



RESEARCH ARTICLE

Compass adjustment by GPS (or any other GNSS receiver) and a single visual reference

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Abstract

This paper proposes a proper compass adjustment method using only a GPS (or any other GNSS receiver) and a single visual reference to enhance the efficiency of compass adjustment. During compass adjustment, the ship proceeds on magnetic courses using a gyroscopic or satellite compass and considering magnetic declination. However, non-magnetic compasses are only compulsory for ships of 500 gross tonnage or upwards (SOLAS V/19.2.5.1). Many ships of less than 500 gross tonnage have only a magnetic compass to indicate heading. In these cases, a minimum of five leading lines or a minimum of five bearings of conspicuous and distant points or sun azimuths are necessary to adjust the compass. This makes compass adjustment more laborious and time consuming. To expedite this process, a reliable and practical method was developed to use the courses over ground given by a GNSS receiver and a single visual reference instead of the headings provided by a gyroscopic or satellite compass. The method is valid for all ships, but is primarily intended for those equipped with only a magnetic compass to indicate heading.

1. Introduction

The objective of this paper is to propose a proper compass adjustment method using only a GNSS receiver and a single visual reference for ships equipped with only a magnetic compass to indicate heading.

Compass adjustment is required for the correct operation of the magnetic compass, namely the first nautical equipment mentioned in SOLAS V/19. For many years, the process of compass adjustment has remained stagnant. However, with the emergence of new technologies, magnetic compass applications and adjustment techniques have again become subjects of research. Several works focus on the improvement of magnetic compass performance. Recently, Androjna et al. published a compendium on the current use of the magnetic compass (Androjna et al., 2021). Other authors have taken a closer look at specific items. For example, Felski applies the least squares method to determine residual deviations (Felski, 1999); Basterretxea updates the residual deviations according to latitude (Basterretxea Iribar et al., 2014); Martínez-Lozares obtains the deviations in real time (Martínez-Lozares, 2009a, 2009b) and Lushnikov updates the table of residual deviations for any single course (Lushnikov, 2011). The present paper follows this line of research by tackling the efficiency of compass adjustment.

The paper is divided into seven sections. Section 2 explains compass adjustment on ships of less than 500 gross tonnage. In Sections 3–5, the proposed method is developed, discussed and verified, respectively, while in Section 6 the proposed method is completed by applying Lushnikov's method (Lushnikov, 2011). Section 7 describes the application of the complete method. Finally, conclusions are drawn in Section 8.

2. Introduction to compass adjustment

The process of compass adjustment has two phases: actual compass adjustment or compensation and creation of the table of residual deviations (deviation table).

2.1. Deviation equation

The deviation equation commonly applied is

$$\Delta = A + B \cdot \sin \zeta' + C \cdot \cos \zeta' + D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta', \quad (1)$$

where Δ is the deviation, ζ' is the compass course (ζ indicates the magnetic course, see Subsection 2.3 and Section 3), and A , B , C , D and E are the approximate coefficients, with the exact coefficients being the sines of the approximate ones (Gaztelu-Iturri Leicea, 1999; National Geospatial-Intelligence Agency, 2004).

Course deviation comprises three parts: constant deviation, A , which does not depend on the course; semicircular deviation, $B \cdot \sin \zeta' + C \cdot \cos \zeta'$, which depends on the course; and quadrantal deviation, $D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta'$, which depends on twice the course. The second and the third are called semicircular and quadrantal deviations because they are repeated with a different sign every 180° and 90° , respectively, where 180° corresponds to half a circle (semicircle) and 90° to a quarter of a circle (quadrant).

Semicircular deviation depends mainly on the ship's hard iron, which has permanent magnetism and is corrected with magnets. On the other hand, constant and quadrantal deviations depend solely on the ship's soft iron, which does not have permanent magnetism, but is induced according to its orientation within the earth's magnetic field.

Considering $\zeta' = 0^\circ, 90^\circ, 180^\circ$ and 270° , the expressions of the deviations on the cardinal courses are obtained as

$$\Delta_n = A + C + E \quad (2)$$

$$\Delta_e = A + B - E \quad (3)$$

$$\Delta_s = A - C + E \quad (4)$$

$$\Delta_w = A - B - E. \quad (5)$$

Therefore, these deviations depend on the constant (coefficient A), semicircular (coefficients B , C) and part of the quadrantal (coefficient E) deviation, with the semicircular one being the main deviation.

2.2. Compensating device

Semicircular deviation of magnetic compasses on ships of less than 500 gross tonnage can be compensated in two ways. Many compasses have a mechanism that adjusts the position of longitudinal and transversal magnets by using an anti-magnetic screwdriver to turn one screw for longitudinal magnets and one for transversal ones. If the compass does not have this device, the magnets must be stuck directly on the compass or in its vicinity.

Quadrantal deviation can be compensated using soft iron correctors, such as small spheres or cylinders, or boxes where several soft iron plates can be placed. However, as this is not a common practice, this paper does not consider the compensation of this deviation. Note, however, that the effect of the quadrantal and constant deviations is always included in the residual deviations.

A common compensating device consists of one (or two) longitudinal and one (or two) transversal magnets that can rotate vertically around their centres, as shown in Figure 1. Note that the longitudinal magnets are inside the transversal rotating cylinder and the transversal magnets are inside the longitudinal rotating cylinder.

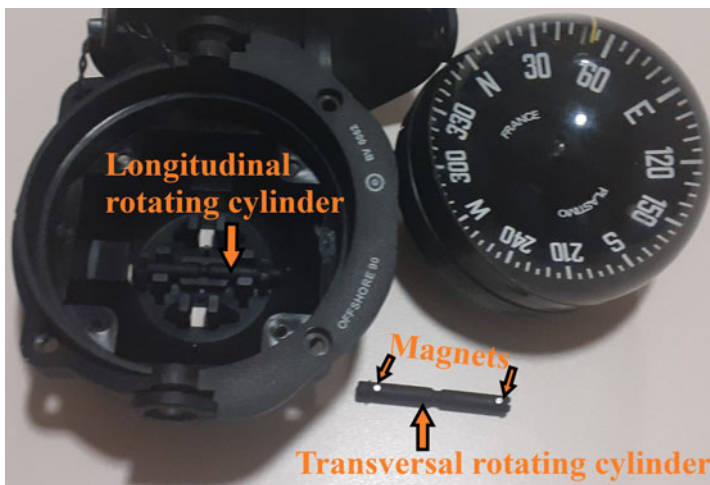


Figure 1. Example of compensating device.

The horizontal component of the magnetic moment of the magnets is used to adjust the compass. If a magnet is completely vertical, the horizontal component (longitudinal or transversal, depending on the type of magnet) of its magnetic moment is zero. If it is completely horizontal, the horizontal component is equal to its own magnetic moment, with a polarity that can be changed by turning the magnet 180°. On the other hand, if the magnet is fitted at a vertical angle of less than 90°, the horizontal component is smaller than its own magnetic moment and smaller the larger the angle.

2.3. Traditional method of compensation

Compensation is typically accomplished by proceeding on the four cardinal magnetic headings, a manoeuvre known as *swing* (Gaztelu-Iturri Leicea, 1999; National Geospatial–Intelligence Agency, 2004). If the ship is equipped with a gyroscopic or satellite compass, a magnetic heading is followed by keeping the corresponding true course, TC (i.e. $TC = \zeta + \delta$, where ζ is the magnetic course and δ is the magnetic declination).

