

Tidal disturbances of small cohesionless bodies: limits on planetary close approach distances

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Abstract. The population of Near-Earth Objects contains small bodies that can make very close passages to the Earth and the other planets. Depending on the approach distance and the object's internal structure, some shape readjustment or disruption may occur as a result of tidal forces. A real example is the comet Shoemaker Levy 9 which disrupted into 21 fragments as a result of a close approach to Jupiter, before colliding with the planet during the next passage in July 1994. We have recently developed an exact analytical theory for the distortion and disruption limits of spinning ellipsoidal bodies subjected to tidal forces, using the Drucker-Prager strength model with zero cohesion. This model is the appropriate one for dry granular materials such as sands and rocks, for rubble-pile asteroids and comets, as well as for larger planetary satellites, asteroids and comets for which the cohesion can be ignored. Here, we recall the general concept of this theory for which details and major results are given in a recent publication. In particular, we focus on the definition of "material strength": while it has great implications this concept is often misunderstood in the community of researchers working on small bodies. Then, we apply our theory to a few real objects, showing that it can provide some constraints on their unknown properties such as their bulk density. In particular it can be used to estimate the maximum bulk density that a particular object, such as 99942 Apophis, must have to undergo some tidal readjustments during a predicted planetary approach. The limits of this theory are also discussed. The cases where internal cohesion cannot be ignored will then be investigated in the near future.

Keywords. Self-gravitation; asteroid, shape

1. Introduction

In our Solar System, a great number of small bodies are on trajectories that make them pass very close to a planet. This is also the case for small planetary satellites, such as the ones recently discovered by the Cassini mission around Saturn. For asteroids and comets in orbit around the sun, the close trajectory to a planet occurs in a fly-by with a short time interval. For instance, the comet Shoemaker Levy 9 made a very close approach to Jupiter which resulted in its disruption into 21 fragments, which in turn collided with Jupiter during the next passage in July 1994. A particular population of bodies whose members can undergo close Earth approaches is the population of Near Earth Objects (NEOs). which can pass as close as a few Earth radii. In this case, depending on the internal structure of the small body, tidal forces may be strong enough to cause shape readjustment or even the total disruption of the body. Tidal effects have also been considered as a potential formation mechanism of binary NEOs, although recent studies suggest that they may not be sufficient to explain the proportion of binaries in the NEO population. Since some very close approaches to the Earth can be predicted in

advance for some asteroids, it is interesting to predict whether an object may undergo some shape readjustment or disruption at its closest distance; and which constraints on its internal properties, such as its bulk density, may allow or prevent such an occurrence. An important example is the well-known asteroid (99942) Apophis which is predicted to come as close as 5.6 Earth radii to the Earth. We have recently developed an exact analytical theory for the distortion or disruption limits of solid spinning ellipsoidal bodies subjected to tidal forces. The details of this theory as well as its major results are described in Holsapple & Michel (2006). Most NEOs whose shapes are known can be well approximated to at least first order by ellipsoids; that is the assumption of our model. We use the zero cohesion Drucker-Prager model of strength. That model characterizes the behavior of dry granular materials such as sands and rocks, rubble pile asteroids and comets, as well as all larger planetary satellites. We will describe in more detail the concept of strength later in this paper. We then note that our theory uses the same approach as the studies of spin limits for solid ellipsoidal bodies given in Holsapple (2001). It is a static theory that predicts conditions for break-up as well as the nature of the deformations at the limit state. However, it does not track the dynamics of the body as it comes apart. Some other studies treat this problem, and we also plan to investigate it in the future. Here, we just limit our analysis to the minimum distance at which a change can happen, but not the subsequent evolution of a body when those changes happen.

The remainder of this paper is organized as follows. Section 2 describes in some details the concept of material strength and the failure criterion used to determine whether a solid body may be affected by external forces. In section 3, we summarize our analytical theory for the tidal disruption limits of cohesionless solid spinning ellipsoids. Section 4 is devoted to the application of this theory to real cases. Conclusions and perspectives are then given in Section 5.

