paper dealt, and which, he says, was suggested to him by the analysis used by Poisson in the article of his *Théorie de la Chaleur*, quoted above.

Questions of priority are usually somewhat difficult to answer; but while it seems clear that the theorem generally quoted as Green's was given independently of Green, yet the importance which he rightly attached to it, and the splendid use to which he put it, amply justify us in keeping to the customary mode of citation.

Some new Properties of the Triangle.

By J. S. MACKAY, M.A., LL.D.

[The substance of this paper will be included in Dr Mackay's paper on the triangle in the first volume of the *Proceedings*, now about to be published.]

Second Meeting, 13th December 1889.

R. E. ALLARDICE, Esq., M.A., Vice-President, in the Chair.

A special case of three-bar motion.

By Professor STEGGALL.

The questions involved in the consideration of three-bar motion have attracted a good deal of attention (*Proceedings of Mathematical Society of London* passim, and elsewhere); but I am not aware of any complete account of the figures that can be derived from such a motion. The present paper gives a complete list of all the different kinds of curve that are obtained by a tracing point at the middle of the middle bar, the two outer bars being equal.

It may be advisable to briefly obtain the general equation to the curve traced by any point on the middle bar, without any condition of equality in the lengths of the other two.

Let $2a$ be the distance of the fixed centres, b, $2c$, d the lengths of the three bars in order, h the distance of the tracing point from the middle of the middle bar measured from the bar b, θ, ϕ, ψ the angles which the bars in order make with the line joining the fixed centres: take this line as axis of x, and a perpendicular through its middle point as that of y.

We have at once, on reference to a diagram,

$$b\cos\theta + (h+c)\cos\phi = a + x,$$
$$b\sin\theta + (h+c)\sin\phi = y,$$
$$d\cos\psi + (h-c)\cos\phi = x - a,$$
$$d\sin\psi + (h-c)\sin\phi = y.$$

Whence

$$b^2 = y^2 + (a+x)^2 + (h+c)^2 - 2(h+c)\{(x+a)\cos\phi + y\sin\phi\},$$
$$d^2 = y^2 + (x-a)^2 + (h-c)^2 - 2(h-c)\{(x-a)\cos\phi + y\sin\phi\}.$$

Whence

$$4\{(hx+ac)\cos\phi + hy\sin\phi\} = 2(x^2+y^2+a^2+h^2+c^2) - b^2 - d^2,$$
$$4\{(cx+ah)\cos\phi + cy\sin\phi\} = 4ax + 4hc - b^2 + d^2.$$

Now the eliminant of

$$P\cos\phi + Q\sin\phi = R$$
$$P'\cos\phi + Q'\sin\phi = R'$$

is

$$R^2(P'^2+Q'^2) + R'^2(P^2+Q^2) - 2RR'(PP'+QQ') = (PQ'-QP')^2.$$

Whence, on substitution, we easily obtain (calling $2a^2+2h^2+2c^2-b^2-d^2$, A, and $4hc+d^2-b^2$, B)

$$\begin{aligned}
&r^6 \,.\, 4c^2\\
&-r^4x \,.\, 8ach\\
&+r^4 \,.\, 4\{c(cA-hB)+a^2h^2\}\\
&-r^2x^2 \,.\, 16a^2c^2\\
&-r^2x \,.\, 4a\{B(c^2-h^2)+4a^2ch\}\\
&+x^3 \,.\, 32a^3ch\\
&+r^2 \,.\, \{4a^2h(hA-cB)+(cA-hB)^2-16a^2(c^2-h^2)^2\}\\
&+x^2 \,.\, 8a^2\{2a^2c^2-c(cA-hB)-h(hA-cB)+2(c^2-h^2)^2\}\\
&+x \,.\, 2a(hA-cB)(cA-hB-4a^2c)\\
&+a^2(hA-cB)^2=0.
\end{aligned}$$

It may be worth while to write this at full length: the result is

$$\begin{aligned}
&r^6 \,.\, 4c^2\\
&-r^4x \,.\, 8ach\\
&+r^4 \,.\, 4\{a^2(h^2+2c^2)+2c^2(c^2-h^2)-b^2c(c-h)-d^2c(c+h)\}\\
&-r^2x^2 \,.\, 16a^2c^2\\
&+r^2x \,.\, 4a\{(h^2-c^2)(4hc+d^2-b^2)-4a^2ch\}\\
&+x^3 \,.\, 32a^3ch
\end{aligned}$$

https://doi.org/10.1017/S0013091500030455 Published online by Cambridge University Press

$$+r^2 \,.\, \{4a^4(2h^2+c^2) - a^2[8(c^2-h^2)^2 + 4b^2(c-h)^2 + 4d^2(c+h)^2] + [2c(c^2-h^2) - b^2(c-h) - d^2(c+h)]^2\}$$
$$+x^2 \,.\, 8a^2\{b^2(c-h)^2 + d^2(c+h)^2 - 2a^2h^2\}$$
$$-x \,.\, 2a\, \{2h(a^2+h^2-c^2) + b^2(c-h) - d^2(c+h)\}\{2c(a^2+h^2-c^2) + b^2(c-h) + d^2(c+h)\}$$
$$+a^2\{2h(a^2+h^2-c) + b^2(c-h) - d^2(c+h)\}^2 = 0.$$

The same equation has been obtained by a slightly different method of elimination by Professor W. W. Johnson (*Messenger of Mathematics*, vol. V., 1875, p. 50).