On the east (or west) magnetic heading, the deviation is nullified by setting $\zeta' = 90^\circ$ (or 270°) with longitudinal magnets because they are perpendicular to the earth’s magnetic field and can alter the compass course, while transversal magnets are in the same direction as the earth’s magnetic field and cannot therefore alter the compass course. Next, on the north (or south) magnetic heading, the deviation is also nullified by setting $\zeta' = 0^\circ$ (or 180°) but with transversal magnets, which are perpendicular to the earth’s magnetic field. The effect of the ship’s hard iron (coefficients B and C) is thus minimised but not completely eliminated because the deviations on the cardinal courses also depend on the constant (coefficient A) and part of the quadrantal deviation (coefficient E) (see Subsection 2.1). Consequently, residual magnetic effects remain after the compensation, i.e.

$$(3) \Rightarrow \Delta e = 0 = A + B' - E \tag{6}$$

or

$$(5) \Rightarrow \Delta w = 0 = A - B' - E \tag{6 bis}$$

$$(2) \Rightarrow \Delta n = 0 = A + C' + E \tag{7}$$

or

$$(4) \Rightarrow \Delta s = 0 = A - C' + E, \tag{7 bis}$$

where B' and C' are the new coefficients corresponding to the hard iron and smaller than the original coefficients B and C .

Next, we continue the swing. On the west (or east) magnetic heading, we have

$$(5) \Rightarrow \Delta w = A - B' - E \tag{8}$$

or

$$(3) \Rightarrow \Delta e = A + B' - E, \tag{8 bis}$$

and on the south (or north) magnetic heading, we have

$$(4) \Rightarrow \Delta s = A - C' + E \tag{9}$$

or

$$(2) \Rightarrow \Delta n = A + C' + E. \tag{9 bis}$$

Consequently,

$$(6)-(8) \text{ or } (6 \text{ bis})-(8 \text{ bis}) \Rightarrow \Delta e - \Delta w = 2B' \Rightarrow B' = \frac{1}{2}(\Delta e - \Delta w) \tag{10}$$

$$(7)-(9) \text{ or } (7 \text{ bis})-(9 \text{ bis}) \Rightarrow \Delta n - \Delta s = 2C' \Rightarrow C' = \frac{1}{2}(\Delta n - \Delta s). \tag{11}$$

Assuming that Δe (or Δw) and Δn (or Δs) are exactly zero, expressions (10) and (11) show that only half the deviations on the west (or east) and on the south (or north) must be nullified with the longitudinal and transversal magnets, respectively, to eliminate coefficients B' and C' .

2.4. Residual deviations

From the previous expressions, we obtain

$$(6) + (8) \text{ or } (6 \text{ bis}) + (8 \text{ bis}) \Rightarrow \Delta e + \Delta w = 2A - 2E \tag{12}$$

$$(7) + (9) \text{ or } (7 \text{ bis}) + (9 \text{ bis}) \Rightarrow \Delta n + \Delta s = 2A + 2E. \tag{13}$$

Hence,

$$(12) + (13) \Rightarrow A = \frac{1}{4}(\Delta n + \Delta e + \Delta s + \Delta w) \tag{14}$$

$$(13)-(12) \Rightarrow E = \frac{1}{4}(\Delta n + \Delta s - \Delta e - \Delta w). \tag{15}$$

Thus, expressions (14) and (15) give coefficients A and E , respectively.

If half the deviations on the west (or east) and on the south (or north) are not nullified, expressions (10) and (11) give coefficients B' and C' , which are residual coefficients B and C .

By contrast, if half the deviations on the west (or east) and on the south (or north) are nullified or minimised, the coefficients corresponding to the hard iron are altered again, nullifying or minimising coefficients B' and C' . In this case, new deviations and new coefficients are obtained, i.e.

$$\Delta'w = A - B'' - E \tag{16}$$

or

$$\Delta'e = A + B'' - E \tag{16 bis}$$

$$\Delta's = A - C'' + E \tag{17}$$

or

$$\Delta'n = A + C'' + E, \quad (17 \text{ bis})$$

where B'' and C'' are zero or have very small values.

The magnets alter coefficients B and C but not the other coefficients. Hence, expressions (14) and (15) are valid to determine coefficients A and E . Once coefficients A and E , and deviations $\Delta'w$ (or $\Delta'e$) and $\Delta's$ (or $\Delta'n$), are known, coefficient B'' is calculated from expression (16) or (16 bis) and coefficient C'' is analogously calculated from expression (17) or (17 bis).

Thus, residual coefficients B and C are B' and C' or B'' and C'' depending on whether the deviations on the third and fourth courses of the swing, i.e. west (or east) and south (or north), are altered.

We now know coefficients A , B , C and E but not coefficient D . To obtain coefficient D , it is necessary to complete the swing on a fifth course, which must be a quadrantal one, i.e. NE, SE, SW or NW, with the deviation equations (1) for these courses ($\zeta' = 45^\circ, 135^\circ, 225^\circ$ and 315°) being

$$\Delta ne = A + B \cdot 0 \cdot 707 + C \cdot 0 \cdot 707 + D \quad (18)$$

$$\Delta se = A + B \cdot 0 \cdot 707 - C \cdot 0 \cdot 707 - D \quad (19)$$

$$\Delta sw = A - B \cdot 0 \cdot 707 - C \cdot 0 \cdot 707 + D \quad (20)$$

$$\Delta nw = A - B \cdot 0 \cdot 707 + C \cdot 0 \cdot 707 - D \quad (21)$$

where $0 \cdot 707$ is a sufficient approximation of the sine and cosine of 45° . Note that $\sin 2\zeta'$ is always ± 1 and $\cos 2\zeta'$ is always zero. For this reason, coefficient E does not appear in the deviations on the quadrantal courses.

Expression (18), (19), (20) or (21) is used to obtain coefficient D , depending on which the fifth course is.

Once all coefficients, A , B , C , D and E , are known, the deviation on different compass courses, typically each 10 or 15 degrees from the north, is calculated by applying the deviation equation with a spreadsheet. Finally, the obtained deviation table is attached to the certificate of compass adjustment, in compliance with SOLAS V/19.2.1.3; IMO Resolution A.382 (X), Annex I.3; ISO Standard 25862:2019, Annex G.7 and the corresponding national regulations (IMO, 1977; ISO, 2019). According to ISO Standard 25862:2019, Annex G.1, the deviation on any course must not exceed 4° for ships of a length less than $82 \cdot 5$ m.

3. Approach and development of the method

The method aims to determine coefficients A , B , C , D and E of the deviation equation (1) by comparing the compass courses with the courses over ground (COGs), indicated by a GNSS receiver. It is based on type of courses and triangle of speeds, as shown in Figure 2. References TN, MN and CN correspond to the true, magnetic and compass north (i.e. the origins of the true course, TC; magnetic course, ζ ; and compass course, ζ' , respectively, where δ is the magnetic declination, Δ the deviation, and S the vessel's speed through the water). The parameters of the external forces are set, α , and drift, d , where set is expressed as a magnetic course, and β is the course difference due to the external forces, $\beta = \text{COG} - \text{TC}$ (Moncunill Marimón et al., 2020).

3.1. Deviation equation referred to course over ground

According to Figure 2, by the law of sines (Moncunill Marimón et al., 2020),

$$\frac{S}{\sin \gamma} = \frac{d}{\sin \beta} \Rightarrow \sin \beta = \frac{d}{S} \cdot \sin \gamma.$$

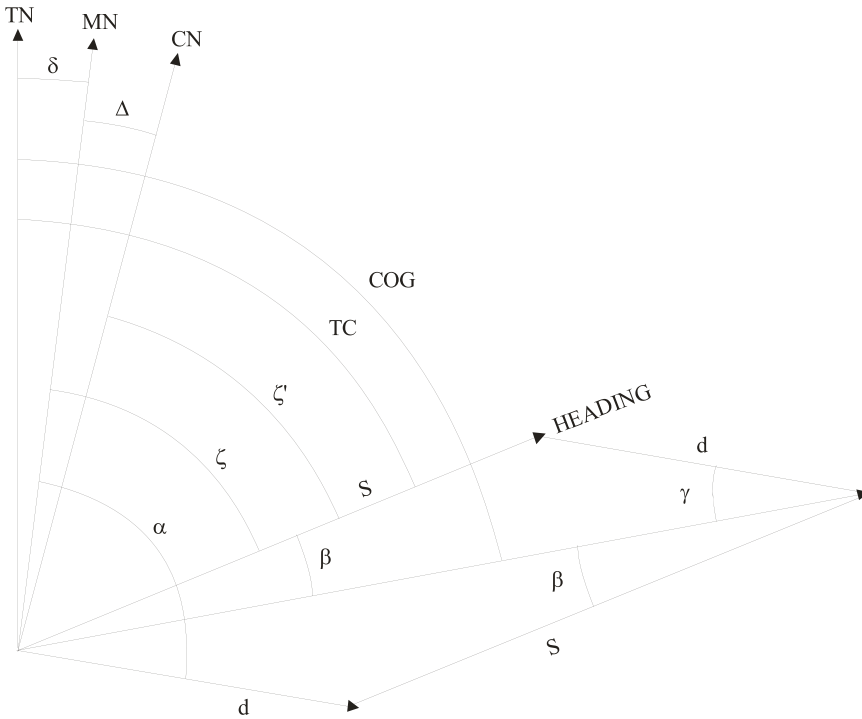


Figure 2. Types of courses and triangle of speeds.

