

SEMIGROUP VARIETIES WITH THE  
AMALGAMATION PROPERTY

G.T. CLARKE

The purpose of this thesis is to determine which varieties of semigroups have the weak or strong amalgamation property. Amalgamation questions for classes of semigroups which form varieties of universal algebras have been studied in the case of groups, inverse semigroups and bands by Neumann [7], Hall [5] and Imaoka [6] respectively. Modulo questions of group theory and of the structure of the lattice of varieties of completely simple semigroups, we determine which varieties of completely regular semigroups have the weak or strong amalgamation property; and, modulo questions of group theory, we determine which varieties of semigroups have the weak or strong amalgamation property.

We first consider completely regular semigroup varieties containing the variety of semilattices, and we show that such a variety has the weak [strong] amalgamation property if and only if it consists entirely of orthodox normal bands of abelian groups. To obtain this result we show that a variety of completely regular semigroups has the congruence extension property if and only if it consists entirely of orthodox normal bands of abelian groups. We then obtain a partial answer to the question of which varieties of completely simple semigroups have the weak or strong amalgamation property.

Next we consider classes of inflations of completely regular semigroups as varieties of universal algebras, showing that such a variety has the weak [strong] amalgamation property if and only if the corresponding

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variety of completely regular semigroups has the same property. We give an example to show that the corresponding result for inverse semigroups does not hold.

Finally we consider semigroup varieties, showing that the class of all inflations of semigroups from a given variety is also a semigroup variety. We characterize those identities which determine varieties of inflations of completely regular semigroups. We then deduce that a variety of semigroups has the weak [strong] amalgamation property if and only if it consists entirely of inflations of completely regular semigroups and the subvariety of all its completely regular semigroups has the same property. Specializing this result to commutative varieties, we have that a variety of commutative semigroups has the weak [strong] amalgamation property if and only if it is a variety of inflations of semilattices of abelian groups.

Most of the material in Chapters 2 and 4 of this thesis appears also in [1], [2], [3] and [4].

### References

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Department of Mathematics,  
Monash University,  
Clayton,  
Victoria 3168,  
Australia.