Chapter V

MAGNETIC SELF-ORGANIZATION OF SOLAR/STELLAR PLASMA

Starspot detection and properties

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Abstract. I review the currently available techniques for the starspots detection including the one-dimensional spot modelling of photometric light curves. Special attention will be paid to the modelling of photospheric activity based on the high-precision light curves obtained with space missions MOST, CoRoT, and Kepler. Physical spot parameters (temperature, sizes and variability time scales including short-term activity cycles) are discussed.

Keywords. stars: activity, stars: magnetic fields, stars: spots, stars: late-type, techniques: photometric,techniques: spectroscopic.

1. Introduction

History of the "starspot hypothesis" which had gone back to 1667 when the French astronomer Ismael Boulliau used it to explain the periodic light variability of Omicron Ceti (in a wrong way!) was described in review by Strassmeier (2009). Photometric observations of stars with actually existing starspots started in 1947 when Kron (1947) observed four eclipsing systems later called RS CVn binaries. European and Russian astronomers had reached the same conclusion on starspots more or less independently in 1965-1967. Hall (1972) postulated the starspot model to explain rotational modulation of the RS CVn stars light curves due to dark starspots moving in and out of view.

As in Strassmeier (2009) we report that the NASA/Harvard ADS database now lists 417 articles (from 1972 to August 2012) with the word starspot(s) in the title, and a total of 1,525 articles mentioned it in the abstract.

2. Doppler imaging

Most reliable information about the surface inhomogeneities of the stars can be obtained with the Doppler imaging technique.

For overviews of the Doppler imaging (DI) (including starspot properties obtaind with this technique) see reviews by Strassmeier (2002), Strassmeier (2009), Strassmeier (2011), Berdyugina (2005), Piskunov (2008), Rice (2002).

Vogt & Penrod (1983) were the first who discussed the technique for imaging the surfaces of cool rapidly rotating spotted stars. Vogt *et al.* (1987) presented improved version of the Doppler imaging technique based on the principals of the maximum entropy image reconstruction and examined the effects of noise, finite spectral resolution, uncertain stellar parameters etc. through a variety of test cases.

The use of combined spectroscopy and photometry has been used extensively in papers by Strassmeier and colleagues (e.g. references can be found in Strassmeier (2002)).

In addition to it basic variations in Doppler imaging technique can be in methods of solving ill-posed nature of DI procedure, single-line or multi-line approach, treatment of molecular lines, treatment of temperatures, surface brightness or filling factors as an unknown object, errors of image recovery etc. Several modifications of DI deal with the binary stars imaging.

As an extension of the temperature and abundance mapping of the stellar surface, a magnetic Zeeman Doppler Imaging (ZDI) was developed. ZDI method is based on the analysis of high-resolution spectropolarimetric data and allows for disentangling magnetic field distribution on the stellar surface due to different Doppler shifts of Zeeman-split local line profiles in the spectrum of a rotating star (details can be found in Rice (2002), Berdyugina (2005), Piskunov (2008), Strassmeier (2009), Strassmeier (2011)).

Other different aspects of the history and modern status of Doppler imaging techniques can be found in the mentioned above reviews.

3. Light-curve inversions

The light curve shape variation due to rotational modulation is known to be a classical tracer to study stellar surface inhomogeneities and its many related phenomena as was found many years ago. With the advent of automated photoelectric telescopes full time-series light curves spanning continuously over many stellar rotations and even over many observing seasons became available.

As an example we can mention that in Strassmeier *et al.* (2008) we presented the first scientific stellar time-series optical photometry from Dome C in Antarctica and analyzed approximately 13000 CCD frames acquired in July 2007 with the optical pilot telescope of the International Robotic Antarctic Infrared Telescope named sIRAIT. The prime targets were the chromospherically active, spotted binary star V841 Cen and the non-radially pulsating δ Scuti star V1034 Cen.

Space telescopes - MOST, COROT and KEPLER, significantly increased the photometric precision. Such extremely high-precision time series datasets require more and more sophisticated reduction and analysis tools. Modern model independent inversion techniques that linearize an ill-posed problem and uniquely convert a one dimensional light curve into a spot map are now state-of-the art. Applications with a pre-defined spot number and spot shape in some cases are not wrong but are likely to be bad approximations and in some cases can be unrealistic.

