

Moonshine Beyond the Monster

The Bridge Connecting Algebra,
Modular Forms and Physics

Moonshine forms a way of explaining the mysterious connection between the monster finite group and modular functions from classical number theory. The theory has evolved to describe the relationship between finite groups, modular forms and vertex operator algebras. *Moonshine Beyond the Monster*, the first book of its kind, describes the general theory of Moonshine and its underlying concepts, emphasising the interconnections between modern mathematics and mathematical physics.

Written in a clear and pedagogical style, this book is ideal for graduate students and researchers working in areas such as conformal field theory, string theory, algebra, number theory, geometry and functional analysis. Containing almost two hundred exercises, it is also a suitable textbook for graduate courses on Moonshine and as supplementary reading for courses on conformal field theory and string theory. This title, first published in 2006, has been reissued as an Open Access publication on Cambridge Core.

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