But $\beta + \gamma = \alpha - \zeta$ since they are opposite angles of a parallelogram. Then,

$$\sin \beta = \frac{d}{S} \cdot \sin(\alpha - \zeta - \beta).$$

Developing,

$$\sin \beta = \frac{d}{S} \cdot \sin(\alpha - \zeta) \cdot \cos \beta - \frac{d}{S} \cdot \cos(\alpha - \zeta) \cdot \sin \beta$$

$$\sin \beta \cdot \left[1 + \frac{d}{S} \cdot \cos(\alpha - \zeta) \right] = \frac{d}{S} \cdot \sin(\alpha - \zeta) \cdot \cos \beta \Rightarrow \tan \beta = \frac{\frac{d}{S} \cdot \sin(\alpha - \zeta)}{1 + \frac{d}{S} \cdot \cos(\alpha - \zeta)}.$$

Given that $(a + b) \cdot (a - b) = a^2 - b^2$, by multiplying the numerator and denominator by $1 - d/S \cdot \cos(\alpha - \zeta)$, we obtain

$$\tan \beta = \frac{\frac{d}{S} \cdot \sin(\alpha - \zeta) \cdot \left[1 - \frac{d}{S} \cdot \cos(\alpha - \zeta) \right]}{1 - \frac{d^2}{S^2} \cdot \cos^2(\alpha - \zeta)}.$$

Because d^2 is much smaller than S^2 , the denominator can be considered 1. Also, since β is a small angle, its tangent can be replaced by its sine, which in turn can be replaced by $\beta \cdot \sin 1^\circ$. Thus,

$$\beta \cdot \sin 1^\circ = \frac{d}{S} \cdot \sin(\alpha - \zeta) - \frac{d}{S} \cdot \sin(\alpha - \zeta) \cdot \frac{d}{S} \cdot \cos(\alpha - \zeta)$$

$$\beta = \frac{d}{S} \cdot \csc 1^\circ \cdot \sin(\alpha - \zeta) - \frac{d}{S} \cdot \csc 1^\circ \cdot \sin(\alpha - \zeta) \cdot \frac{d}{S} \cdot \csc 1^\circ \cdot \cos(\alpha - \zeta) \cdot \sin 1^\circ,$$

where

$$\begin{aligned} \sin(\alpha - \zeta) &= \sin \alpha \cdot \cos \zeta - \cos \alpha \cdot \sin \zeta \\ \cos(\alpha - \zeta) &= \cos \alpha \cdot \cos \zeta + \sin \alpha \cdot \sin \zeta . \end{aligned}$$

Let

$$\left. \begin{aligned} x &= \frac{d}{S} \cdot \csc 1^\circ \cdot \cos \alpha \\ y &= \frac{d}{S} \cdot \csc 1^\circ \cdot \sin \alpha \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} \frac{d}{S} \cdot \csc 1^\circ \cdot \sin(\alpha - \zeta) &= -x \cdot \sin \zeta + y \cdot \cos \zeta \\ \frac{d}{S} \cdot \csc 1^\circ \cdot \cos(\alpha - \zeta) &= x \cdot \cos \zeta + y \cdot \sin \zeta . \end{aligned} \right.$$

Consequently,

$$\begin{aligned} \frac{d}{S} \cdot \csc 1^\circ \cdot \sin(\alpha - \zeta) \cdot \frac{d}{S} \cdot \csc 1^\circ \cdot \cos(\alpha - \zeta) &= (-x \cdot \sin \zeta + y \cdot \cos \zeta) \cdot (x \cdot \cos \zeta + y \cdot \sin \zeta) \\ &= -x^2 \cdot \sin \zeta \cdot \cos \zeta - x \cdot y \cdot \sin^2 \zeta + x \cdot y \cdot \cos^2 \zeta \\ &\quad + y^2 \cdot \sin \zeta \cdot \cos \zeta \\ &= x \cdot y \cdot \cos 2\zeta - \frac{1}{2}(x^2 - y^2) \cdot \sin 2\zeta \end{aligned}$$

and

$$\beta = -x \cdot \sin \zeta + y \cdot \cos \zeta - x \cdot y \cdot \sin 1^\circ \cdot \cos 2\zeta + \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ \cdot \sin 2\zeta .$$

Since the magnetic and compass courses are similar,

$$\beta = -x \cdot \sin \zeta' + y \cdot \cos \zeta' - x \cdot y \cdot \sin 1^\circ \cdot \cos 2\zeta' + \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ \cdot \sin 2\zeta' .$$

The deviation is the difference between the magnetic course and the compass course, i.e. $\Delta = \zeta - \zeta'$. On the other hand, the magnetic course is the difference between the true course and the magnetic declination, i.e. $\zeta = \text{TC} - \delta$. Hence, $\Delta = \text{TC} - \delta - \zeta'$, and TC is the difference between COG and β , i.e. $\text{TC} = \text{COG} - \beta$. Therefore,

$$\Delta = \text{COG} - \beta - \delta - \zeta' ,$$

and the deviation equation (1) is

$$\Delta = A + B \cdot \sin \zeta' + C \cdot \cos \zeta' + D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta' .$$

Thus,

$$\text{COG} - \beta - \delta - \zeta' = A + B \cdot \sin \zeta' + C \cdot \cos \zeta' + D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta' .$$

Now let the pseudo-deviation, Ψ , be defined as the difference between the COG and the compass course, i.e. $\Psi = \text{COG} - \zeta'$. Then,

$$\begin{aligned} \Psi &= A + B \cdot \sin \zeta' + C \cdot \cos \zeta' + D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta' + \beta + \delta \\ \Psi &= A + B \cdot \sin \zeta' + C \cdot \cos \zeta' + D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta' - x \cdot \sin \zeta' \\ &\quad + y \cdot \cos \zeta' - x \cdot y \cdot \sin 1^\circ \cdot \cos 2\zeta' + \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ \cdot \sin 2\zeta' + \delta . \end{aligned} \tag{22}$$

3.2. Compensation and calculation of coefficients of deviation equation

Particularising expression (22) for the four cardinal compass courses (Moncunill Marimón et al., 2020), we obtain

$$\Psi_n = A + C + E + y - x \cdot y \cdot \sin 1^\circ + \delta \tag{23}$$

$$\Psi_e = A + B - E - x + x \cdot y \cdot \sin 1^\circ + \delta \tag{24}$$

$$\Psi_s = A - C + E - y - x \cdot y \cdot \sin 1^\circ + \delta \tag{25}$$

$$\Psi_w = A - B - E + x + x \cdot y \cdot \sin 1^\circ + \delta \tag{26}$$

If we apply the traditional method of compensation (see Subsection 2.3) but consider the COGs provided by a GNSS receiver instead of the true courses provided by a gyroscopic or satellite compass, the first COGs must be $90^\circ + \delta$ (or $270^\circ + \delta$) and $0^\circ + \delta$ (or $180^\circ + \delta$). Then, when the ship proceeds on these COGs, the compass course must be altered by the longitudinal and transversal magnets to obtain the following compass courses, respectively: 90° (or 270°) and 0° (or 180°), resulting in $\Psi_e = \delta$ (or $\Psi_w = \delta$) and $\Psi_n = \delta$ (or $\Psi_s = \delta$). Thus,

$$(24) \Rightarrow \Psi_e = \delta = A + B' - E - x + x \cdot y \cdot \sin 1^\circ + \delta \text{ (or analogously for } \Psi_w) \tag{27}$$

$$(23) \Rightarrow \Psi_n = \delta = A + C' + E + y - x \cdot y \cdot \sin 1^\circ + \delta \text{ (or analogously for } \Psi_s) \tag{28}$$