2. What do we mean by material strength?

The term “strength” is often used in imprecise ways, and it is important that this concept is well understood. The description provided here is largely inspired from Holsapple & Michel (2006) and Holsapple (2006). We believe that it is important to present it in different places to ensure that a large part of our community speaks the same language.

Materials such as rocks, dirt and ice, which constitute small bodies of our Solar System, are complex and characterized by several kinds of strength. Generally, the concept of “strength” is a measure of an ability to withstand stress. But stress, as a tensor, can take on many different forms. One of the simplest is a uniaxial tension, and the tensile strength is often (mis)used to characterize material strength as whole. Thus, while it is common to equate “zero tensile strength” to a fluid body, that is not correct. In fact, while “fluid” has zero tensile strength, “solids” may also. For instance, dry sand has no tensile strength. However, dry sand and granular materials in general can withstand considerable shear stress when they are under pressure: that is why we can walk on dry sand but not on water. Here comes into play a second kind of strength: the shear strength which measures the ability to withstand pure shear. The shear strength in a granular material under confining pressure comes from the fact that the interlocking particles must move apart to slide over one another, and the confining pressure resists that. Then a third kind of strength, the compressive strength, governs the ability to withstand compressive uniaxial stress. Thus, a general material has tensile strength, shear strength at zero pressure (technically the “cohesion”) and compressive strength. In geological materials such as soils and rocks, the failure stresses depend strongly on the confining pressure; as a result, these three strength values can be markedly different. A cohesionless body,

such as the one considered in this paper, is simply a solid body whose cohesion (shear strength at zero pressure) is null, but that does not mean it does not have any shear strength under confining pressure (provided by the self-gravity for large bodies). Hence, a body can be cohesionless but nevertheless solid.

The Drucker-Prager (DP) model is a common model for geological materials, as is the Mohr-Coulomb criterion (MC) (the difference between these two models is not relevant for our study). The DP model assumes that the allowable shear stress depends linearly on the confining pressure. The shear stress magnitude is measured by the square root of the second invariant of the deviator stress. Thus, the DP model is similar to models for linear friction and is defined by two constants: one characterizes the “cohesion” (shear strength at zero pressure); and the second characterizes the dependence on the confining pressure and is related to the so-called angle of friction. Those two constants then determine the tensile and compressive strengths. When the cohesion is zero, so is the tensile strength; but not the compressive strength. Physically, the pressure dependence is, as already explained, the consequence of the interlocking of the granular particles and not the friction of the surfaces of the particles. In fact, a closely packed mass of uniform rigid *frictionless* spherical particles has an angle of friction about 23° , so the term angle of friction is somewhat a misnomer. Angle of *interlocking* would be more correct, but we will keep the standard name to avoid confusion.

Figure 1 gives a representation of the DP model. Using the three principal stresses $\sigma_1, \sigma_2, \sigma_3$ (positive in tension) of a general three-dimensional stress state, the pressure (positive in tension) is given as:

$$p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad (2.1)$$

and the square root of the second invariant of the deviator stress is:

$$\sqrt{J_2} = \frac{1}{\sqrt{6}} \sqrt{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \quad (2.2)$$

Then, the DP failure criterion is generally given as:

$$\sqrt{J_2} \leq k - 3sp \quad (2.3)$$

which is illustrated as a straight line with slope $3s$ and intercept k on Fig. 1. Clearly negative pressure (compression) increases the allowable $\sqrt{J_2}$ when s is positive.

For the special case of a pure shear stress only, $\sqrt{J_2}$ is just that shear stress and the pressure p is zero. On Fig. 1, the uniaxial tension strength σ_T has $\sqrt{J_2} = 3^{-1/2}\sigma_T$ and $p = \sigma_T/3$. The uniaxial compression strength σ_C has $\sqrt{J_2} = -3^{-1/2}\sigma_C$ and $p = \sigma_C/3$.

