Why Galaxies Care About AGB Stars:
A Continuing Challenge through Cosmic Time
Proceedings IAU Symposium No. 343, 2019
F. Kerschbaum, M. Groenewegen & H. Olofsson, eds.

doi:10.1017/S1743921318005033

On cylindrically symmetric solutions of polarized radiative transfer equation

Juris Freimanis

Engineering Research Institute "Ventspils International Radio Astronomy Centre",
Ventspils University College, Inzenieru iela 101, Ventspils, LV-3600, Latvia
email: jurisf@venta.lv

Abstract. While the observed polarization maps of spatially resolved post-AGB objects usually require numerical modelling of radiative transfer, it is useful to have known analytical solutions of the polarized radiative transfer equation (PRTE) as benchmarks of computer codes in the simplest model cases. We consider two such solutions: cylindrically symmetric Green's function for an infinite medium and cylindrically symmetric inner eigenfunctions of PRTE.

Keywords. Radiative transfer, polarization, methods: analytical

1. Introduction

Strongly polarized radiation, with high spatial resolution mapping over the surface of often asymmetric gas-dust envelopes of AGB stars and post-AGB objects, has been observed in visible light (Ohnaka *et al.* 2017) and in the near infrared (Su *et al.* 2003). Physical interpretation of such observations includes numerical solution of the polarized radiative transfer equation (PRTE); however, it is desirable to test the numerical computer codes against the known analytic solutions of PRTE in the simplest model cases.

Here we review two options for an analytic or semi-analytic solution of PRTE in cylindrical symmetry, namely, Greens function for an infinite medium (Freimanis 2009) and cylindrically symmetric inner eigenfunctions.

2. The physical conditions and radiative transfer equation

Let us assume that the multiply scattering (polydisperse, dusty) medium within Euclidean space, and radiation field in it, obeys the following physical conditions:

• The conditions of validity of the radiative transfer equation in a polydisperse medium (Mishchenko, Travis & Lacis (2006)) are fulfilled;

• The medium is statistically homogeneous, isotropic and stationary. It is either the whole space or an infinitely long cylinder with nonreflecting surface;

• The effective refractive index of the medium is independent of polarization, and there is no circular birefringence nor circular dichroism;

• The radiation field is stationary and cylindrically symmetric, with the same symmetry axis as that of the medium.

Let us introduce a standard spatial cylindrical coordinate system $(\tau \equiv \alpha r, \Phi, \zeta)$, with α being the scalar extinction coefficient, and let us characterize the direction of propagation of radiation by standard spherical angles $(\vartheta = \cos^{-1} \mu, \varphi)$, with polar axis in the radial direction. Let us define the plane going through the radial direction and the direction of propagation of radiation as the linear polarization reference plane.

Describing Stokes vector $\mathbf{I}(\tau,\mu,\varphi)$ in circular polarization representation (Mishchenko 2006), the polarized radiative transfer equation is (Freimanis 2014)

$$\mu\frac{\partial\mathbf{I}(\tau;\mu,\varphi)}{\partial\tau}+\frac{(1-\mu^2)\sin^2\varphi}{\tau}\frac{\partial\mathbf{I}(\tau;\mu,\varphi)}{\partial\mu}-\frac{\mu\sin 2\varphi}{2\tau}\frac{\partial\mathbf{I}(\tau;\mu,\varphi)}{\partial\varphi}$$
$$+\frac{\sin 2\varphi}{2\tau}\mathbf{U}\mathbf{I}(\tau;\mu,\varphi)=-\mathbf{I}(\tau;\mu,\varphi)+\mathbf{B}_0(\tau;\mu,\varphi)$$
$$+\frac{1}{4\pi}\int_0^{2\pi}\sum_{s=-\infty}^{+\infty}\mathrm{e}^{-is(\varphi-\varphi')}d\varphi'\int_{-1}^{1}\mathbf{p}_s(\mu,\mu')\mathbf{I}(\tau;\mu',\varphi')d\mu', \tag{1}$$

where $\mathbf{U}$ is a constant diagonal 4×4 matrix, 4×4-matrices $\mathbf{p}_s(\mu,\mu')$ describe the scattering in the medium, and 4-vector $\mathbf{B}_0(\tau;\mu,\varphi)$ is the primary source function.

3. Green's function for infinite medium

If the medium is infinite, then the solution of equation (1) is given by its Greens function (Freimanis 2009). Its main advantage is that it is mathematically exact. Its main drawbacks are: i) it is applicable only for a homogeneous infinite medium, ii) it is extremely complex and hardly suitable for practical computations.

4. Cylindrically symmetric eigenfunctions

Cylindrically symmetric eigenfunctions of the homogeneous version of equation (1) were found heuristically earlier (Freimanis 2014), but without strict proof that they are eigenfunctions indeed. Now this has been proved, and the proof will be presented in Freimanis (2018). There are convergent inner eigenfunctions bounded in the vicinity of the cylindrical symmetry axis, and very strongly divergent outer eigenfunctions; more treatable expressions are to be found for them. We propose to use inner eigenfunctions for semi-analytic radiative transfer modelling in homogeneous cylinders of infinite length but finite radius, illuminated from outside. The advantage of this approach is that eigenfunctions are mathematically much simpler than the Green's function. The drawback of this method is that neither some orthogonality properties of eigenfunctions, nor the completeness of such eigenfunctions in some Banach space has been proved.

Acknowledgements

This work and participation of the author at IAUS 343 were financed by the ERDF project No. 1.1.1.1/16/A/213. Cofinancing by the basic budget of Ventspils International Radio Astronomy Centre and by Ventspils City Council was received as well. The author expresses his gratitude to all these entities.

References

Freimanis, J. 2009, *Journal of Quantitative Spectroscopy & Radiative Transfer*, 110, 1307
Freimanis, J. 2014, in: H.-W. Lee, Y.W. Kang & K.-C. Leung (eds.), *ASP-CS*, 482, p. 265
Freimanis, J., 2018, *Journal of Quantitative Spectroscopy & Radiative Transfer*, in preparation
Mishchenko, M.I., Travis, L.D., & Lacis, A.A. 2006, *Multiple Scattering of Light by Particles. Radiative Transfer and Coherent Backscattering.* Cambridge *et al.*, Cambridge University Press, 478 pp.
Ohnaka, K., Weigelt, G., & Hofmann, K.-H. 2017, *A&A*, 597, A20
Su, K.Y.L., Hrivnak, B.J., Kwok, S., & Sahai, R. 2003, *AJ*, 126, 848

https://doi.org/10.1017/S1743921318005033 Published online by Cambridge University Press