Next, we continue the swing on the other cardinal courses but by steering the ship on compass courses, which are easier to handle than are COGs, and we observe the corresponding COGs to obtain the pseudo-deviations. Then, we have

$$(26) \Rightarrow \Psi_w = A - B' - E + x + x \cdot y \cdot \sin 1^\circ + \delta \text{ (or analogously for } \Psi_e) \tag{29}$$

$$(25) \Rightarrow \Psi_s = A - C' + E - y - x \cdot y \cdot \sin 1^\circ + \delta \text{ (or analogously for } \Psi_n) \tag{30}$$

Consequently,

$$(27)-(29) \Rightarrow B' = \frac{1}{2}(\Psi_e - \Psi_w) + x \tag{31}$$

$$(28)-(30) \Rightarrow C' = \frac{1}{2}(\Psi_n - \Psi_s) - y \tag{32}$$

$$(27) + (29) \Rightarrow \Psi_e + \Psi_w = 2A - 2E + 2 \cdot x \cdot y \cdot \sin 1^\circ + 2\delta \tag{33}$$

$$(28) + (30) \Rightarrow \Psi_n + \Psi_s = 2A + 2E - 2 \cdot x \cdot y \cdot \sin 1^\circ + 2\delta \tag{34}$$

$$(33) + (34) \Rightarrow A = \frac{1}{4}(\Psi_n + \Psi_e + \Psi_s + \Psi_w) - \delta \tag{35}$$

$$(34)-(33) \Rightarrow E = \frac{1}{4}(\Psi_n + \Psi_s - \Psi_e - \Psi_w) + x \cdot y \cdot \sin 1^\circ. \tag{36}$$

Expressions (31), (32), (35) and (36) give coefficients B' , C' , A and E , respectively. Coefficient A depends solely on the pseudo-deviations and the magnetic declination, which are known data. The other coefficients depend on the pseudo-deviations, which are known data, but also on parameters x and y , which are not known. However,

$$x = \frac{d}{S} \cdot \csc 1^\circ \cdot \cos \alpha \cong 57 \cdot 3 \cdot \frac{d}{S} \cdot \cos \alpha \quad y = \frac{d}{S} \cdot \csc 1^\circ \cdot \sin \alpha \cong 57 \cdot 3 \cdot \frac{d}{S} \cdot \sin \alpha$$

Thus,

$$x \cdot y \cdot \sin 1^\circ = \frac{d}{S} \cdot \csc 1^\circ \cdot \cos \alpha \cdot \frac{d}{S} \cdot \csc 1^\circ \cdot \sin \alpha \cdot \sin 1^\circ = \frac{d^2}{S^2} \cdot \csc 1^\circ \cdot \frac{1}{2} \sin 2\alpha$$

$$x \cdot y \cdot \sin 1^\circ \cong 28 \cdot 65 \cdot \frac{d^2}{S^2} \cdot \sin 2\alpha. \tag{37}$$

Finally, we complete the swing by proceeding on a quadrantal course, for example NE:

$$(22) \Rightarrow \Psi_{ne} = A + B' \cdot 0 \cdot 707 + C' \cdot 0 \cdot 707 + D - x \cdot 0 \cdot 707 + y \cdot 0 \cdot 707 + \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ + \delta. \quad (38)$$

Let $\bar{A} = \frac{1}{4}(\Psi_n + \Psi_e + \Psi_s + \Psi_w)$, $\bar{B} = \frac{1}{2}(\Psi_e - \Psi_w)$ and $\bar{C} = \frac{1}{2}(\Psi_n - \Psi_s)$. Then,

$$A = \bar{A} - \delta \quad (39)$$

$$B' = \bar{B} + x \quad (40)$$

$$C' = \bar{C} - y \quad (41)$$

Replacing (39), (40) and (41) in (38), we obtain

$$\begin{aligned} \Psi_{ne} &= \bar{A} + \bar{B} \cdot 0 \cdot 707 + \bar{C} \cdot 0 \cdot 707 + D + \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ \\ D &= \Psi_{ne} - \bar{A} - \bar{B} \cdot 0 \cdot 707 - \bar{C} \cdot 0 \cdot 707 - \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ, \end{aligned} \quad (42)$$

where

$$\begin{aligned} \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ &= \frac{1}{2} \cdot \left(\frac{d^2}{S^2} \cdot \csc^2 1^\circ \cdot \cos^2 \alpha - \frac{d^2}{S^2} \cdot \csc^2 1^\circ \cdot \sin^2 \alpha \right) \cdot \sin 1^\circ = \frac{1}{2} \cdot \frac{d^2}{S^2} \cdot \csc 1^\circ \cdot \cos 2\alpha \\ \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ &\cong 28 \cdot 65 \cdot \frac{d^2}{S^2} \cdot \cos 2\alpha, \end{aligned} \quad (43)$$

and analogously for the other quadrantal courses.

3.3. Residual deviations and verification of compensation

The residual deviations cannot be determined because, except A , the coefficients of the deviation equation depend on factors x and y , which are unknown. Consequently, we cannot check whether any residual deviation exceeds 4° (in accordance with ISO Standard 25862:2019, Annex G.1). If one does, coefficients B' or C' must be completely nullified. Section 7 explains how to nullify coefficients. In Section 4, coefficients D and E are obtained, and in Section 6, residual coefficients B and C are calculated to finally check the deviation table and make the necessary readjustments.

4. Discussion of method

Expressions (31) and (32) are not reliable for the calculation of residual coefficients B and C because an imprecise d/S ratio can lead to a considerable error. It is observed, however, that, at a sufficient speed, the d^2/S^2 ratio is very small, so that expressions (37) and (43) can be considered zero. Thus, coefficients D and E can be determined solely from the pseudo-deviations, i.e.

$$D = \Psi_{ne} - \bar{A} - \bar{B} \cdot 0 \cdot 707 - \bar{C} \cdot 0 \cdot 707, \quad (44)$$

or analogously for the other quadrantal courses, where

$$\bar{A} = \frac{1}{4}(\Psi_n + \Psi_e + \Psi_s + \Psi_w), \bar{B} = \frac{1}{2}(\Psi_e - \Psi_w), \bar{C} = \frac{1}{2}(\Psi_n - \Psi_s)$$

and

$$E = \frac{1}{4}(\Psi_n + \Psi_s - \Psi_e - \Psi_w). \quad (45)$$

The maximum error in the calculation of coefficients D and E is shown for the S/d ratio in Table 1.

Table 1. Maximum error of coefficients *D* and *E* for the *S/d* ratio.

<i>S/d</i>	5	6	7	8	9	10	11	12
Max. error: $28 \cdot 65 \cdot d^2/S^2$	1 · 146	0 · 796	0 · 585	0 · 448	0 · 354	0 · 287	0 · 237	0 · 199

These results show that coefficients *D* and *E* can be determined solely from the pseudo-deviations but with a small error that is negligible for sufficiently high speeds, i.e. for a ratio of *S/d* equal to or greater than 8, which causes an error less than $0 \cdot 5^\circ$ for each coefficient. Assuming a drift of 1 knot or less, the minimum speed is 8 knots, and assuming a drift of $0 \cdot 5$ knots or less, the minimum speed is 4 knots. A suitable minimum speed could be 7 knots.