Several approachers were developed for the light-inversion techniques. First, "parametric" numerical techniques were used by Dorren (1987), Strassmeier (1988). Technique which is taking into account time evolution of starspots was used by Strassmeier, Bopp (1992). We can mention also "zonal" model with (near-)equatorial inhomogeneous or bands of spots by Eaton & Hall (1979) and Alekseev & Gershberg (1996).

More detailed information on the spot pattern from light curves can be obtained in the case of eclipsing binaries by the eclipse mapping technique. This method employs the opportunity to scan the stellar disk by the eclipsing star. The inversion techniques based on the Maximum Entropy and Tikhonov regularisation methods were reviewed by Berdyugina (2005).

Modelling of light-curves is clearly less informative than DI techniques which are based on high resolution spectroscopic observations. Continuous and frequent photometric data give us conclusions basically on longitudinal spot patterns and their long-term evolution.

In our paper (Savanov & Strassmeier (2008)) we adopted the new technique from our recently published Doppler-imaging inversion code based on quasi-optimal filtering of the objects principal components. It is also applicable to multi-color light curves. We applied it for the first time to a decade-long time series of precise V and I-band photometry of the spotted star HD291095 (V1355Ori).



Figure 1. (A) Light curve of Corot-Exo-2a. (B) Phases of the active longitudes. Filled circles show more active areas and open circles less active ones. (C) Variations of the surface spottedness of Corot-Exo-2a. (D, E) Variations of the brightness variation amplitude and of the mean, minimum(min), and maximum(max) values for each of the 32 datasets. The horizontal dashed lines in (B) and (C) mark the preferred phases, separated by 0.5 of the period. The slanted dashed lines in (B) display the motion of the active regions on the stellar surface in longitude. The vertical dashed lines in B - E correspond to flip-flop phases for the active longitudes.

Savanov (2009) analyzed light curves from the MOST satellite for the two active dwarfs ε Eri and κ Cet. Our maps of the stellar surface-temperature inhomogeneities were obtained with no a priori assumptions about the shape, configuration, and number of spots. We find variations of the surface temperature inhomogeneities on time scales close to their rotation periods.

Unique photometry of the active G7 star Corot-Exo-2a (a young analog of our Sun possessing a Jupiter-type planet) obtained during continuous 142-day observations with the Corot space telescope have enabled us to study surface-temperature inhomogeneities of the star and to trace their evolution over almost five months (Savanov (2010a)). The observational material was divided into 32 datasets, each covering one complete rotational period of the star. We analyzed each individual light curve using the our code, which reconstructs the stars temperature inhomogeneities from its light curve in a two-temperature approximation. We identified five time intervals that can be interpreted as activity cycles (corresponding to four position switches for active areas). The durations of the activity time intervals were 53 days, 34 days, 21 days, 20 days, and at least 15 days, respectively. The pattern of the activity longitude flip-flops is similar to those observed



Figure 2. Distribution of the filling factor f with longitude on the stellar surface and with time. The longitudes 0° -360° are marked with solid vertical lines and repeated three times to underscore the motion of the active longitudes. The filled circles connected with lines show the positions of the most active area.

earlier for other stars, with the exception that, in the case of Corot-Exo-2a, the timescale for the longitude flip-flops is tens of days rather than years. We have detected motion of the active areas in longitude which can be interpreted as being due to differential rotation of the star. In the absence of information on the latitudes of the active areas on the stellar surface we can conclude only that either the active areas are located at nearby latitudes (but not at the same latitude) or the stars differential rotation is small.

We find evidence of another feature we noted earlier in our analysis for other stars: the times of the longitude flip-flops coincide with extrema of the brightness variation amplitude. During the 142 days spanned by the observations the evolution of active formations on the surface of Corot-Exo-2a led to various changes: there existed and developed either single active regions that replaced each another, two strong regions, or two weak regions. We have obtained clear evidence that active longitude switching can have a complex character. With the exception of the Sun, this is the first case when the high quality of observations and the availability of a continuous series of observations have enabled this level of detail when following the appearance and development of temperature inhomogeneities on the surface of a star. We established timescales for the stars activity variations of 17-20, 28-32, 33-38, and 51-55 days which characterize changes of the brightness amplitude, spotted surface area, positions of the active regions, and brightness variations.