The DP criterion can be made to match the MC one in all combinations of pressure plus uniaxial compression if the parameters s and k are related to the cohesion and the angle of friction ϕ used in the MC model. In particular, the slope s is related to the angle of friction of the MC model ϕ by:

$$s = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)} \quad (2.4)$$

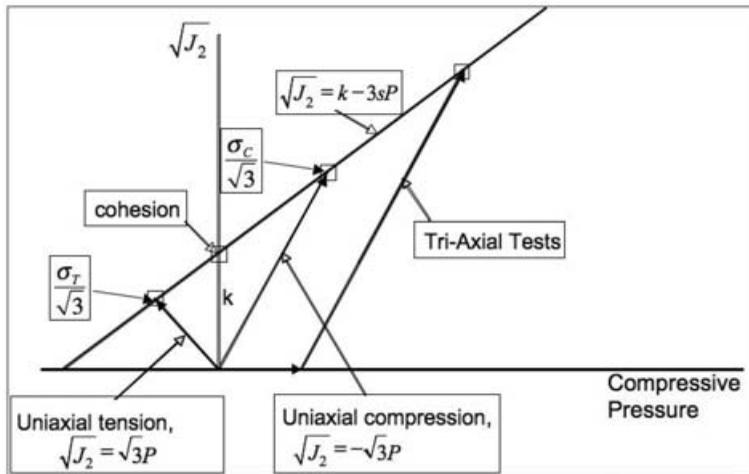


Figure 1. The Drucker-Prager failure criterion. The abscissa is positive in compression. The four small squares indicate the failure condition in, respectively from the left: tension, shear, compression and a confined compression or tri-axial test. The intercept at zero pressure at the value k is called the cohesion and the slope of the line passing through k is $3s$. From Holsapple (2006).

The intercept k of the DP model is also the shear stress τ for failure in pure shear. Technically the term “cohesion” means this intercept value of shear stress strength at zero pressure. When the cohesion is zero, so is the tensile strength, and vice versa; both cases would have the envelope starting at the origin in Fig. 1. Thus our theory applies to bodies for which those two measures are zero, while the plot shows the more general case where they are non-zero.

The MC model defines a maximum shear stress directly, which is determined by the difference of the maximum and minimum principal stresses. As a consequence, to use this criterion in algebraic manipulations involving general stress states, one must first determine the principal stresses and which is the largest and which is the smallest. The result is a difference in the algebra of the results in six different regimes, where the three principal stress components take on different orderings. An example of the six possible cases of the ordering of the stress magnitudes is given in Fig. 4 of Holsapple (2001). Moreover, there are “corners” in the curves shown where the ordering of the principal stresses changes. In contrast, the DP criterion has a single algebraic relation for all stress states. Thus, although the algebraic form of that relation is more complicated than the MC criterion, there is no need to consider the six different possibilities of the ordering of the stress magnitudes. The algebraic complexity of the DP model is of little consequence when an algebraic manipulation program such as *Mathematica* is used. So, in our theory, we use the DP failure criterion, noting that the differences between the two models are small (see, e.g. Fig. 1 in Holsapple & Michel, 2006).

3. Tidal disruption theory for cohesionless bodies

Using these concepts of strength and the DP failure criterion, we now move on to give a brief description of our theory for the limit distance for tidal readjustment or disruption of general ellipsoid solids (for details, see Holsapple & Michel, 2006).

The equilibrium problem of an ellipsoid body has been presented in Holsapple (2001). Three stress equilibrium equations must be satisfied by the stresses σ_{ij} in any body in

static equilibrium with body forces b_i , which are given as (using repeated index summation convention):

$$\frac{\partial}{\partial x_j} \sigma_{ij} + \rho b_i = 0 \tag{3.1}$$

We use an x, y, z coordinate system aligned with the ordered principal axes of the ellipsoid. In the problems here, the body forces arise from mutual gravitational forces, centrifugal forces, and/or tidal forces; they all have the simple linear forms $b_x = k_x x$, $b_y = k_y y$, $b_z = k_z z$. The full expressions of k_x, k_y and k_z are explicitly given in Holsapple & Michel (2006).

