# Correspondence

#### DEAR EDITOR,

I was pleasantly surprised to stumble across yet another example of the same discovery made independently in different parts of the globe: The divisibility test for 19 given in the November 1998 issue of the *Mathematical Gazette* by Humphreys and Macharia was also offered, as part of a more general result, by a high school student in India, Apoorva Khare, in 'Divisibility Tests' by A. Khare, *Furman University Electronic Journal of Undergraduate Mathematics*, Volume 3, 1997, pp 1-5. That is not to take anything away from the H-M article, which I found explained the special case more usefully.

Yours sincerely,

## **DINO SURENDRAN**

University of Zimbabwe, Harare, Zimbabwe

DEAR EDITOR,

In a recent note entitled 'The convergence of a Lucas series' [Math. Gaz. 83 (July 1999) pp. 273-274], T. Koshy undertook to prove that, for integral  $k \ge 2$ , the ratio  $(2k - 1)/(k^2 - k - 1)$  is integral if, and only if, k = 2 or k = 3. Koshy's approach was unnecessarily involved. Here is a more direct proof of this result.

For  $k \ge 2$ ,  $(2k - 1)/(k^2 - k - 1)$  is positive. Furthermore, this ratio can only be integral if  $(k^2 - k - 1) \le (2k - 1)$ , i.e. if  $k(k - 3) \le 0$ . The only solutions for integral  $k \ge 2$  are then obviously k = 2, 3.

Yours sincerely,

## N. GAUTHIER

# Department of Physics, Royal Military College of Canada, Kingston, Ontario, Canada

#### DEAR EDITOR,

With regard to the article [1] by K. Robin McLean, an interesting variation on 'Diffy' is to play 'Quiffy', in which one finds the larger quotient of each number with its successor in the cycle, ending with a cycle of ones e.g.

4	7	2	1
7/4	7/2	2	4
2	7/4	2	16/7
8/7	8/7	8/7	8/7
1	1	1	1

The reason that the process works is simple, since, if we take logarithms of each number, we are then playing 'Diffy' (unsigned differences) and reaching a cycle of zeros ( $\ln 1 = 0$ ).

An interesting sidelight is a very simple proof that, if the cycle of four positive numbers is not equivalent to a purely increasing sequence, the process terminates after at most six steps. (Cycles are equivalent when cycled, reversed, multiplied by a constant, or raised to a power.)