst E(u) increases directly as E(k) but, since direct measurements of intrinsic permeability are rarely made, k must be calculated from determinations of ϕ and the average grain size (e.g. Shimizu, 1970). From data given by Kuroiwa (1968), it can be shown that k increases as exp (15.9 ϕ), which is a typical permeability-porosity relationship for a porous medium. Neglecting any error in the determination of the average grain size, the relative error in permeability is highly sensitive to $E(\phi)$ as shown by

$$E(k) = 15.9\phi E(\phi).$$
 (27)

Again, the need to determine ϕ accurately is apparent.

From Equation (4),

$$E(S^{\star}) \leqslant E(S_{\mathbf{m}}) + \frac{S_{\mathbf{w}i}}{\mathbf{I} - S_{\mathbf{w}i}} E(S_{\mathbf{w}i}), \qquad (28)$$

where the "mobile-water saturation" is given by

$$S_{\mathbf{m}} = S_{\mathbf{w}} - S_{\mathbf{w}i}.\tag{29}$$

As a consequence, $E(S^*)$ is most sensitive to the measurement of S_m since $S_{wi}/(I-S_{wi})$ is a small number. When the error in S^* is expressed in this way, the advantages of using a system which measures only the mobile water are apparent. This result suggests that for the purpose of calculating the flux of water, a controlled capillary withdrawal of the mobile liquid might be better than measurement of the total liquid saturation. For the determination of the material properties, however, the total liquid saturation is required.

 $E(S^{\star})$ can also be expressed as

$$E(S^{\star}) \leqslant \frac{S_{\mathbf{w}}}{S_{\mathbf{w}} - S_{\mathbf{w}\mathbf{i}}} E(S_{\mathbf{w}}) + \frac{(\mathbf{I} - S_{\mathbf{w}}) S_{\mathbf{w}\mathbf{i}}}{(\mathbf{I} - S_{\mathbf{w}\mathbf{i}})(S_{\mathbf{w}} - S_{\mathbf{w}\mathbf{i}})} E(S_{\mathbf{w}\mathbf{i}}), \tag{30}$$

which shows that $E(S^*)$ becomes very large as S_w approaches S_{wi} . Apparently, the error in calculating flux from measurements of S_w becomes very large as flux becomes vanishingly small.

From direct determinations of S_w , S_{wi} and ϕ , the error in calculating the flux of water is bounded by

$$E(u) \leqslant 15.9\phi E(\phi) + \frac{3S_{\mathsf{w}}}{S_{\mathsf{w}} - S_{\mathsf{w}i}} E(S_{\mathsf{w}}) + \frac{3S_{\mathsf{w}i}(1 - S_{\mathsf{w}})}{(1 - S_{\mathsf{w}i})(S_{\mathsf{w}} - S_{\mathsf{w}i})} E(S_{\mathsf{w}i}).$$
(31)

The upper bound of E(u) is approximated at large fluxes by

$$E(u) \leq 5E(\phi) + 6E(S_{\mathrm{w}}) + 2.5E(S_{\mathrm{wi}}), \qquad (32)$$

and the error increases with decreasing flow rates. Again, the need to determine the liquid content accurately is apparent. Likewise, the need to make an accurate porosity determination is shown.

If method (ii) is used to determine the liquid saturation, even in the absence of errors in the measurements of m_s , v_s or S_{w1} , for large flow rates

$$E(u) = 36.6E(v_{\rm i}),\tag{33}$$

and E(u) increases rapidly as u decreases.

This large error in the calculated value of flux precludes the possibility of getting a meaningful estimate of the flow field from measurements of the ice volume and explains why reproducible results have not been generally obtained with a melting calorimeter.

If method (iii) is used and the volume of liquid is sensed directly, at large flow rates

$$E(u) \leq 6.03 E(v_{\rm w}) + 19.2 E(v_{\rm s}) + 15.7 E(m_{\rm s}).$$
(34)

Even in the absence of error in measuring v_s , m_s or S_{w1} , the error in calculating the flux of water from the measured volume of water is six times greater than the error in the measurement itself. Assuming equal errors in the measurements of v_w and v_1 , the advantage of working directly with the volume of water is apparent. It is important, however, that the volume of ice is generally about six times greater than the volume of liquid hence its error of measurement might be somewhat lower. These errors must be considered, for example, in choosing between the melting and freezing calorimeters.

