years of insurance,* and as an illustration of a method of comparison which in a larger experience might afford some indication as to the effect of withdrawals upon the average vitality of a body of assured lives.

The problem of the calculation of annual premiums for term assurances may be briefly referred to. If it is a correct assumption that a body of assured lives is deteriorated by withdrawals, it is obvious that select tables compiled from an experience chiefly consisting of whole-term assurances are not directly applicable to the calculations of annual premiums for more numerously discontinued assurances. It is possible, however, to provide for more numerous withdrawals of healthy lives than are covered by the tables by using the formula

$$P^1_{x\overline{n}|} = \frac{M_{[x]} - M_{[x]+n} - p(M_{[x+1]} - M_{[x+1]+n-1}) - q(1-p)(M_{[x+2]} - M_{[x+2]+n-2}) - \&c.}{N_{[x]} - N_{[x]+n} - p(N_{[x+1]} - N_{[x+1]+n-1}) - q(1-p)(N_{[x+2]} - N_{[x+2]+n-2}) - \&c.}$$

where, p, q, &c., represent the extra proportions of healthy lives withdrawing at the end of the first, second, &c., years.

Now it is questionable whether the term-assurance experience of life offices is sufficient to admit of the construction of a special set of select tables for the calculation of premiums, and whether, in any case, it would be worth while to construct such tables, but the experience will, of course, readily lend itself to the calculation of rates of non-renewal. The excess of these rates over the rates of non-renewal shown by the whole-term table, would then afford an indication of the values to be given in the above formula to the quantities p, q, &c.

In conclusion, I may just mention one other problem to the solution of which rates of non-renewal appear to be directly applicable —the determination of a superior limit to the amount allowable in commutation of an annual commission.

<div align="center">

I am, Sir,

Your obedient Servant,

R. TODHUNTER.

</div>

39, *King Street, Cheapside, E.C.,*
20 *February* 1897.

<div align="center">

Mr. King's Inter-Valuation Formula for "Exposed to Risk."

To the Editor of the Journal of the Institute of Actuaries.

</div>

Sir,—In reading Mr. Ackland's interesting paper on "Methods for deducing the Rates of Mortality and Withdrawal", I notice that, in referring to Mr. King's Inter-Valuation Method, Mr. Ackland states that there is a possible error of twelve months in the age at

* To obtain the exposed to risk of non-renewal, I have followed Mr. Sanderson in deducting the deaths from the exposed to risk of death. It is assumed that lives which were assured up to the close of the observations, but withdrew immediately afterwards, are included in the "withdrawals", and not in the "existing."

entry, and concludes by saying : "It will probably, however, give
"on the whole good results ; but is clearly inferior to the Method of
"Nearest Ages."

As this point has already given rise to some discussion, I was
surprised to find that no further mention was made of it after the
reading of the paper at the Institute.

The subject is certainly of interest to a large number of Students,
and I therefore venture to make a few suggestions which, I hope,
may prove of some assistance to them.

In volume xxvii, p. 218, Mr. King states that his formula was
devised more particularly for taking out the mortality experience of
a company between two valuation epochs ; and he explains how this
may be done with a minimum of trouble by making use of the
classification which will already have been prepared for valuation
purposes.

As explained by Mr. King, the age at entry is taken as the
nearest age at the nearest 31 December, so that all entries are
considered as taking place at the end of a year. Thus, if a life is
actually exposed for less than six months in the year of entry, it is
not tabulated as an exposure for that year, and if for more than six
months, then it is tabulated as a full year's exposure ; therefore the
limit of error, as to the period of exposure in the initial year of entry,
is six months.

To take an example. Suppose a man born 30 June 1820,
enters in July 1860, his age will be taken as 41, but by the formula
he is not assumed to enter until 31 December 1860, when 41 is
really his nearest age, and throughout the remaining history of the
policy, the age tabulated for each year of exposure coincides with the
nearest age of the life, and there can never be an error of more than
six months in the age.

The formula, like others, involves an error both as regards the
duration in the initial year of exposure and the age at which the
policy is tabulated ; but it is only where these errors are in the same
direction that there is the possibility of an apparent, but not real,
error of twelve months, and it is, I think, decidedly misleading
to saddle the whole of this error on to the age, without making any
mention whatever of the duration.

Perhaps the most striking feature of the method is the automatic
way in which it meets the difficulty of the unequal distribution of
entrants. Thus, suppose, as is often the case, there is a considerable
rush of entrants towards the close of the year, so that the average
date of entry is, for instance, 1 September instead of 1 July, as it
would be if entries were equally distributed. In this case, if the
Institute method were applied, the period of exposure in the initial
year would be taken as six months instead of four, thus causing the
mortality shown for the year of entry to be lighter than is really the
case. By Mr. King's method, however, this difficulty is provided
for, since the larger proportion of entrants at the close of the year,
the larger will be the proportion of lives taken at the succeeding
age, and consequently the fewer will there be credited with exposure
in the initial year. In this way, the average period tabulated is,
undoubtedly, a very close approximation.

The method of treating the withdrawals is similar to that used for the entrants; and there is, consequently, the same limit of error; but, since most of the withdrawals will occur on an anniversary of the date of entry, the effect will be to make the period of exposure tabulated to exactly coincide with the actual experience.

In cases of death, the age is necessarily taken as the nearest at the *previous* 31 December, thus, as pointed out by Mr. Whittall, giving rise to the apparent possible anomaly of a policy taken at age x at entry and dying at age $x-1$.

In considering this point, it must not be forgotten that the method is intended for the formation of aggregate tables, and that there will be a balance of errors from age to age. It may also be pointed out, that whatever the youngest age may be at which premiums are quoted by the office, the mortality experience will necessarily commence at one year younger; and, although a life may be tabulated as dying at a younger age than that at which it is assumed to enter, yet there will always be a proportion of the entrants also referred back to that previous age to form the exposures from which the death will be assumed to have occurred.

With Endowment Assurance Policies, the method of classification adopted at the valuation would, probably, prevent the results being used in the same way as with the Whole-Life Policies; but even in those cases, if at any valuation, all the Endowment Policies in force were retabulated (for mortality investigation cases only) in a similar manner to the Whole-Life Policies, it would then be a very simple matter to keep a record of all entrants and withdrawals, and so obtain the results required.

For the purposes for which Mr. King's method was intended, the saving of labour is very considerable. When once the policies have been classified at nearest ages at a valuation, we shall be able to trace the greater proportion of these for *exact* periods of exposure with a very small amount of trouble; and, looking at the method from this point of view, I think it will be admitted that it is undoubtedly the best which has yet been suggested.

I am,
Yours obediently,
J. BURN.

Prudential Assurance Company,
    Holborn Bars,
        June 1896.