We note that the behavior of a regions position near flip-flop times can be cyclic: after a flip-flop, a non-active area moves in the direction of the rotation, whereas, before a flip-flop, the more active area moves in the opposite direction. The high quality of the available observations enables us detailed studies of the spatial motions of spots on the stellar surface which have been earlier analyzed only for the Sun.



Figure 3. (a) Light curve of KOI 877. The solid vertical lines correspond to transits of the planet rotating around the star with a 5.96-day period, while dashed lines correspond to transits of the planet with a 12.04-day period. (b) Dynamic map of temperature inhomogeneities on the surface of KOI 877 (constructed for 2.48 rotations of the star) assuming inclinations to the line of sight $i = 80^{\circ}$ (upper) and $i = 88^{\circ}$ (lower).

Savanov (2011a) studied the activity of the KOI 877 and KOI 896 systems which at that time were among five ones classified as candidate multi-planetary systems (i.e., two or more planets may be present). Unique high-accuracy photometric data for these objects obtained during continuous 33.5-day observations by the Kepler Telescope enabled us to study the surface temperature inhomogeneities of these stars and trace the evolution of spots on their surfaces over two to three rotations. We have found evidence for the existence of two active longitudes on the surfaces of KOI 877 and KOI 896 separated by 165° and 135° respectively. In KOI 877, we observe two spots with comparable areas, while, in KOI 896, the area of one of the spots is larger than the area of the other. The area of the spotted surface is about 0.6% - 1.1% of the total visible surface of the objects. We suggest that we have detected a switch between the activity of the two active longitudes between the first and second rotation of KOI 877. Maps of surface temperature inhomogeneities can be used to derive high-accuracy estimates of the parameters of planetary system.

4. Spot properties

What we would like to know about starspots? We should mention the following characteristics: spot sizes, temperatures and filling factors, lifetime and decay times, morphology etc.



Figure 4. (A) Light curve of XX Tri. (B) Phases of the active longitudes. Filled circles show more active areas and open circles less active ones. (C) Variations of the surface spottedness. The horizontal lines mark the preferred phases. The vertical dashed area corresponds to flip-flop phases for the active longitudes.

The spots sizes according the reviews and literary sources range between 0.1-0.15 of the total stellar surface. Among the record holders are stars with the largest observed light-curve amplitude (the weak-line T Tauri type star V410 Tau, RS CVn-type stars HD12545 and II Peg). For these stars one can suggest the presence of cool spotted areas covering more than 20% of the entire stellar surface or more than 40% of the stellar disk.

The activity of V410 Tau, a star with periodic brightness variations was studied in numerous papers (see in Strassmeier (2009)). Savanov (2012a) investigated the evolution of spots on the stars surface over 46 years by reconstructing inhomogeneities of the stellar surface based on published photometric measurements. Analysis of the distribution of filling factors f as functions of time and longitude on the stars surface can be used to trace changes in the location of the dominant active area. In some cases, a second active area (longitude) separated by about 180° was detected. The position of the active area was stationary and the position of minimum brightness remained unchanged during a time interval of 4800 days (about 13 years), which ended by the end of 2002. The star possesses considerable cool spots on its surface, its fractional area is typically 32% and varies between 27% and 40%.

According to Strassmeier (1999) the K0III component of the RS CVn binary XX Tri has huge high latitude - polar spot of size 10000 times large than the largest sunspot group ever observed. Results of our light-curve inversion procedure applied to the published in literature photometric measurements of XX Tri are presented in Fig.4 (spot areas are in the range 29%-36%).

Kiraga (2012) discovered one of the new variables, ASAS 063656-0521.0, which presented large photometric changes due to presence of starspots. This star is an optical counterpart to the X-ray source 1RXS J063656.7-052104 and is classified as a rotational variable with a highest seasonal brightness variability (up to $\Delta V = 0.8$ mag). Fig.5 illus-



Figure 5. (Top) Light curve of ASAS 063656-0521.0. (Middle) Phases of the active longitudes. Filled circles show more active areas and open circles less active ones. (Bottom) Variations of the surface spottedness . The horizontal lines mark the preferred phases. The vertical dashed area corresponds to flip-flop phases for the active longitudes.

trates results of our analysis of ASAS 063656-0521.0 with its huge spot of size changing with time from 22 % to 50 %.