Then, for the limit states sought, the stresses must satisfy the DP failure criterion (2.3) at all points x, y and z , which, in terms of principal stresses, can be written for a cohesionless body as:

$$\frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = s^2[\sigma_1 + \sigma_2 + \sigma_3] \tag{3.2}$$

Also, the surface tractions are zero on the surface points of the ellipsoidal body surface defined by: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$. Holsapple (2001) solves this problem, showing that the distribution of stress in that limit state just at uniform global failure has the simple form:

$$\begin{aligned} \sigma_x &= -\rho k_x a^2 \left[1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 - \left(\frac{z}{c}\right)^2 \right], \\ \sigma_y &= -\rho k_y b^2 \left[1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 - \left(\frac{z}{c}\right)^2 \right], \\ \sigma_z &= -\rho k_z c^2 \left[1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 - \left(\frac{z}{c}\right)^2 \right], \end{aligned} \tag{3.3}$$

and the shear stresses in this coordinate system are all zero (note that the x, y, z stresses being principal stresses, they can be used for the 1, 2, 3 stresses in the previous equations). The body force constants k_x, k_y, k_z depend on the body forces, so those body forces must be such that the DP failure criterion is not violated. That condition determines the limit states.

Note that the fact that all the stresses have a common functional dependence on the coordinates is a necessary consequence of the fact that the DP and MC failure criteria are homogeneous functions of the stress components. Putting the expressions of these components into the DP criterion(3.2), one can see that the common functional dependence $1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 - \left(\frac{z}{c}\right)^2$ will cancel out of the Eq. (3.2). That is because the limit stress state has simultaneous failure at all points. Thus, we can omit that functional dependence and focus on finding the combinations of the leading multipliers of the three terms of 3.3 that satisfy the failure criterion. We define the dimensionless spin and dimensionless distance by:

$$\Omega = \frac{\omega}{\sqrt{\pi \rho G}}, \quad \delta = \left(\frac{\rho}{\rho_p}\right)^{1/3} \frac{d}{R} \tag{3.4}$$

then, as detailed in Holsapple & Michel (2006), failure will occur when:

$$\frac{1}{6}[(c_x - c_y)^2 + (c_y - c_z)^2 + (c_z - c_x)^2] = s^2[c_x + c_y + c_z]^2 \tag{3.5}$$

where, for arbitrary spin and when the long axis points towards the Earth:

$$\begin{aligned} c_x &= \left(-A_x + \frac{1}{2}\Omega^2 + \frac{4}{3}\delta^{-3} \right), \\ c_y &= \beta^2 \left(-A_y + \frac{1}{2}\Omega^2 - \frac{2}{3}\delta^{-3} \right), \\ c_z &= \alpha^2 \left(-A_z - \frac{2}{3}\delta^{-3} \right) \end{aligned} \quad (3.6)$$

and A_x , A_y and A_z are the components of the self-gravitational potential of a homogeneous ellipsoidal body of uniform mass density ρ in the body coordinate system expressed as: $U = \pi\rho G(A_0 + A_x x^2 + A_y y^2 + A_z z^2)$ (e.g. Chandrasekhar 1969).

Holsapple & Michel (2006) give a similar form when the long axis points along the trajectory at its closest approach. The criterion expressed in these forms can then be used to solve for the dimensionless distances δ at the failure condition as a function of the aspect ratios α and β (which determine the A_x , A_y and A_z), the mass ratio p of the secondary to the primary, and for any value of the constant s related to the angle of friction. The solution always has the dimensionless form:

$$\delta = \frac{d}{R} \left(\frac{\rho}{\rho_p} \right)^{1/3} = F[\alpha, \beta, p, \phi, \Omega] \quad (3.7)$$

so that the mass density only occurs with this cube root. The number of independent variables is reduced by one when the scaled spin is zero or the spin-locked value, and by another one when $p = 0$, i.e. when the mass of the secondary is negligible compared to that of the primary, which is the case for an asteroid flying by a planet.