If method (iv) is used with a remote-sensing system, the error in calculating flux would be

$$E(u) \leqslant 13.6E(v_{\rm i}) + 5.63E(v_{\rm w}) + 19.2E(v_{\rm s}). \tag{35}$$

Clearly this method comparcs favorably with any of the others although large errors could still occur in the calculation unless very accurate measurements are possible.

4. DESTRUCTIVE SAMPLING TECHNIQUES

With this background, a systematic discussion of the various types of "saturometers" can be made. Excluding those which cannot give accurate results *a priori*, we consider two types those which must take a sample from the snow cover and those which can get the necessary information *in situ*.

4a. Freezing calorimeter

Radok and others (1961) discussed the inherent advantage of the freezing calorimeter. They point out that in calculating "snow quality" from measurements with a melting calorimeter, it is necessary to take the small difference between large numbers, hence large errors are possible. In the freezing calorimeter, however, the calculation is different and the errors are not likely to be so large. This conclusion is identical to that derived above for techniques which measure v_w versus those which measure v_i .

Leaf (1966) reports achieving an accuracy of 1% liquid by weight or $\frac{1}{2}$ % by volume with this technique. This suggests that S_w could be calculated to within an accuracy of 1%, a feat which would only be possible if there were essentially no errors in measuring v_s and m_s and if v_w was calculated to within 1% from the calorimetry data. The latter seems unlikely in view of the difficulty of handling the snow samples without causing any phase changes, but nevertheless this method is promising and hopefully further refinements will be made. The inherent advantage of a direct measurement of v_w probably compensates for the fact that freezing is more difficult than melting. Perhaps this method's major disadvantage is the fact that it is a destructive sampling technique, hence some disturbance of the flow field will necessarily be associated with its use.

4b. Centrifuges

Much attention has been given to the use of centrifuges in soil physics, petroleum reservoir engineering, and snow hydrology. The attractive feature of a centrifuge is that large accelerative forces are exerted on the fluids, thereby decreasing the time necessary to drain a porous sample. Stallman (1964) shows that

$$L' = Lg/g', \tag{36}$$

$$t' = t(g/g')^2,$$
 (37)

and

$$g' = \omega^2 r, \tag{38}$$

where primed length L', time t' and acceleration g' are for the centrifuge's frame of reference and unprimed L and t are their equivalents in a sample subjected only to Earth's gravity g. When g' = 1000g, a 10 mm long sample centrifuged for 40 min is equivalent to the drainage of a 10 m column for 76 years provided the bottom of the sample remains in contact with the liquid during the 40 min of centrifuging. Unfortunately these scaling laws do not apply to the centrifuges currently in use by snow hydrologists because the samples do not remain in contact with the extracted liquid so there is some uncertainty about how the snow centrifuge should be scaled. Furthermore, Slobod and others (1951) state that displacement occurs only down to the "connate value" or irreducible water saturation. If Slobod and others are correct, only the mobile water can be extracted by this method and the total liquid-water content cannot be found by centrifuging! This does not necessarily rule out the use of centrifuges since, even if only the mobile component of the water were removed and measured, this could be useful information. It must be noted, however, that centrifuging snow samples at 60 revolutions per second can cause some consolidation. The time of consolidation effectively increases by a factor of $(g'/g)^2$ (Terzaghi, 1942).

Terzaghi (1942) described centrifuges by noting that, if the accelerative force is increased $\mathcal N$ times, the body forces are increased to $\mathcal NS_w$ per unit volume while the retentive forces remain constant. Therefore the height of capillary rise is decreased to $N^{-1}h_c$, hence the capillary end effect is reduced. Terzaghi also states that this scaling is invalid when the water is discontinuous because the weight of the particles becomes small enough that the surface tension can balance it. Contrary to Slobod and others, Terzaghi believes that the discontinuous moisture in centrifuging is less than the discontinuous moisture in gravity drainage, hence more than just the mobile water is extracted. Unfortunately, there are different interpretations of just how much liquid can be extracted by centrifuging. This question is more important for snow than for most other porous materials because of the limitations placed on the centrifuging of snow by the problems of melting and compacting during the process. To provide some insight into the question of how much liquid can be extracted by centrifuging, samples of hydrophilic glass beads were soaked with water and centrifuged under conditions similar to those possible in a field situation. Although glass beads were used in place of snow in these tests, the results are applicable to snow because all of the important parameters which affect the retention of water in snow were closely simulated in these tests. The samples were well soaked to ensure complete wetting and then hand-centrifuged for 60 s at about 7 revolutions per second to reduce their liquid content to about that of a freely draining snow cover. The samples were then centrifuged by a machine for 60 s at 33.3 revolutions per second plus the 30 s necessary to accelerate and decelerate the machine. The residual water present following centrifuging, as determined by oven-drying the samples, was highly dependent on the size of the glass beads (see Fig. 2) over the range of grain sizes common in natural snow covers



Fig. 2. The residual water saturation left behind after centrifuging decreases with increasing particle size. Much uncertainty is connected with the use of centrifuges because of this residual water.

