of non-deterministic matrices (or Nmatrices) and used as semantics tool for characterizing some logics which cannot be characterized by a single finite matrix. Carnielli and Coniglio introduced the semantics of swap structures for **LFI**s (Logics of Formal Inconsistency), which are Nmatrices defined over triples in a Boolean algebra, generalizing Avron's semantics. In this thesis, we will introduce a new method of algebraization of logics based on multialgebras and swap structures that is similar to classical algebraization method of Lindenbaum-Tarski, but more extensive because it can be applied to systems such that some operators are non-congruential. In particular, this method will be applied to a family of non-normal modal logics and to some **LFI**s that are not algebraizable by the very general techniques introduced by Blok and Pigozzi. We also will obtain representation theorems for some **LFI**s and we will prove that, within out approach, the classes of swap structures for some axiomatic extensions of **mbC** are a subclass of the class of swap structures for the logic **mbC**.

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René Gazzari, ***Formal Theories of Occurrences and Substitutions***, University of Tübingen, Germany, 2020. Supervised by Peter Schroeder-Heister and Reinhard Kahle. MSC: 03F03, 00A30, 03F07. Keywords: Occurrence, substitution, substitution function.

Abstract

Gazzari provides a mathematical theory of occurrences and of substitutions, which are a generalisation of occurrences constituting substitution functions. The dissertation focusses on term occurrences in terms of a first order language, but the methods and results obtained there can easily be carried over to arbitrary kinds of occurrences in arbitrary kinds of languages.

The aim of the dissertation is twofold: first, Gazzari intends to provide an adequate formal representation of philosophically relevant concepts (not only of occurrences and substitutions, but also of substitution functions, of calculations as well as of intuitively given properties of the discussed entities) and to improve this way our understanding of these concepts; second, he intends to provide a formal exploration of the introduced concepts including the detailed development of the methods needed for their adequate treatment.

The dissertation serves as a methodological fundament for consecutive research on topics demanding a precise treatment of occurrences and as a foundation for all scientific work dealing with occurrences only informally; the formal investigations are complemented by a brief survey of the development of the notion of occurrences in mathematics, philosophy and computer science.

The notion of occurrences. Occurrences are determined by three aspects: an occurrence is always an occurrence *of* a syntactic entity (its *shape*) *in* a syntactic entity (its *context*) *at* a specific *position*. Context and shape can be any meaningful combination of well-known syntactic entities as, in logic, terms, formulae or formula trees. Gazzari's crucial idea is to represent the position of occurrences by *nominal forms*, essentially as introduced by Schütte [2]. The nominal forms are a generalisation of standard syntactic entities in which so called *nominal symbols* $*_k$ may occur. The position of an occurrence is obtained by eliminating the intended shape in the context, which means to replace the intended shape by suitable nominal symbols.

Standard occurrences. Central tool of the theory of nominal terms (nominal forms generalising standard terms) is the *general substitution function* mapping a nominal term $\mathtt{t}$ and a sequence $\vec{\mathtt{t}}$ of them to the result $\mathtt{t}[\vec{\mathtt{t}}]$ of replacing simultaneously the

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nominal symbols $*_k$ in the first argument by the respective entries $\mathtt{t}_k$ of the second argument.

A triple $\mathfrak{o} = \langle t, s, \mathtt{t} \rangle$ is a *standard occurrence*, if an application of the general substitution function on the position $\mathtt{t}$ and the shape s results in the context t of that occurrence. As $*_0$ can occur more than once in $\mathtt{t}$, arbitrary many single occurrences in the context t of the common shape s can be subsumed in $\mathfrak{o}$.[1] Gazzari illustrates the appropriateness of his approach by solving typical problems (counting formally the number of specific occurrences, deciding whether an occurrence lies within another) which are not solvable without a good theory of occurrences.

Multi-shape occurrences. The *multi-shape occurrences* $\mathfrak{o} = \langle t, \vec{s}, \mathtt{t} \rangle$ are the generalisation of standard occurrences, where the shape $\vec{s}$ is a sequence of standard terms. Such occurrences subsume arbitrary non-overlapping single occurrences in the context t.

Gazzari addresses the non-trivial identity of such occurrences and their independence. The latter represents formally the idea of non-overlapping occurrences and is a far-reaching generalisation of disjointness as discussed by Huet with respect to single occurrences. Independent occurrences can be merged into one occurrence; an occurrence can be split up into independent occurrences.

Substitutions. A substitution $\mathbf{s} = \langle t, \vec{s}, \mathtt{t}, \vec{s}\,', t' \rangle$ satisfies that both $\mathfrak{o} = \langle t, \vec{s}, \mathtt{t} \rangle$ and $\mathfrak{o}' = \langle t', \vec{s}\,', \mathtt{t} \rangle$ are occurrences such that the shapes have the same length. Such a substitution represents the replacement of $\vec{s}$ in t at $\mathtt{t}$ by $\vec{s}\,'$ resulting in t'. This means that a substitution is understood as a process and not as a (specific type of a) function.