We also want to stress that the limit distance to the primary corresponds to the distance below which a secondary with a given shape cannot exist, because the failure criterion would be violated. However, it does not mean that below this distance, the secondary would disrupt, and a flow rule is required to indicate the nature of any readjustment (or disruption). Then, if those changes lead to a new configuration that is within failure at the given distance, a shape change is indicated. Otherwise, if the new shape still violates the failure criterion, a global disruption is indicated. Such analysis goes beyond the scope of this paper but is developed in Holsapple & Michel (2006).

4. Application to some real NEOs

In this section, we show how our theory can be used to estimate the tidal limit distance to Earth of some Near-Earth Objects whose required physical properties for this theory have been estimated. In Holsapple & Michel (2006), we used the asteroid Apophis as an example, and estimated its mass densities that might allow a tidal disruption or readjustment during its predicted close approach.

4.1. Limit distance to the Earth of Itokawa

The asteroid (25143) Itokawa was visited by the Japanese space mission Hayabusa during fall 2005. The goal of the Hayabusa mission was to collect some samples of this asteroid and bring them back to Earth. The return of the spacecraft is anticipated in 2010, and whether or not some samples have been collected will only be known at that time. Nevertheless, this mission has provided the first detailed images of such a small asteroid. Fujiwara *et al.* (2006) review the main results of the mission and indicate that the size of Itokawa is 535x294x209 meters and its bulk density is 1.9 ± 0.13 g/cm³, which is lower

than expected for coherent S-type asteroids (usually around 2.7 g/cm^3). Its rotation period of 12.1324 hours is also longer than usual ones for bodies of this size.

Thanks to this mission, we have almost all the necessary parameters to estimate the limit distance to the Earth which Itokawa can approach without undergoing any readjustment or disruption. We set $p = 0$ (Itokawa's mass is negligible with respect to Earth's mass), its aspect ratios are $\alpha = 0.39$ and $\beta = 0.55$. We also set the Earth's bulk density to $\rho_p = 5.515 \text{ g/cm}^3$. Since Itokawa's density is $\rho = 1.9 \text{ g/cm}^3$, the factor $(\rho/\rho_p)^{1/3}$ in the expression of δ (3.7) is equal to 0.701, and we can divide the right hand side of (3.7) to express the distance in term of d/R only. Note that the classical Roche limit (for a fluid and ignoring the fact that Itokawa does not have the required Roche fluid shape) would then equal $d/R = 2.455/0.701 = 3.50$. The only unknown is the angle of friction and we can determine the limit distance that it can approach as a function of this parameter. The results are shown in Figure 2, for the two special orientations at the closest approach: the long axis pointing towards the Earth or the long axis pointing along the trajectory. As expected, the distance decreases for increasing angle of friction. The minimum limit distance corresponds to an angle of friction of 90° and is about 1.4 Earth radii from Earth's center. But, for an angle of friction of 30° , which is typical for rocky bodies, this distance is about 2.5 Earth radii from Earth's center for either orientation, distinctly closer than the classical Roche value.

Note that both solutions approach infinity if the angle of friction is less than 15° , indicating failure at any distance. That is because Itokawa could not have its observed spin even without tidal forces if the angle of friction were less than 15° : if so it would collapse into a more spherical body; see Holsapple (2001). The case with the long axis pointed down actually has near-Earth solutions down to about 7° . That is because at those distances the tidal forces can add additional tensile stresses along the long axis, partially offsetting the excessive compression from self-gravity. However, there is no way that portion of the curve can be accessed, since at further distances where the tidal forces are not yet effective, the asteroid has to have the higher friction angle of 15° .

Also we note that a close approach could change the spin of the asteroid. However, as shown in Holsapple & Michel (2006), the spin has little effect on the closest approach distance as long as it is not close to the limit spin.

Thus, since all currently possible predicted Earth approaches of Itokawa until 2100 are always situated well above those distances (see the NEODys web site), tidal readjustments or disruptions due to Earth approaches are not expected for this object in the near future.