It is obvious that using DI small spots can be resolved on very rapidly rotating stars, e.g. like Wolter, Schmitt & van Wyk (2005) presented for the ultra rapidly rotating K3 dwarf BO Mic with spots of size as small as 0.5 %. Now-times small spots of comparable sizes can be discovered from the light-curve inversions of the high precision photometry obtained with space telescopes like Kepler.

Spot temperatures can be constrained in Doppler imaging but not all DI codes invert the spectral line profiles into a temperature map. The problem of cross-talk between spot temperature and spot size is most evident for the photometric spot modelling. In DI some uncertainties in determinations of spot temperature can be associated with the size of resolution element on the stellar disk.

General conclusion is that the cooler the star the smaller is the difference between the temperatures of spots and quite photosphere. We can refer to the review by Berdyugina (2005) according which the mentioned difference can be up to 2000 K for the F stars and decreases to 200 K for M dwarfs. Several different techniques can be used in the case of determinations of spots temperatures. Line-ratios and photometric variations were used by Frasca *et al.* (2008) to obtain spot temperatures. Carefully reproducing this technique using a new set of spectroscopic observations of active stars, O'Neal (2006) find that the LDR technique encounters difficulties, specifically by overestimating spot temperatures (because the atomic lines blend with titanium oxide absorption in cooler spots) and by not tightly constraining the filling factor of spots. Assuming that molecular absorption band heads can be only formed at temperatures below the normal photosphere and employing the TiO band heads at 705 and 886 nm O'Neal *et al.* (2004) determined starspot parameters by fitting TiO bands using spectra of inactive G and K stars as

proxies for the unspotted photospheres of the active stars and spectra of M stars as proxies for the spots.

Savanov & Strassmeier (2005) presented a new Doppler-imaging inversion code that is based on quasi-optimal filtering of the objects principal components of a Fisher information matrix. The new code allowed to perform the reconstruction of stellar surface temperature maps using molecular features like TiO, CO, OH, CN etc. which are numerous in spectra of late-type dwarfs. It is possible to use atomic and molecular features simultaneously in the restoration. Authors concluded that molecular input is important for stars with cool spots with temperatures below 4000 K and becomes mandatory for stars with effective temperatures of and below 4250 K. Even for the unspotted photosphere of 5000 K precise line profile calculations should take into account the absorption from CN, CH, etc.

Berdyugina (2010) selected several lines of Fe I and Ti I and many of TiO, CaH and FeH with prominent Stokes V profiles in the spectra of the active red dwarf AU Mic (M1Ve). These lines cover a wide range of temperature and magnetic sensitivity and also effectively form at different heights and locations in the stellar atmosphere. This provides a unique opportunity for probing directly the interior of starspots, basically independent on the distance to the star or its size, i.e. clearly beyond current direct spatial resolution on stellar surfaces. Spots on AU Mic were found in a near-equatorial zone, four spots were identified: one of positive and three of negative polarity. They are 500-700K cooler than the photosphere and harbour a maximum magnetic field of 5.3 kG. The smallest spot is comparable in absolute dimensions (70 Mm) with very large sunspots (60 Mm), and it practically disappears at the bottom of the atmosphere. The spot temperature decreases with height from 3100K to 2400K, with the gradient, which is significantly larger than that in sunspots. The magnetic field strength (modulus) in spots reaches more than 5 kG, which is much stronger than the equipartition level.

On the other side, another experiment in inverting the TiO molecular feature at 705.4 nm showed (Rice, Strassmeier & Kopf (2011)) that one can obtain good agreement of the surface temperature scale but that it is difficult to get a reliable, well resolved image from molecular features, especially with stars of large v sin i, because of the overlapping of the wings in adjacent molecular lines in the band. Such overlapping in the wings obscures information necessary in DI for reconstructing features in the latitudes of the equatorial region. In particular case of V410 Tauri authors obtained separate Doppler images from atomic lines as well as from the TiO 705.5-nm lines and found an average temperature difference between these images about 150 K in the sense that the TiO-based temperatures appeared to be generally cooler.

