For the first system the triangles are

$$[\overline{2N-1}\,.\,(2n-\overline{2N-1})],\ [2n\,(n-\overline{2N-1})],$$
$$[2n\,(n-\overline{2N-1})+(2N-1)^2], \qquad \text{(iii)}$$

where N is the ordinal number of the set of triangles in question, and n is any number, not necessarily an integer and not necessarily positive.

For formula (i), $a=2N-1$, an odd integer; $b=n-(2N-1)$.

For formula (ii), $a=(2N-1)/\sqrt{2}$, $b=\sqrt{2}\,(n-\overline{2N-1})$.

For the second system the triangles are

$$[2N(2n-\overline{N-1})],\ [4n\,(n-\overline{N-1})-\overline{2N-1}],$$
$$[4n\,(n-\overline{N-1})+2N(N-1)+1]. \qquad \text{(iv)}$$

For formula (i), $a=\sqrt{2}\,N$, $b=\{2\,(n-N)+1\}/\sqrt{2}$.

For formula (ii), $a=N$, an integer; $b=2\,(n-N)+1$.

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Note on Isogonal Conjugates.

If T, U are any pair of isogonal conjugates with respect to a triangle ABC (circumcentre O, orthocentre H), then

$$OU=(TH/T\Phi)\,.\,(\text{circumradius});$$

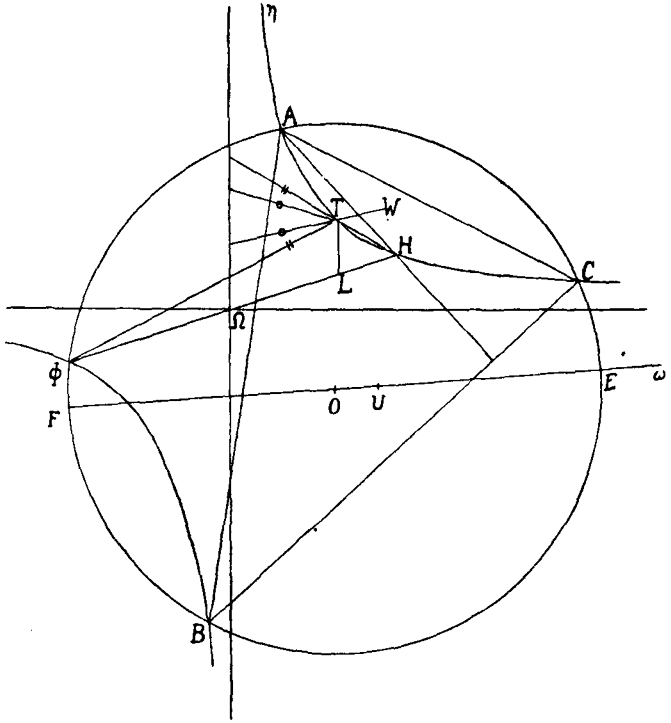
where Φ is the fourth point of intersection of the circumcircle with the rectangular hyperbola $ABCHT$ (whose centre Ω is the middle point of $H\Phi$).

It has been established by the method of isogonal transformation that if T is any point on a fixed rectangular hyperbola $ABCH\Phi$, then the point U (the isogonal conjugate of T) always lies on a fixed circumdiameter EOF.

Now AT, AU are equally inclined to the bisector of the angle A; hence the cross ratio of the pencil formed by joining A to any four positions of T is equal to the cross ratio of the four corresponding positions of U on EOF.

This may be expressed by

$$A\{T\} = \{U\}.$$



The isogonal conjugates of, H, T, Φ, η lying on the rectangular hyperbola are O, U, ϖ, E respectively lying on EOF (η, ϖ being points at infinity on the rectangular hyperbola and EOF).

Therefore $A\{HT\Phi\eta\} = \{OU\varpi E\}$.

But $A\{HT\Phi\eta\} = \eta\{HT\Phi\eta\} = \{HL\phi\Omega\}$ estimating on $H\Omega\Phi$ and drawing TL parallel to $\Omega\eta$.

Thus $\quad \{OU\varpi E\} = \{HL\Phi\Omega\}$

or $\quad OU.\varpi E : OE.\varpi U = HL.\Phi\Omega : H\Omega.\Phi L$

$$= HL.\Omega\Phi : H\Omega.L\Phi$$

and $\quad OU : OE = HL : L\Phi = TH : T\Phi$

since TH, $T\Phi$ being supplemental chords are equally inclined to TL.

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Corollary.

Draw TW equal and parallel to OU so that $UOTW$ is a parallelogram. Then $OU : TH = OE : T\Phi$, or $TW : TH = \Phi O : \Phi T$.

But TH, $T\Phi$ are equally inclined to be asymptotes; also OUE, $O\Phi$ are equally inclined to ΦE, ΦF (parallels to the asymptotes). Thus the angle WTH between TH and TW (OUE) is equal to the angle $O\Phi T$ between $T\Phi$, $O\Phi$.

The triangles WTH, $O\Phi T$ are therefore similar,

and $$WH : WT(OU) = OT : O\Phi$$

or $$OT . OU = O\Phi . WH$$
$$= 2\, O\Phi . NZ$$

as WH = twice join of middle point of OW (also middle point Z of TU) to middle point of OH (N the Nine Point centre).

This is Ramaswami Aiyar's theorem.

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