4.2. Limit distance to the Earth of Geographos

The asteroid (1620) Geographos was the target of radar observations in 1994 when the asteroid was 7.2 million kilometers from Earth. A planetary radar instrument at the Deep Space Network's facility in Goldstone was used and the details have been published by Ostro *et al.* (1996). The images show a highly elongated object whose representation by an homogeneous ellipsoid leads to a size approximately $4.7 \times 1.9 \times 1.9 \text{ km}$, as estimated by Hudson & Ostro (1999). Its rotation period is 5.22 hours and it is an S-type Near-Earth asteroid, so that we assume a typical bulk density for non-porous bodies of this taxonomic type, about 2.7 g/cm^3 . Of course, if porous, the density could be less.

We have thus again all the parameters required to repeat the same exercise as for Itokawa.

First, we can determine the required properties at its present state removed from external gravitational forces. With the aspect ratio of 0.35 and the period of 5.22 hours, the required angle of friction to withstand gravitational collapse along the long axis is

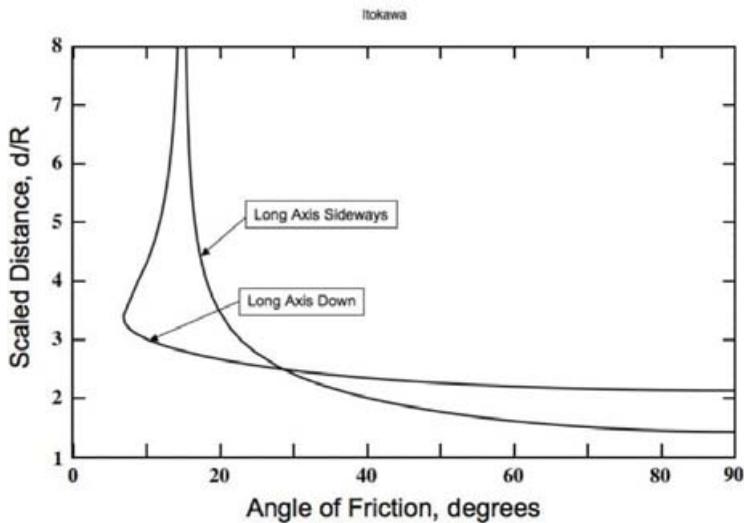


Figure 2. Limit distance (in Earth radii) of Itokawa to the Earth as a function of its angle of friction for two different orientations at closest approach.

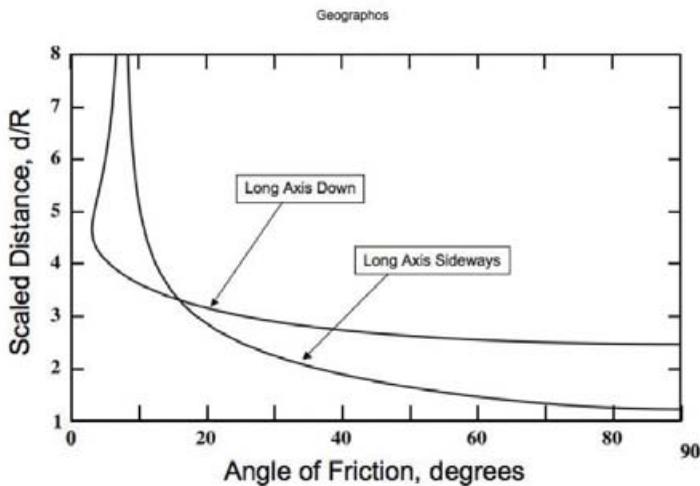


Figure 3. Limit distance (in Earth radii) of Geographos to the Earth as a function of its angle of friction.

7.7°. Note that this is a very different answer than the points plotted for Geographos in Fig. 6 of Holsapple (2001). That is a consequence of using much better dimensions for Geographos: in the region in spin-shape space occupied by this very elongated asteroid, a small difference in shape makes a large difference in required friction angle.

Then we consider a close approach to Earth. The results are shown in Figure 3.