It should be emphasised that the method is not based on an exact calculation of the effect of the external forces, which is variable and has no exact vector behaviour, but on determining which coefficients are affected by this effect and which are not.

5. Verification of the method

The method was verified by performing a swing on a recreational fishing ship in the Bay of Santoña (Cantabria, Spain). The swing was carried out on 24 May 2021 between 1255 and 1335 local time, once the ship was inside the bay (outside the harbour and its channel). The wind was from the west, force 5–6 on the Beaufort scale, generating waves of approximately 1 · 5 m in the bay. The cloud cover was 7 oktas stratocumulus mainly as well as clouds of greater vertical development, causing intermittent rains of moderate intensity. The ship was navigating at about 7 knots, an adequate speed as stated in Section 4.

5.1. Equipment

An integral magnetic compass, IMC (Martínez-Lozares, 2009a, 2009b), was used to find the deviations in real time. The true course input to the IMC was obtained from the ship's satellite compass (Figure 3 shows the satellite compass antenna with its clover-shaped base). The compass course input to the IMC was obtained using a magnetic sensor (Figure 4 shows the magnetic compass and the sensor being adjusted, and Figure 5 shows the magnetic compass with the sensor already adjusted).

The IMC installed in a PC, with the inputs of both courses obtained from the magnetic compass, *C*, and satellite compass, *G*, can be seen in Figure 6. It is observed how the position obtained from the GPS given by the IMC is used to calculate the magnetic declination, δ , with the US National Oceanic and Atmospheric Administration (NOAA) calculator (Figure 7). The true course, the compass course and the magnetic declination provide the deviation value at all times, i.e. $\Delta = G - \delta - C$.

5.2. Data collection

Using the satellite compass, the ship proceeded on the eight main true headings, i.e. N, NE, E, SE, S, SW, W and NW. For each heading, the COG was observed and the compass course was recorded by the IMC. The reading of the COGs followed the same technique as the observation of draughts in wave conditions or of the compass course in gyrocompass navigation: observation of variations in data (draught, compass course or COG in this case), estimation of an average value and, for the courses, observation of the value of the data to be compared (gyroscopic and compass courses, or true course and COG in this case) at different times to check the average. To facilitate the comparison of headings, the IMC was selected in *G* mode, i.e. showing the true course determined by the satellite compass as main course information. Figure 8 shows the IMC and the GPS receiver when a COG was being obtained.



Figure 3. Ship on which the swing was carried out (the blue hull one).



Figure 4. Ship's magnetic compass while adjusting the IMC sensor.

The deviations recorded by the IMC after the swing are shown in [Figure 9](#). The COGs for each true heading were

000 046 091 136 179 226 269 315 .

5.3. Data processing: obtaining coefficients from deviations

The column *arithmetic* in [Figure 9](#) corresponds to the deviations calculated by direct comparison between the true course and the compass course and taking into account the magnetic declination. The deviations of the intermediate courses (other than the eight main courses) could have been determined when the ship changed course during the swing, assuming there was sufficient course stabilisation or, more likely, there were previous records. Therefore, only the deviations of the main courses are considered here.



Figure 5. Ship's magnetic compass with the IMC sensor already adjusted.



Figure 6. IMC with the compass course, true course obtained from the satellite compass, position obtained from GPS and magnetic declination obtained from the NOAA calculator.

From the deviations on the cardinal courses, we obtain

$$(10) \Rightarrow B = \frac{1}{2}(\Delta e - \Delta w) = \frac{1}{2}(-8 \cdot 377 - 5 \cdot 624) = -7 \cdot 0005^\circ$$

$$(11) \Rightarrow C = \frac{1}{2}(\Delta n - \Delta s) = \frac{1}{2}(-1 \cdot 177 - 3 \cdot 620) = -2 \cdot 3985^\circ$$

$$(14) \Rightarrow A = \frac{1}{4}(\Delta n + \Delta e + \Delta s + \Delta w) = \frac{1}{4}(-1 \cdot 177 - 8 \cdot 377 + 3 \cdot 620 + 5 \cdot 624) = -0 \cdot 0775^\circ$$

$$(15) \Rightarrow E = \frac{1}{4}(\Delta n + \Delta s - \Delta e - \Delta w) = \frac{1}{4}(-1 \cdot 177 + 3 \cdot 620 + 8 \cdot 377 - 5 \cdot 624) = 1 \cdot 299^\circ.$$

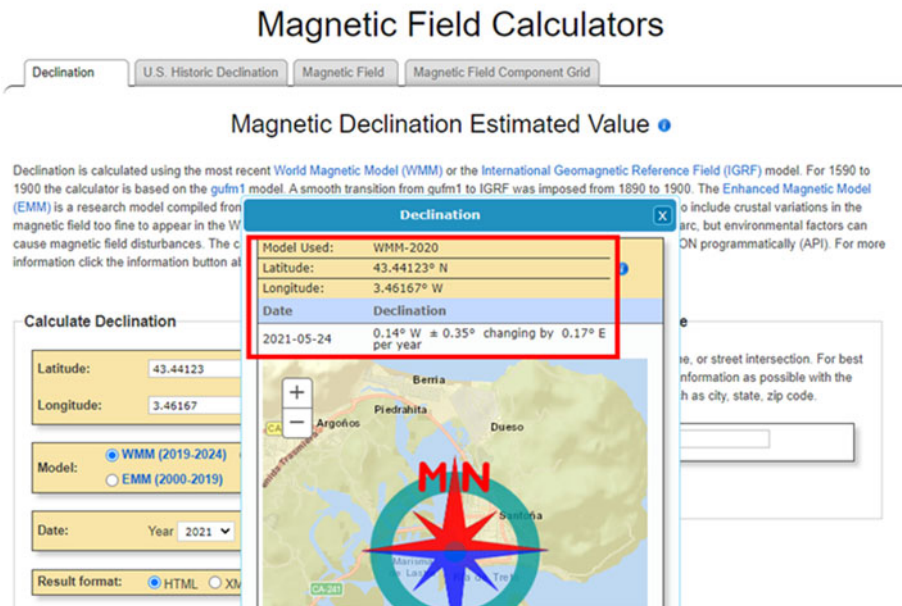


Figure 7. NOAA calculator: magnetic declination for position and date of the swing. (Source: <https://www.ngdc.noaa.gov/geomag/calculators/magcalc.shtml>).



Figure 8. True course and COG comparison. True course from the satellite compass is shown on the IMC.

The column *deviation* corresponds to the deviations determined by the deviation equation (1), which considers the calculated coefficients *A*, *B*, *C* and *E* and coefficient *D* obtained from a quadrantal deviation that, in this case, is $\Delta se = -4 \cdot 279^\circ$. From expression (19), we have

$$\Delta se = A + B \cdot 0 \cdot 707 - C \cdot 0 \cdot 707 - D \Rightarrow D = A + B \cdot 0 \cdot 707 - C \cdot 0 \cdot 707 - \Delta se$$

$$D = -0 \cdot 0775 - 7 \cdot 0005 \cdot 0 \cdot 707 + 2 \cdot 3985 \cdot 0 \cdot 707 + 4 \cdot 279 = 0 \cdot 9479^\circ,$$

	arithmetic	deviation
000°	-1.177°	-1.177°
015°	6.523°	-2.607°
030°	4.123°	-4.184°
045°	2.023°	-5.776°
060°	-4.077°	-7.168°
075°	-0.677°	-8.111°
090°	-8.377°	-8.377°
105°	-12.578°	-7.818°
120°	-6.478°	-6.411°
135°	-4.279°	-4.279°
150°	-3.179°	-1.671°
165°	1.021°	1.079°
180°	3.620°	3.620°
195°	4.620°	5.650°
210°	10.521°	6.970°
225°	14.721°	7.516°
240°	15.921°	7.356°
255°	11.324°	6.654°
270°	5.624°	5.624°
285°	2.524°	4.465°
300°	1.724°	3.316°
315°	15.123°	2.229°
330°	26.023°	1.175°
345°	14.823°	0.069°

Figure 9. Deviations recorded by the IMC after the swing.

and the mean coefficient D is

$$(18)-(19) + (20)-(21) \Rightarrow D = \frac{1}{4}(\Delta ne - \Delta se + \Delta sw - \Delta nw)$$

$$D = \frac{1}{4}(2 \cdot 023 + 4 \cdot 279 + 14 \cdot 721 - 15 \cdot 123) = 1 \cdot 475^\circ. \tag{46}$$

The coefficients obtained from the deviations are

$$A = -0 \cdot 0775^\circ \quad B = -7 \cdot 0005^\circ \quad C = -2 \cdot 3985^\circ \quad D = 1 \cdot 475^\circ \quad E = 1 \cdot 299^\circ$$

5.4. Data processing: obtaining coefficients from pseudo-deviations

Applying the magnetic declination, δ , and the deviations, Δ , obtained from the IMC to the true courses, TC, the compass courses, ζ' , are determined, and with them, the pseudo-deviations, i.e. $\Psi = \text{COG} - \zeta'$ (see Table 2).

