

Filtrations and valuations on rings

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The concept of a multiplicative filtration on a ring [4] is generalized so as to include among filtered rings, rings with valuation, pseudovaluation and semivaluation. The generalized filtration induces a topology on the ring, and the Hausdorff completion of the resulting topological ring [1, Chapters 1, 2, 3] is investigated as an inverse limit.

Next a generalized valuation and pseudovaluation on a ring are defined, and they are shown to induce a filtration on the ring which can then be completed. Conversely, a wide class of filtrations on a ring is shown to define pseudovaluations on the ring. And the completion of a ring with respect to another class of valuations is found to have no divisors of zero.

The rest of the thesis treats special cases of valuations: semivaluations on fields, and Manis [3] and Harrison [2] valuations on commutative rings. Results include that the completion of a (commutative) ring with respect to a Manis valuation is an integral domain, and necessary and sufficient conditions are obtained for the completion to be a field; the completion of a (commutative) ring with respect to a Harrison valuation is a field if the value group is archimedean ordered; and the completion of the rational numbers with respect to a semivaluation induced by the integers is a ring whose elements can be uniquely represented by

expressions of the form $\sum_{i=1}^{\infty} i! \alpha_i$ where $\alpha_1 \in \mathbb{Q}$, $0 \leq \alpha_1 < 2$; and, for each $i \in I \setminus \{1\}$, $\alpha_i \in \{0, 1, \dots, i\}$.

Received 21 June 1971. Thesis submitted to Monash University, February 1971. Degree approved, June 1971. Supervisor: Dr C.F. Moppert.

References

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- [3] Merle E. Manis, "Valuations on a commutative ring", *Proc. Amer. Math. Soc.* 20 (1969), 193-198.
- [4] D.G. Northcott, *Lessons on rings, modules and multiplicities* (Cambridge University Press, Cambridge, 1968).