The minimum limit distance corresponds always to an angle of friction of 90° and for the long axis pointing down, is about 2.3 Earth radii from Earth's center. But for an angle of friction of 30°, this distance is about 2.9 Earth radii from Earth's center. For the case of a sideways closest approach, the distance is a mere 1.2 radii, or almost impacting. At the friction angle of 7.7° both curves approach infinity; again that is the free-space value

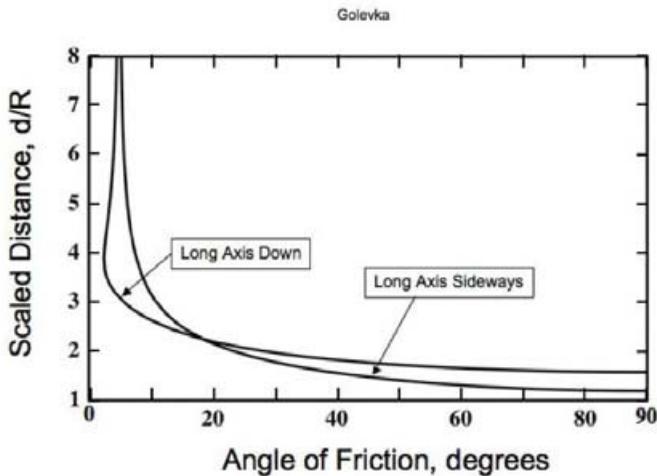


Figure 4. Limit distance (in Earth radii) of Golevka to the Earth as a function of its angle of friction.

required for its spin. Again these values for Geographos rule out any tidal disruption in the near future.

4.3. Limit distance to the Earth of Golevka

Radar ranging from Arecibo (Puerto Rico) has been performed by Chesley *et al.* (2003) for the near-Earth asteroid (6489) Golevka. For the first time, the magnitude of the thermal Yarkovsky effect was measured for a small asteroid. Since this effect is a function of the asteroid's mass and surface thermal characteristics, its direct detection helped constrain the physical properties of Golevka, such as its bulk density, and refine its orbital path. Based on the strength of the detected perturbation, the authors estimated the bulk density of Golevka to be about 2.7 g/cm^3 . Its size is estimated to be about $0.35 \times 0.25 \times 0.25 \text{ km}$ and its rotation period is 6.0264 hours.

So, for Golevka, we can again apply the same exercise as previously. However, it should be noted that this asteroid has a very strange angular shape, and the use of an average ellipsoid certainly introduces some error. In any case, Figure 4 shows the results. For an angle of friction of 30° the asteroid can approach as close as 1.8–2.0 Earth radii from Earth's center, depending on the orientation, suggesting that this asteroid is also safe with respect to tidal disruption by the Earth in the near future.

5. Conclusion and Perspectives

We have developed a theory for cohesionless ellipsoids that gives explicit and exact results for any ellipsoidal shape and combination of gravity, spin, tidal forces, giving closed-form solutions for the limit distances to a planet for such bodies. We find that a body with the expected physical properties of a geological solid for which the cohesion can be ignored can exist in arbitrary ellipsoidal shapes and much closer to a primary than a fluid body. Applying our theory to a few NEOs whose physical properties have been determined, we find that none of these examples should undergo a tidal disruption or readjustment in the near future, as their predicted future Earth approaches are well above the limit distance for such an event.

We want to stress that our theory is static and is therefore limited to the determination of the conditions for the onset of disruption. Thus, the nature of the resulting dynamics of a body if it disintegrates is not addressed by this theory. In particular, the resulting motions are affected by how the body breaks, and the resultant history of the broken particles. For instance, although a body may disrupt once the limit distance is reached, it may reassemble as a result of gravitational reaccumulation into a single piece, so that the final outcome is not the disruption that was predicted by our theory. Some indications of such behaviors are given by Walsh & Richardson (2006) and we also plan to address this problem in future work. Our next step in the theory will finally be to consider small bodies for which the cohesion cannot be neglected, as has been done recently by Holsapple (2006) for the case of pure spin limits.

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