Flip-flops were first noticed from light-curve phase jumps of the spotted giant star FK Comae (Jetsu, Pelt & Tuominen (1993)). Its explanation is related to the existence of two active longitudes separated by about of 0.5 in phase or about 180 degrees in longitude. Stars with long enough datasets show several changes in the position of the most active region within these two longitudes. The average time between such flip-flops is referred as a flip-flop cycle and is in the range of a few years up to a decade. An observational overview and an inventory of stellar targets with such cycles was given by Berdyugina (2005). This flip-flop phenomenon has also been claimed to exist on the Sun Berdyugina & Usoskin (2003).

Savanov (2010b) used continuous 156-day COROT photometric observations of the F dwarf HD 181906 to analyze temperature inhomogeneities on the stellar surface and to follow their evolution. For the first time in studies of active regions and active longitudes, we find that the phases of the active longitudes on the surface of HD 181906 are concentrated close to two systems of active longitudes. In each system, the active



Figure 6. (A) Light curve of HD 181906. (B) Phases of the active longitudes (the filled and empty circles represent more active and less active regions, respectively). (C) Variations in the coverage of the surface with spots. (D, E) Variations of the amplitude of brightness variations and the mean ("Mean"), minimum("Min"), and maximum ("Max") values for each of the 60 datasets. The horizontal solid and dashed straight lines in panel (B) represent the two systems of active longitudes; the active longitudes in each system are separated by 0.5 in phase; the relative phase separation of the systems is 0.3. The shaded regions in (B) correspond to time intervals when the first system of active longitudes was observed; the second system was observed during unmarked time intervals. The vertical solid lines in (B)(E) mark the times of longitude switches.

longitudes are separated by 180°, with the shift between the systems being 100°. During the observing period switches between the systems of active longitudes occurred quasi-periodicly on a time scale of 30-35 days about two-thirds of the time, while these switches occurred less frequently about one-third of the time. The positions of active regions switched either every 2025 days or every 40-45 days. The periodicity of brightness and amplitude variations is of the order of 110 day. Variations in the spot coverage and changes in the active longitudes have shorter time scales (about 55-75 days). All these parameters are variable on time scales of 25-38 days. A wavelet analysis of the periodicity of the brightness variations indicates that all the above processes are quasiperiodic; activity on all time scales became less pronounced in the last third of the observing period.

It is assumed that idealized sunspots appear to follow a mean decay law in the form proposed by Petrovay, Martinez Pillet & van Driel-Gesztelyi (1999). The lifetime of the large starspots can be much longer than that of the sunspots, even years instead of weeks for sunspots (e.g., Strassmeier (2009), Hussian (2002)). As the spatial resolution obtained with Doppler imaging is not good enough for distinguishing whether the large cool region is a one single spot or a group of spots. In this case the individual spots could exhibit much shorter lifetimes than the group itself. Numerical simulations (e.g. Içik, Schüssler & Solanki (2007)) shown that in a rapidly rotating active star the expected spot lifetime is few months. Longer spot lifetimes are possible for the sub-giants.

Long-term photometric monitoring of starspot modulation reveals changes in the seasonal rotation period which indicate the presence of differential rotation on stellar surfaces and changes in spot latitudes. Characteristics of differential rotation can be estimated from observations with various methods, e.g. by Fourier analysis of light curves, cross-correlation of successive stellar Doppler images, spot tracking, Fourier transform of rotationally broadened line profiles, parameter fit in Doppler and Zeeman Doppler imaging, through asteroseismology (Berdyugina (2005)). Barnes *et al.* (2005) collected measurements from different sources and methods and obtained results which indicated a trend towards decreasing surface differential rotation with decreasing temperature also imply power-law correlation between the rotation and surface differential rotation with a power of 0.15.

The time span of Doppler images even for the most studied stars is not long enough to see the latitude changes over the spot cycle and to recover stellar butterfly diagrams. Light curve modelling is based on a longer time scale but spot latitudes obtained from the model are not reliable. A new approach for recovering stellar butterfly diagrams was suggested by Berdyugina (2005). According to this approach knowing the surface differential rotation and phase migration of active longitudes one can recover mean spot latitudes during the course of sunspot-like cycles.

