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DIRAC OPERATORS ON ORIENTIFOLDS

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Motivated by Wigner's theorem, a canonical construction is described that produces an Atiyah–Singer Dirac operator [5, Section II.6] with both unitary and anti-unitary symmetries. This Dirac operator includes the Dirac operator for KR-theory [1] as a special case, filling a long-standing gap in the literature. The conditions under which this construction can be made are investigated, and the obstruction is identified as a class within a generalisation of equivariant Čech cohomology. An associated geometric K-homology theory [3] is constructed, along with a homomorphism into an appropriate generalisation of analytic K-homology. More broadly, this thesis demonstrates that difficulties surrounding the interaction of K-orientation and anti-linear symmetry can be naturally resolved by building on Wigner's theory of corepresentations. Potential applications include the classification of D-brane charges in orientifold string theories [6, Section 5.2], the construction of index invariants for topological insulators [4], and the formulation of a Baum–Connes conjecture [2] for discrete groups with a distinguished order-2 subgroup.

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