(Wakahama, 1968). This result shows that different amounts of water will be extracted from different parts of a snow pack due to the occurrence of different grain sizes. Therefore, even as a *relative* measure of the amount of water present and the amount of water flowing, the centrifuge can give misleading results.

In the past Yosida (1967) and LaChapelle (1956) have noted the partial retention of water by snow samples during centrifuging. Hopefully, the results given here will discourage the widespread use of centrifuges in snow hydrology. Centrifuges have often been used as a measure of the spatial variability of the flow field of water in snow (e.g. Langham, 1974) but as shown by Equation (12), the error in flow is six times as large as the error in S_w . Unfortunately, the error in S_w as determined by centrifuging is affected by such things as grain size, melting, compaction, and time and rate of spinning. Under any circumstances, some water will be retained by the snow sample, a fact which precludes the quantitative use of the information obtained from a centrifuge. LaChapelle (1956) showed that some qualitative information can be obtained with a centrifuge, but the use of centrifuges for purposes such as determining the spatial and temporal variability of the flow field in normal snow covers is risky. Much of the infered non-uniformity in the flow field is simply due to the inherent limitations of the centrifuge.

4c. Solution method

Bader (1948) proposed a simple method in which a dilute solution of sodium hydroxide is added to a known quantity of wet snow and the temperature depression is measured. As long as the temperature depression is small (≈ 1.5 deg) and no significant errors occur in measuring the solution weight or sample weight, an accurate determination of the free water content F can be made. The relative error is

$$E(F) = \frac{\beta m_0 c_0 + 0.0062 (m_s + c_0) T^2}{\beta m_0 c_0 - 0.0062 (m_s + c_0) T^2 - c_0 T} E(T),$$
(39)

or, for a typical case described by Bader (1950),

$$E(F) \approx 1.5 E(T). \tag{40}$$

Thus Bader's method offers a quick and easy alternative to calorimetry. Assuming accurate weight, volume and temperature measurements, useful information on the free water content should be obtainable. Probably the biggest error in this method would be introduced by inaccurate measurements of the molal concentration m_0 of the solution of sodium hydroxide. Neglecting other errors,

$$E(F) = \frac{\beta c_0 m_0}{TF} E(m_0), \qquad (41)$$

or, for a typical case,

$$E(F) \approx 3.1 E(m_0). \tag{42}$$

This shows that the solution must be prepared carefully.

5. Non-destructive measuring devices

While some of these destructive sampling techniques give reasonably accurate information about the liquid water in a sample, they all suffer the serious disadvantage of disturbing the flow field by the act of removing the sample. This fact precludes repeated sampling to determine the temporal variations at a point in a snow cover. Repeated sampling to determine the local spatial variations is also suspect because of disturbances to the flow field caused by the creation of new surfaces within the flow field. Accordingly, the use of *in situ* or remote-sensing devices is necessary to obtain the most useful information, and several such devices are reviewed here. Other possible methods, which have not yet been applied to snow, include nuclear magnetic resonance (NMR), time-domain reflectometry, Raman scattering (see Miller, 1972), and acoustic methods.

5a. Dielectric devices

The large contrast between the dielectric constants of liquid water and ice at megahertz frequencies has provided the basis for measuring the liquid-water content of various materials at least since the early 1930's when meters were used to determine the liquid content of wheat. Perhaps Gerdel (1954) was the first to apply these devices to snow using a capacitance probe operating at a frequency of 1.5 MHz. Although the dielectric constant of snow is very sensitive to small changes in the volume of liquid water present, it is difficult to make a strict interpretation of the dielectric constant of the solid-liquid-gaseous mixture because of the importance of shape factors on the contribution of each phase to the dielectric constant of the mixture.

