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BOOK REVIEWS

KIRK, W. A. AND SIMS, B. (EDS) *Handbook of metric fixed point theory* (Kluwer Academic Publishers, 2001), xii+703 pp., 0 7923 7073 2 (hardback), £135.

This handbook consists of 19 chapters, which are devoted to the various branches of metric fixed point theory.

The contents are as follows.

This is a marvellous handbook. It is well written, carefully planned, and contains fascinating material. The authors of the chapters are without exception excellent specialists in the topics discussed. Each chapter includes a glossary that provides succinct definitions of most of the terms from that chapter. Individual topics are covered in sections. Each chapter of the book includes a list of references.

The handbook will be a useful source for anyone who is interested in mathematical analysis and, in particular, for those researchers whose primary interest is in metric fixed point theory and its applications. It may not be a book which will be read from cover to cover, but it is very likely that both students and researchers in metric fixed point theory will find the particular information they seek in this detailed and self-contained exposition.

The handbook is an incredible achievement. The editors should certainly be congratulated for managing to organize the work of so many different individuals into such a coherent and usable structure.

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HERTLING, CLAUS *Frobenius manifolds and moduli spaces for singularities* (Cambridge University Press, 2002), 280 pp., 0 521 81296 8 (hardback), £45 (US$60).

Frobenius manifolds, created by Dubrovin in 1991 from rich theoretical physics material, have been found since in many different fragments of mathematics—quantum cohomology and mirror symmetry, complex geometry, symplectic geometry, singularity theory, integrable systems—raising hopes for unifying them into one picture. It also became clear that the notion of Frobenius manifold is not broad enough to cover *all* objects of the associated working categories; say, on the B-side of the mirror symmetry it applies only to extended moduli spaces of *Calabi–Yau* manifolds, the latter forming a rather small subcategory of the category of complex manifolds. In 1998, Hertling and Manin introduced a weaker notion of F-*manifold*, which is, by definition, a pair (M,μ) consisting of a smooth supermanifold M and a smooth $\mathcal{O}_M$-linear associative graded commutative multiplication on the tangent sheaf, $\mu:\otimes^2_{\mathcal{O}_M}\mathcal{T}_M\to\mathcal{T}_M$, satisfying the integrability condition

$$[\mu,\mu]=0,$$

where the 'bracket', $[\mu,\mu]:\otimes^4_{\mathcal{O}_M}\mathcal{T}_M\to\mathcal{T}_M$, is given explicitly by

$$\begin{aligned}[\mu,\mu](X,Y,Z,W) := {} & [\mu(X,Y),\mu(Z,W)] - \mu([\mu(X,Y),Z],W) \\ & - (-1)^{(|X|+|Y|)|Z|}\mu(Z,[\mu(X,Y),W]) - \mu(X,[Y,\mu(Z,W)]) \\ & - \mu(X,[Y,\mu(Z,W)]) - (-1)^{|Y|(|Z|+|W|)}\mu[X,\mu(Z,W)],Y) \\ & + (-1)^{|Y||Z|}\mu(X,\mu(Z,[Y,W])) + \mu(X,\mu([Y,Z],W)) \\ & + (-1)^{|Y||Z|}\mu([X,Z],\mu(Y,W)) + (-1)^{|W|(|Y|+|Z|)}\mu([X,W],\mu(Y,Z)).\end{aligned}$$

A non-trivial part of the above definition is an implicit assertion that $[\mu,\mu]$ is a tensor, i.e. $\mathcal{O}_M$-polylinear in all four inputs. It is here where the assumption that μ is both graded, commutative and associative plays a key role.

The book under review gives a very detailed analysis of the category of F-manifolds. In particular, it is shown that any Frobenius manifold is an F-manifold; any F-manifold with semi-simple product μ can be made into a Frobenius manifold. Most importantly, the author gives a careful and rigorous survey of how F-manifolds turn up naturally in singularity theory.

The style of the book is not impressive. Sometimes the author struggles with his English. Nevertheless, the book is clean, rigorous and readable. The researchers in the areas of singularity theory, complex geometry, integrable systems, quantum cohomology, mirror symmetry and