From the pseudo-deviations in Table 2, we have

$$(44) \Rightarrow \bar{A} = \frac{1}{4}(\Psi n + \Psi e + \Psi s + \Psi w)$$

$$\bar{A} = \frac{1}{4}(-1 \cdot 312 - 7 \cdot 512 + 2 \cdot 485 + 4 \cdot 489) = -0 \cdot 4625^\circ$$

Table 2. *Determination of the pseudo-deviations.*

	N	NE	E	SE	S	SW	W	NW
TC	000	045	090	135	180	225	270	315
$-\delta$	0 · 135	0 · 135	0 · 135	0 · 135	0 · 135	0 · 135	0 · 135	0 · 135
ζ	000 · 135	045 · 135	090 · 135	135 · 135	180 · 135	225 · 135	270 · 135	315 · 135
$-\Delta$	1 · 177	-2 · 023	8 · 377	4 · 279	-3 · 620	-14 · 721	-5 · 624	-15 · 123
ζ'	001 · 312	43 · 112	098 · 512	139 · 414	176 · 515	210 · 414	264 · 511	300 · 012
COG	000	046	091	136	179	226	269	315
Ψ	-1 · 312	2 · 888	-7 · 512	-3 · 414	2 · 485	15 · 586	4 · 489	14 · 988

Table 3. *Differences between coefficients A, D and E determined from the deviations and the pseudo-deviations.*

	A	D	E
Δ	-0 · 0775	1 · 475	1 · 299
Ψ	-0 · 3275	1 · 725	1 · 049
Dif	0 · 250	0 · 250	0 · 250

$$(39) \Rightarrow A = \bar{A} - \delta = -0 \cdot 4625 + 0 \cdot 135 = -0 \cdot 3275^\circ$$

$$(44) \Rightarrow \bar{B} = \frac{1}{2}(\Psi_e - \Psi_w) = \frac{1}{2}(-7 \cdot 512 - 4 \cdot 489) = -6 \cdot 0005^\circ$$

$$(44) \Rightarrow \bar{C} = \frac{1}{2}(\Psi_n - \Psi_s) = \frac{1}{2}(-1 \cdot 312 - 2 \cdot 485) = -1 \cdot 8985^\circ$$

$$(45) \Rightarrow E = \frac{1}{4}(\Psi_n + \Psi_s - \Psi_e - \Psi_w) = \frac{1}{4}(-1 \cdot 312 + 2 \cdot 485 + 7 \cdot 512 - 4 \cdot 489) = 1 \cdot 049^\circ.$$

Analogously to expression (46), we have

$$D = \frac{1}{4}(\Psi_{ne} - \Psi_{se} + \Psi_{sw} - \Psi_{nw})$$

$$D = \frac{1}{4}(2 \cdot 888 + 3 \cdot 414 + 15 \cdot 586 - 14 \cdot 988) = 1 \cdot 725^\circ. \tag{47}$$

The coefficients determined from the pseudo-deviations are

$$A = -0 \cdot 3275^\circ \quad D = 1 \cdot 725^\circ \quad E = 1 \cdot 049^\circ \quad \bar{B} = -6 \cdot 0005^\circ \quad \bar{C} = -1 \cdot 8985^\circ .$$

5.5. Data analysis

1. Coefficients A, D and E determined from the pseudo-deviations are very similar to those obtained from the deviations (Table 3 shows these differences in absolute value). By contrast, the differences between coefficients B and C determined from the deviations and \bar{B} and \bar{C} , respectively, are greater than the differences between coefficients A, D and E.

The same value of 0 · 25 is a coincidence. Note that if Dif = $\Psi - \Delta$, the difference for coefficients A and E is negative, while for coefficient D it is positive.

Table 4. Differences between coefficient D determined from the deviations and the pseudo-deviations for each quadrantal course.

	NE	SE	SW	NW
Δ	8 · 7456	0 · 9479	8 · 1534	-11 · 9469
Ψ	8 · 9351	0 · 0514	10 · 4639	-12 · 5504
Dif	0 · 1895	0 · 8965	2 · 3105	0 · 6035

2. Analogously to expression (44), we have

$$D = \Psi_{ne} - \bar{A} - \bar{B} \cdot 0 \cdot 707 - \bar{C} \cdot 0 \cdot 707 \tag{44}$$

$$D = \bar{A} + \bar{B} \cdot 0 \cdot 707 - \bar{C} \cdot 0 \cdot 707 - \Psi_{se} \tag{48}$$

$$D = \Psi_{sw} - \bar{A} + \bar{B} \cdot 0 \cdot 707 + \bar{C} \cdot 0 \cdot 707 \tag{49}$$

$$D = \bar{A} - \bar{B} \cdot 0 \cdot 707 + \bar{C} \cdot 0 \cdot 707 - \Psi_{nw}. \tag{50}$$

Thus, we can compare each coefficient D calculated from the deviations by expressions (18), (19), (20) and (21) and from the pseudo-deviations by expressions (44), (48), (49) and (50). The deviations are obtained from column *arithmetic* in Figure 9 and the pseudo-deviations from Table 2.

On the NE heading,

$$D = \Delta_{ne} - A - B \cdot 0 \cdot 707 - C \cdot 0 \cdot 707 \text{ (deviations)}$$

$$D = 2 \cdot 023 + 0 \cdot 0775 + 7 \cdot 0005 \cdot 0 \cdot 707 + 2 \cdot 3985 \cdot 0 \cdot 707 = 8 \cdot 7456^\circ$$

$$D = \Psi_{ne} - \bar{A} - \bar{B} \cdot 0 \cdot 707 - \bar{C} \cdot 0 \cdot 707 \text{ (pseudo - deviations)}$$

$$D = 2 \cdot 888 + 0 \cdot 4625 + 6 \cdot 0005 \cdot 0 \cdot 707 + 1 \cdot 8985 \cdot 0 \cdot 707 = 8 \cdot 9351^\circ.$$

Table 4 shows the values of coefficient D determined from the deviations and the pseudo-deviations for each quadrantal course and their differences.

3. Coefficient D takes different values. As can be seen, it is similar for the opposite headings NE and SW, i.e. about 8° , while it is different for the opposite headings SE and NW, i.e. about 1° and -12° , respectively, and the mean coefficient D is about $1 \cdot 5^\circ$. This is because the compass was not adjusted and therefore, other higher order deviations, such as sextantal and octantal deviations, which depend on the triple and quadruple of the course, respectively, occurred, in agreement with the observations in Smith and Evans (1861). However, if coefficients B and C are reduced beforehand, this should not happen.

5.6. Conclusion

No matter how the method is applied, the results are satisfactory, i.e. coefficients A , D and E obtained from the deviations and pseudo-deviations are highly similar. Even when coefficient D varies with the quadrantal course, the results from the deviations and pseudo-deviations are similar for the same heading, except for the SW, but even in this case, the difference is not significant.