Ambach and Denoth (1975) have recently improved the capacitance probes used in snow by designing an instrument which operates at frequencies up to 20 MHz. At higher frequencies, the effect of grain size is minimized but the snow density must be known to calculate the free water content from the dielectric constant (Ambach and Denoth, 1975). The need to measure the snow density, a distinct disadvantage of this approach, arises because the mixing formulae (used to account for the shape factors) require the use of the value of the dielectric constant extrapolated to an infinite frequency. The dielectric constant then has only a real part, hence the density must be determined separately in order to supply enough information to calculate both the porosity and liquid saturation. For the purposes of designing an *in situ* instrument, the lack of information about the imaginary part of the dielectric constant is very unfortunate. Nevertheless, the dielectric devices are useful instruments with a standard error of about 0.5% by volume (Thomas, 1966; personal communication from A. Denoth).

At microwave frequencies, the real and imaginary parts of the dielectric constant can be determined simultaneously, thus providing all of the information necessary to calculate the porosity and water saturation in a known volume of wet snow (Sweeny and Colbeck, 1974). Unfortunately, the sophisticated microwave equipment is not easily adapted to field situations, so dielectric measurement by capacitance (Ambach and Denoth, 1975) is probably the best available at this time. Perhaps the best hope for developing a system to obtain information about porosity and water saturation in wet snow is the active microwave system. Linlor and others (1974) describe the application of such a system for obtaining profiles of snow wetness. The ultimate system would determine profiles of solid and liquid contents in a snow cover, thus providing information about the degree of layering, propagation of melt-water waves, state of ripening, depth, etc.

5b. Other devices

The soil-water tensiometer of Richards and Gardner (1936) measures the negative gage pressure in the liquid phase in an unsaturated soil. In snow, the negative pressure, or "tension", is determined primarily by the liquid-water saturation and grain size. For a thoroughly wetted snow where the grain size is stable but the liquid saturation changes with the flux of water, tension measurements provide a direct indication of the flow rate (Colbeck, 1976). This correlation between flow rate and tension at a point provides a useful method for observing the flow field without providing any direct knowledge of the porosity or water saturation. For example, Wankiewicz (unpublished) used tensiometers to make extensive observations of the flow of water in a deep mountain snow-pack. Unfortunately, tensiometers are difficult to use in a large-grained, porous medium like snow, hence *in situ* measurements with a capacitance probe may be more successful. Also, capacitance probes are free from the freezing problems of tensiometers; the capacitor can be positioned before the seasonal snow falls and left in position during freeze-thaw cycles.

Methods currently in use to determine the water equivalent of snow include terrestrial gamma-ray surveys (Peck and Bissell, 1973) and nuclear profiling gages. The use of nuclear sources which are attenuated by the snow was introduced by Gerdel and others (1950) and refined by Smith (e.g. Smith and others, 1965). The resolution of these gages is approaching the point of development necessary to detect changes in the flow rate of water. However, even in the absence of any error in ϕ or S_{wi} , Equation (12) shows that E(u) is typically six times as large as $E(S_w)$. If at most a ten per cent error in flow rate is desired, the liquid-water saturation would have to be determined to within a relative error of less than two per cent. Typically this means a measurement of total water mass accurate to within two parts per thousand, a feat which is beyond the capability of the nuclear gages currently in use.

6. Conclusions

Liquid-water saturation and porosity are two of the most basic pieces of information about a snow cover. These two parameters largely control such important properties as reflectivity, rheology, and water flow rates. Nevertheless, adequate methods for measuring these properties do not exist and further developments are necessary.

Besides the inherent limitations of destructive sampling techniques, the two most commonly used sampling devices have physical limitations. The centrifuge does not extract all of the liquid, a fact which has been frequently cited in the past. The problem with the centrifuge is that the water left in the sample is a complicated function of structural parameters such as grain size. For example, the water left behind in a sample of glass beads depends on the grain size over the range of sizes commonly observed for snow (see Fig. 2). This fact precludes the use of a centrifuge to infer accurately the flow field in a snow cover. From data taken by the melting calorimeter, both the water saturation and porosity can be calculated. While the calculated value of porosity may be fairly accurate, the calculated value of water saturation is generally highly inaccurate since any error in the measurements is magnified by the nature of the calculation for the liquid-water saturation (see Fig. 1). Accordingly, the use of either the melting calorimeter or centrifuge to determine the spatial or temporal variations in the flow field would be risky. What appear to be variations in flow might just be due to errors inherent in the methods.

Although the response of a tensiometer has been related experimentally to the liquid flow rate in snow, the high-frequency capacitors seem to offer more advantages for making *in situ* determinations of the liquid-water saturation for research studies. The advantages of the capacitance probes include easy coupling with the snow and a lack of freeze-thaw problems. Eventually efficient methods must be developed for interrogating the snow cover remotely in order to provide the necessary information for hydrological forecasting practices.

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200

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