

## A COMMENT ON STAR-ORDERING AND TAIL-ORDERING

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In a recent letter, Deshpande and Kochar (1983) gave the following theorem.

*Theorem.* Let  $F$  and  $G$  be absolutely continuous distribution functions such that  $F(0) = G(0) = 0$  and  $f(0) \geq g(0) > 0$  where  $f$  and  $g$  are corresponding density functions, then if  $F$  is star-ordered with respect to  $G$  then it is tail-ordered with respect to  $G$ .

One does not need to have absolute continuity, the only condition required is

$$(1) \quad \lim_{x \rightarrow 0^+} \frac{G^{-1}(F(x))}{x} \geq 1.$$

Since  $F^* < G$ , therefore,  $G^{-1}(F(x))/x$  is non-decreasing in  $x$  and (1) implies  $G^{-1}(F(x))/x \geq 1$  for all  $x \geq 0$  and hence  $x((G^{-1}(F(x))/x) - 1)$  is non-decreasing in  $x$  for  $x \geq 0$  as it is the product of two non-decreasing non-negative functions. Therefore,  $G^{-1}(F(x)) - x$  is non-decreasing in  $x$  for  $x \geq 0$ , i.e.  $F \prec G$ .

### Reference

DESHPANDE, J. V. AND KOCHAR, S. C. (1983) Dispersive ordering is the same as tail-ordering. *Adv. Appl. Prob.* **15**, 686–687.

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