Identity and independence are addressed again, using and extending the methods developed for occurrences; as before, independent substitutions can be merged and substitutions can be split up into sequences of independent substitutions. Substitutions are used to represent formally calculations (as found in everyday mathematics) and to investigate them.

Sets of substitutions turn out to be set-theoretic functions mapping the affected occurrences $\mathfrak{o}$ and the inserted shapes $\vec{s}\,'$ to the result t' of a substitution $\mathbf{s}$. Such sets are called *explicit substitution functions*. In order to qualify functions which are usually understood as substitution functions (and which are not formulated in a theory of occurrences) as substitution functions, Gazzari develops the concept of an *explication method* transforming such functions into explicit substitution functions. The appropriateness and the (philosophical) limitations of this concept are illustrated with example functions.

Conclusion. Gazzari's theory of occurrences is strong (not restricted to single occurrences), canonical (nominal forms are a canonical generalisation of the underlying syntactic entities) and general (presupposing the grammar for the underlying syntactic entities, suitable nominal forms are easily defined and the theory of occurrences is immediately carried over). Another advantage is a kind of methodological pureness: positions are generalised syntactic entities (and not extraneous, as sequences of natural numbers) and can be treated, in particular, with the well-known methods developed for the underlying syntactic entities.

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[1] Single occurrences are state of the art in Computer Science; cf., for example, Huet [1].

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ZACH NORWOOD, ***The Combinatorics and Absoluteness of Definable Sets of Real Numbers***, University of California, Los Angeles, USA, 2018. Supervised by Itay Neeman. MSC: 03E60, 03E57.

Abstract

This thesis divides naturally into two parts, each concerned with the extent to which the theory of $L(\mathbf{R})$ can be changed by forcing.

The first part focuses primarily on applying generic-absoluteness principles to how that definable sets of reals enjoy regularity properties. The work in Part I is joint with Itay Neeman and is adapted from our paper *Happy and mad families in* $L(\mathbf{R})$, JSL, 2018. The project was motivated by questions about *mad families*, maximal families of infinite subsets of ω of which any two have only finitely many members in common. We begin, in the spirit of Mathias, by establishing (Theorem 2.8) a strong Ramsey property for sets of reals in the Solovay model, giving a new proof of Törnquist's theorem that there are no infinite mad families in the Solovay model.

In Chapter 3 we stray from the main line of inquiry to briefly study a game-theoretic characterization of filters with the Baire Property.

Neeman and Zapletal showed, assuming roughly the existence of a proper class of Woodin cardinals, that the boldface theory of $L(\mathbf{R})$ cannot be changed by proper forcing. They call their result the Embedding Theorem, because they conclude that in fact there is an elementary embedding from the $L(\mathbf{R})$ of the ground model to that of the proper forcing extension. With a view toward analyzing mad families under AD^+ and in $L(\mathbf{R})$ under large-cardinal hypotheses, in Chapter 4 we establish *triangular* versions of the Embedding Theorem. These are enough for us to use Mathias's methods to show (Theorem 4.5) that there are no infinite mad families in $L(\mathbf{R})$ under large cardinals and (Theorem 4.9) that AD^+ implies that there are no infinite mad families. These are again corollaries of theorems about strong Ramsey properties under large-cardinal assumptions and AD^+, respectively. Our first theorem improves the large-cardinal assumption under which Todorcevic established the nonexistence of infinite mad families in $L(\mathbf{R})$. Part I concludes with Chapter 5, a short list of open questions.

In the second part of the thesis, we undertake a finer analysis of the Embedding Theorem and its consistency strength. Schindler found that the the Embedding Theorem is consistent relative to much weaker assumptions than the existence of Woodin cardinals. He defined *remarkable cardinals*, which can exist even in L, and showed that the Embedding Theorem is equiconsistent with the existence of a remarkable cardinal. His theorem resembles a theorem of Harrington–Shelah and Kunen from the 1980s: the absoluteness of the theory of $L(\mathbf{R})$ to ccc forcing extensions is equiconsistent with a weakly compact cardinal. Joint with Itay Neeman, we improve Schindler's theorem by showing that absoluteness for σ-closed $*$ ccc posets—instead of the larger class of proper posets—implies the remarkability of $\aleph_1^V$ in L. This requires a fundamental change in the proof, since Schindler's lower-bound argument uses Jensen's reshaping forcing, which, though proper, need not be σ-closed $*$ ccc in that context. Our proof bears more resemblance to that of Harrington–Shelah than to Schindler's.

The proof of Theorem 6.2 splits naturally into two arguments. In Chapter 7 we extend the Harrington–Shelah method of coding reals into a specializing function to allow for trees with uncountable levels that may not belong to L. This culminates in Theorem 7.4, which asserts that if there are $X \subseteq \omega_1$ and a tree $T \subseteq \omega_1$ of height ω_1 such that X is codable along T (see Definition 7.3), then $L(\mathbf{R})$-absoluteness for ccc posets must fail.