6. Complete method: obtaining residual coefficients B and C for any single heading

In addition to coefficients A , D and E , two further headings, ζ'_1 and ζ'_2 , are necessary to obtain residual coefficients B and C . First, their deviations, Δ_1 and Δ_2 , must be determined. The ship must head for two visual references with known true bearings or azimuths (for good discrimination, the angle between

both headings must be between 60° and 120°):

$$(1) \Rightarrow \Delta_1 = A + B \cdot \sin \zeta'_1 + C \cdot \cos \zeta'_1 + D \cdot \sin 2\zeta'_1 + E \cdot \cos 2\zeta'_1$$

$$B \cdot \sin \zeta'_1 + C \cdot \cos \zeta'_1 = \Delta_1 - A - D \cdot \sin 2\zeta'_1 - E \cdot \cos 2\zeta'_1.$$

Analogously,

$$B \cdot \sin \zeta'_2 + C \cdot \cos \zeta'_2 = \Delta_2 - A - D \cdot \sin 2\zeta'_2 - E \cdot \cos 2\zeta'_2.$$

Let

$$\eta_1 = \Delta_1 - A - D \cdot \sin 2\zeta'_1 - E \cdot \cos 2\zeta'_1$$

$$\eta_2 = \Delta_2 - A - D \cdot \sin 2\zeta'_2 - E \cdot \cos 2\zeta'_2,$$

where both η_1 and η_2 are known data. Thus, we have the following system of two equations:

$$B \cdot \sin \zeta'_1 + C \cdot \cos \zeta'_1 = \eta_1$$

$$B \cdot \sin \zeta'_2 + C \cdot \cos \zeta'_2 = \eta_2,$$

whose solutions are

$$B = \frac{\eta_1 \cdot \cos \zeta'_2 - \eta_2 \cdot \cos \zeta'_1}{\sin(\zeta'_1 - \zeta'_2)}, C = \frac{\eta_2 \cdot \sin \zeta'_1 - \eta_1 \cdot \sin \zeta'_2}{\sin(\zeta'_1 - \zeta'_2)}.$$

By contrast, Lushnikov proposed a method in which, if coefficients A , D and E are known, only a single heading is required (Lushnikov, 2011). In Subsection 6.2, Lushnikov’s method is applied to obtain residual coefficients B and C .

6.1. Obtaining deviation on visual reference heading

If the ship proceeds to a shore point, its position can be determined from the chart or another source, such as Google Maps. This position is then entered as a waypoint into the GNSS receiver, and the function *GO TO* is used to obtain and compare the true course with the compass course to determine the deviation considering the magnetic declination.

If the ship heads towards the sun, its true azimuth, Z , is the true course, and is calculated by one of the azimuth formulae, e.g.

$$\tan Z = \frac{|\sin \text{LHA}|}{\cos \varphi \cdot \tan \text{Dec} - \sin \varphi \cdot \cos \text{LHA}}, \tag{51}$$

where φ is the ship’s latitude; Dec is the sun’s declination, and LHA is its local hour angle.

Dec and LHA are taken from the nautical almanac and corrected as necessary, bearing in mind that a proper sign convention must be applied in the formula. For example, φ , Dec and Z are positive when they are north and negative when they are south, and regardless of the sign convention, Z is east before noon (LHA greater than 180°) and west after noon (LHA smaller than 180°).

The calculation of azimuths does not require great accuracy for the ship’s position and the sun’s declination. For example, the ship’s position can be considered as that of the green light of the harbour breakwater and the sun’s declination as that of the estimated compensation time to prepare the calculation data. It is possible to determine LHA by considering only the UTC provided by the GNSS receiver when the ship is heading towards the sun, time of meridian passage, MP, taken from the nautical almanac, and longitude of the ship, L , which is positive when it is east and negative when it is west, i.e.

$$\text{LHA} = (\text{UTC} - \text{MP}) \cdot 15 - L \tag{52}$$

Note that in this expression, LHA is negative before noon but this does not affect (51), where the numerator must be in absolute value.

6.2. Simplification of method with single visual reference: application of Lushnikov’s method

Compass needles are oriented towards the horizontal component of the magnetic field at the compass location. The horizontal component of the earth’s magnetic flux density, H , can be found using an earth’s magnetic field calculator, such as the NOAA calculator, or a map, such as the *World Magnetic Model* (WMM). However, the total magnetic flux density at the compass location, H' , includes not only that of the earth but also that of the ship irons, which varies with the course. Each course therefore has a concrete H' , where the directive force (strictly speaking, magnetic flux density) towards the magnetic north is $H' \cdot \cos \Delta$, but also

$$\lambda \cdot H \cdot (1 + \sin B \cdot \cos \zeta - \sin C \cdot \sin \zeta + \sin D \cdot \cos 2\zeta - \sin E \cdot \sin 2\zeta),$$

Gatzelu - Iturri Leicea (1999) and Lushnikov (2011),

where λ is the mean directive force coefficient, specific to each ship, and the sines of the approximate coefficients B, C, D and E are the exact coefficients of the deviation equation, as indicated in Subsection 2.1. Therefore,

$$H' \cdot \cos \Delta = \lambda \cdot H \cdot (1 + \sin B \cdot \cos \zeta - \sin C \cdot \sin \zeta + \sin D \cdot \cos 2\zeta - \sin E \cdot \sin 2\zeta).$$

Since the approximate coefficients are small angles, their sines can be replaced by their values multiplied by $\sin 1^\circ$, i.e. approximately $1/57 \cdot 3$. Hence,

$$H' \cdot \cos \Delta = \lambda \cdot H \cdot (1 + B/57 \cdot 3 \cdot \cos \zeta - C/57 \cdot 3 \cdot \sin \zeta + D/57 \cdot 3 \cdot \cos 2\zeta - E/57 \cdot 3 \cdot \sin 2\zeta)$$

$$\frac{57 \cdot 3 \cdot H' \cdot \cos \Delta}{\lambda \cdot H} = 57 \cdot 3 + B \cdot \cos \zeta - C \cdot \sin \zeta + D \cdot \cos 2\zeta - E \cdot \sin 2\zeta. \tag{53}$$

Then, if only a single visual reference is considered, we have

$$(1) \Rightarrow \Delta v = A + B \cdot \sin \zeta' v + C \cdot \cos \zeta' v + D \cdot \sin 2\zeta' v + E \cdot \cos 2\zeta' v.$$

And since the magnetic and compass courses are similar,

$$B \cdot \sin \zeta v + C \cdot \cos \zeta v = \Delta v - A - D \cdot \sin 2\zeta v - E \cdot \cos 2\zeta v. \tag{54}$$

On the other hand,

$$(53) \Rightarrow B \cdot \cos \zeta v - C \cdot \sin \zeta v = 57 \cdot 3 \cdot \left(\frac{Hv \cdot \cos \Delta v}{\lambda \cdot H} - 1 \right) - D \cdot \cos 2\zeta v + E \cdot \sin 2\zeta v, \tag{55}$$

where Hv is the specific value of H' when the ship proceeds on the magnetic course ζv .

Let

$$\Theta_1 = \Delta v - A - D \cdot \sin 2\zeta v - E \cdot \cos 2\zeta v \tag{56}$$

$$\Theta_2 = 57 \cdot 3 \cdot \left(\frac{Hv \cdot \cos \Delta v}{\lambda \cdot H} - 1 \right) - D \cdot \cos 2\zeta v + E \cdot \sin 2\zeta v. \tag{57}$$

Then, we have the following system of two equations:

$$(54), (56) \Rightarrow B \cdot \sin \zeta v + C \cdot \cos \zeta v = \Theta_1$$

$$(55), (57) \Rightarrow B \cdot \cos \zeta v - C \cdot \sin \zeta v = \Theta_2,$$

whose solutions are

$$B = \Theta_1 \cdot \sin \zeta v + \Theta_2 \cdot \cos \zeta v \tag{58}$$

$$C = \Theta_1 \cdot \cos \zeta v - \Theta_2 \cdot \sin \zeta v. \tag{59}$$

6.3. Obtaining factor Θ_2

Factor Θ_2 has two unknown data, namely Hv and λ . They can be calculated by installing devices at the compass location, i.e. replacing the compass with devices, a difficult task for the ships for which this work is intended. Furthermore, the process requires the ship to proceed on the four cardinal courses, which is correspondingly time consuming (Gaztelu-Iturri Leicea, 1999; National Geospatial–Intelligence Agency, 2004; Lushnikov, 2011).