For a summary of properties of spot cycles we refer to the review by Strassmeier (2009). Relations between the spot cycle and flip-flop cycle which appear to be different for binary components and single stars are discussed by Berdyugina (2005). Savanov (2012b) made an attempt to determine the activity cycles for the coolest M dwarfs using photometric data from the ASAS survey. The data in the sample contains 31 M stars. The time interval covered by the observations reached 2000 - 3000 days or more for most of the objects enabling us to detect activity cycles with durations not exceeding five to eight years. The brightness-variation time scales for the program stars were derived from their calculated amplitude power spectra and results of wavelet analysis. Most of the program stars display periodic variations in their light curves on time scales of hundreds to thousands of days. We find no differences in the activity cycles for stars of different masses (including stars with convective envelopes and fully convective stars). Literature data with a higher photometric accuracy for EY Dra (also from a different dataset of observations) and GJ 411 (derived from different activity indicators) are in agreement with our results. Our analysis indicates that the slope in the logarithmic P_{cuc}/P_{rot} versus $1/P_{rot}$ diagram is equal to $i = 1.03 \pm 0.04$ for all the data and differs from those obtained previously in the literature for other active G-K stars. From the analysis of the P_{cuc} - P_{rot} relation we conclude that the durations of the activity cycles for the studied M dwarfs are independent of their rotation periods. This conclusion differs significantly from that one for comparatively young (active) and older (less active) stars. The data for our program

objects do not fit any of the relations listed above, and probably form a new branch of low-mass stars.

Most of our current knowledge about magnetic fields of cool stars and in starspots is based on Zeeman broadening measurements. Zeeman broadening is best measured for slowly rotating stars, in contrast to ZDI. Magnetic field measurements for active dwarfs and giants were collected by Berdyugina (2005). These results indicate a tendency for cooler dwarfs to have stronger magnetic fields and larger areas covered by them. Also there is a clear contradiction between spot filling factors measured from light curves and magnetic field filling factors measured from spectral lines. This is also supported by results obtained with the ZDI technique, which reveals stronger magnetic fields for intermediate brightness regions (see in Strassmeier (2009)). New aspects of the different techniques available for magnetic field measurements and results of the analyses on coolstar magnetic fields can be found in Reiners (2012).

Planetary transits provide a unique opportunity to investigate the surface distributions of star spots. The occultations of the starspots by a transiting planet can be used as a high-pass spatial filter to map the fine structure of the spot distribution within the occulted band. The method was successfully applied, for example, to CoRoT-2 by Silva-Valio *et al.* (2010), Silva-Valio & Lanza(2011) and provided information on the characteristics of individual spots as well as on stellar differential rotation and the spinorbit alignment of the planetary system. Llama *et al.* (2012) discussed the possibility of recovering spot cycles on stars hosting a transiting planet. The method of using transiting planets to determine changes in spot latitudes requires not only that a planet is transiting, but also on an inclined orbit. Low latitude spots are preferentially recovered by this method, and shorter magnetic cycles will be easier to study within the Kepler lifetime. Transit observations can therefore provide new insight into stellar activity and cycles, and hence are an additional, critical test for stellar dynamo theories.

5. Conclusions

I review the observational tools and diagnostic techniques for studying starspots including Doppler Imaging technique and methods of light-curve inversions. Several examples illustrating the capabilities of light-curve inversion techniques were chosen for our review. Nowadays light-curve inversion technique is a widely used tool for studying geometrical position of temperature inhomogeneities and its evolution on stellar surfaces. The main areas of applications are related to the long term behavior of spots and their evolution. Another very new and fast developing area is the application of light-curve inversion technique to datasets obtained with the space missions like CoRoT and Kepler. Space missions provide us a unique, high quality, practically continuous record of stellar optical variability that allows detailed study of starspot modulation, growth, decay and migration os starspots for stars in a wide range of its fundamental properties (but in the cases when it is done in 'white-light' it just reconstructs geometrical properties, e.g. spot longitudes and/or zonal rotational periods). In the second part of the review the starspot properties including their temperatures, areas, lifetimes, active latitudes and longitudes, etc. are discussed.

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Discussion