In expression (53), coefficients D and E are known data, and coefficients B and C can be considered \bar{B} and \bar{C} , respectively. Then,

$$\frac{57 \cdot 3 \cdot Hv \cdot \cos \Delta v}{\lambda \cdot H} = 57 \cdot 3 + \bar{B} \cdot \cos \zeta v - \bar{C} \cdot \sin \zeta v + D \cdot \cos 2\zeta v - E \cdot \sin 2\zeta v.$$

Hence,

$$\bar{B} \cdot \cos \zeta v - \bar{C} \cdot \sin \zeta v = 57 \cdot 3 \cdot \left(\frac{Hv \cdot \cos \Delta v}{\lambda \cdot H} - 1 \right) - D \cdot \cos 2\zeta v + E \cdot \sin 2\zeta v$$

$$(57) \Rightarrow \Theta_2 = \bar{B} \cdot \cos \zeta v - \bar{C} \cdot \sin \zeta v \tag{60}$$

7. Application of complete method

The complete process to adjust the compass and create its deviation table is as follows.

7.1. Application for all cases

1. Using a GNSS receiver, the ship proceeds on $\text{COG} = 90^\circ + \delta$ (or $270^\circ + \delta$), and with the longitudinal magnets, the compass course is adjusted to 90° (or 270°). Next, using the GNSS receiver again, the ship proceeds on $\text{COG} = \delta$ (or $180^\circ + \delta$), and with the transversal magnets, the compass course is adjusted to 0° (or 180°). Then, Ψ_e (or Ψ_w) = δ and Ψ_n (or Ψ_s) = δ are obtained. Note that to determine the COG, δ can be rounded to the nearest degree because its decimals are taken into account when calculating coefficient A .
2. Using the magnetic compass, the ship proceeds on the other two cardinal courses and a quadrantal one, and the COGs shown by the GNSS receiver are noted down to determine the pseudo-deviations, i.e. Ψ_w (or Ψ_e), Ψ_s (or Ψ_n) and Ψ_{ne} (or Ψ_{se} , Ψ_{sw} or Ψ_{nw}). The quadrantal course does not have to be the last one, as the position of the magnets on the third and fourth cardinal courses does not change. To avoid the impact of errors caused by other higher-order deviations, such as sextantal or octantal deviations, on coefficient D (Smith and Evans, 1861), we recommend choosing the quadrantal course between the first and second cardinal courses.
3. Coefficients A , D and E are calculated using expressions (35), (44) and (45), respectively. The exact value of δ must be inserted into expression (35).

4. The ship proceeds to a visual reference and its deviation is calculated as described in Subsection 6.1. Residual coefficients B and C are then determined from expressions (58) and (59), respectively, where factors Θ_1 and Θ_2 are determined from expressions (56) and (60), respectively. In order to avoid the impact of errors on coefficient D , we recommend proceeding to a visual reference within the same quadrant as the quadrantal course to calculate this coefficient. However, in line with the recommendation in point 2, the choice of the visual reference fixes the quadrantal course and, therefore, the two cardinal courses to which the compass courses are adjusted (point 1). It is thus proposed that the ship proceeds on the cardinal courses that delimit the quadrant containing the visual reference. Next, the compass is adjusted on these courses and the ship proceeds on the quadrantal course or the visual reference course. Finally, the ship proceeds on the other course within the quadrant, and then on the other two cardinal courses.
5. Once residual coefficients B and C are obtained, the deviation on various compass courses, typically every 10 or 15 degrees from the north, is calculated by the deviation equation (1) with a spreadsheet.
6. The process can next be completed if no deviation exceeds 4° , whereas if a deviation exceeds 4° , the largest value between coefficients B and C must be nullified. To increase accuracy, both coefficients can be nullified even if no deviation exceeds 4° . The process for nullifying a coefficient is described in point 7.

7.2. Application when deviation exceeds 4° or when greater accuracy required

7. To nullify coefficient B , we must remember that if the other coefficients were equal to zero, the deviation would be $\Delta e = B$ and the compass course would be $\zeta' = \zeta - \Delta e = 90^\circ - B$ on the east magnetic heading and analogously, $\Delta w = -B$ and $\zeta' = \zeta - \Delta w = 270^\circ + B$ on the west magnetic heading. The procedure is, therefore, to proceed on one of these courses, $90^\circ - B$ or $270^\circ + B$, with the magnetic compass, observe the COG, proceed on this COG and nullify the coefficient by setting $\zeta' = 90^\circ$ or 270° with the longitudinal magnets. To nullify coefficient C , the procedure is to proceed on $\zeta' = 0^\circ - C$ or $\zeta' = 180^\circ + C$, observe the COG, proceed on this COG and nullify the coefficient by setting $\zeta' = 0^\circ$ or $\zeta' = 180^\circ$ with the transversal magnets.
8. When a coefficient is nullified, the deviation table must be obtained as described in point 5, but without considering this coefficient. It is important to ensure that the coefficient is nullified exactly and is not simply reduced to avoid errors in the deviation table. If the magnetic moment of the compensating device's magnets cannot nullify the coefficient completely, then the procedure must be repeated, but this is uncommon.
9. If no deviation of the table obtained in point 8 exceeds 4° , then the procedure can be completed, while if a deviation exceeds 4° , then the other coefficient must be nullified. With the help of the spreadsheet, we can know in advance whether only one or both coefficients must be nullified because the deviation table can be calculated using the deviation equation with all coefficients, without B , without C and without both B and C . Finally, if coefficients B and C are nullified but some deviations of the table exceed 4° , these deviations cannot be reduced unless the compass is fitted in a different location. In this case, the residual table shall be considered final, with the corresponding exemption from compliance with ISO Standard 25862:2019, Annex G.1, if necessary.

7.3. Reliability of method

When the drift is less than one knot, we conclude that a suitable minimum speed could be 7 knots (see Section 4). Even if the drift were one knot, the maximum error of coefficients D and E would only be slightly greater than half a degree ($0 \cdot 585^\circ$ exactly, as indicated in Table 1). Therefore, we can affirm that the method is reliable if the drift does not exceed one knot and the vessel speed is 7 knots or more, a speed that most vessels can reach. Then, the method may not be reliable when the drift is greater than one knot, and usually this can occur in any of the following cases:

1. When there is strong wind. However, with strong wind, compensation can hardly be carried out, since the waves generated by the wind do not allow steering with the accuracy required by the compensation process, especially in small ships, which are those the method is primarily aimed at. Therefore, this limitation is not specific to the method, but to the compensation itself.
2. In rivers or near their mouths due to the stream of the river.
3. In areas with tidal streams, i.e. in estuaries, narrows and other confined spaces affected by tides; but in these cases, compensation is typically carried out outside the harbour, where the ship is not in a confined space and the tidal stream is considerably weaker, except in the case of estuaries.

In summary, the method has some limitations in rivers and confined spaces with tidal streams. Therefore, it must be taken into account that, in some ports or areas, the method is impracticable.

8. Conclusions

A simple, rigorous method for compass adjustment on ships having only a magnetic heading indicator is proposed. Only six headings are required to adjust the compass and create the deviation table, and depending on the vessel, one or two more headings are needed to make proper readjustments. Similarly, only a GNSS receiver and a visual reference are required, eliminating the need for leading lines, peloruses or other equipment, except for support for spreadsheets, such as a mobile phone. Moreover, no current or wind data is required, but a minimum vessel speed must be ensured, e.g. 7 knots.

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