The equation is so unmanageable in this form that some special assumption seems necessary in order to be able to trace the sextic curve; and the assumption in this paper is that $b=d,\ h=0$; in other words, the tracing point bisects the free link, while the fixed links are of equal length.

The curve now becomes

$$r^6 + 2r^4(a^2+c^2-b^2) - 4r^2x^2a^2 + r^2\{a^2(a^2-2c^2-2b^2) + (c^2-b^2)^2\} + 4a^2b^2x^2 = 0.$$

This equation may be readily solved to give r in terms of θ, the vectorial angle, and the result (which may be easily obtained in other ways) is

$$r^2 = a^2\cos 2\,\theta + b^2 - c^2 \pm 2a\sin\theta\,\sqrt{c^2 - a^2\cos^2\theta},$$

or $$\sin\theta = (a^2 - c^2 + b^2 - r^2)/2a\,\sqrt{(b^2 - r^2)}.$$

The minimum values of r^2 are when $\cos\theta = 0, \sin\theta = 1, \cos 2\,\theta = -1$, and in this case $r^2 = b^2 - (a+c)^2$. This expression being only positive if $b > (a+c)$ the division is at once suggested into the cases

I. $b > a+c$
II. $b = a+c$
III. $b < a+c$.

Before proceeding to the separate cases, we may notice that the maximum and minimum values of r^2 are given by the equation

$$rdr/d\theta = 0 = -a^2\sin 2\,\theta + a\cos\theta(c^2 - a^2\cos 2\,\theta)/\sqrt{(c^2 - a^2\cos^2\theta)},$$

whose only solution is $\cos\theta = 0$; and those of y^2 by the equation

$$\sin\theta\cos\theta\{4a^2\cos^2\theta + b^2 - c^2 - 3a^2 \pm a\sin\theta(3c^2 + a^2 - 4a^2\cos^2\theta)/\sqrt{(c^2 - a^2\cos^2\theta)}\} = 0,$$

whose solutions are $\cos\theta = 0, \sin\theta = 0$, and

$$8a^4b^2\cos^4\theta + a^2b^2\{b^2 - 10c^2 - ba^2\}\cos^2\theta + \{(a-c)^3 + b^2a\}\{(a+c)^3 - b^2c\} = 0 \quad \ldots \quad (1)$$

or $$16a^2\cos^2\theta = 10c^2 + 6a^2 - b^2 \pm \{b^2 + 2(c^2 - a^2)\}\sqrt{1 + 8(c^2 - a^2)/b^2} \quad (2)$$

I. $b > a + c$.

We notice that if $a \ngtr c$, θ can have every value, otherwise the maximum value of $\cos\theta$ is c/a: we therefore divide this case into

(i.) $a < c$, (ii.) $a = c$, (iii.) $a > c$.

(i.) $a < c$.

Here θ may have every value giving always two values of r^2 except at $\theta = 0$. We get two detached but intersecting ovals which must be separately described, as may be at once seen on drawing a diagram.

On referring to equation (1) we see that if $b^2 > (a+c)^3/c$, the product of the values of $\cos^2\theta$ is negative, and the negative value of $\cos^2\theta$ is inadmissible, while from equation (2) it is clear that the positive value of $\cos^2\theta$ is greater than unity. Thus the only maximum and minimum heights are on the axis of y. The form of one branch of the curves is given in figure i. In this, and in all the other figures, the three numbers affixed denote the values of the constants a, b. c, taken in that order. As a rule the standard value of b is 12, but for clearness it is taken of various convenient magnitudes.

If $b^2 < (a+c)^3/c$, both values of $\cos^2\theta$ are positive, and the smaller value is less than unity. This gives two maximum values of y on each loop, symmetrically situated with regard to the axis of y; while if $b^2 = (a+c)^3/c$ these two maxima coalesce with the minimum between them, giving a flattish figure to the curve (see figures ii. and iii.), and a point where the tangent meets it in four coincident points.

(ii.) $c = a$.

In this case, since the two loops in general make (on opposite sides) an angle with the axis of x, where they cut, of

$$\tan^{-1}\{(a^2 + b^2 - c^2)/a\sqrt{(c^2 - a^2)}\}$$

the two loops cut the axis of x at right angles, and therefore touch and may each be continued from the other, since the lines BC, AD when they lie along AB may be made to cross or not cross in continuing their motion, as may be desired.

The equation reduces to

$$r^2 = b^2 - 2a^2\sin^2\theta \pm 2a^2\sin\theta\sqrt{(1 - \cos^2\theta)},$$

which, by taking the top half of each with the bottom of the other, becomes

$$r^2 = b^2$$
$$r^2 = b^2 - 4a^2\sin^2\theta.$$

https://doi.org/10.1017/S0013091500030455 Published online by Cambridge University Press

The latter is the inverse of an ellipse with regard to its centre, since $b > 2a$; and it becomes two circles if $b = 2a$.

This curve being written

$$y^2 = \{b^4 - (b^2 - 2r^2)^2\}/16a^2$$

the least value of r^2 is $b^2 - 4a^2, \theta = \pi/2$; and if this is $> b^2/2$, the greatest value of y^2 is when r^2 has this least value; this requires b^2 to be $> 8a^2$; but if $b^2 < 8a^2$, and therefore $b^2/2 > b^2 - 4a^2$, r^2 can be as small as $b^2/2$, and this then gives a *maximum* value of y—viz., $b^2/4a, \sin^2\theta = b^2/8a^2$; and a *minimum* value of y when $\theta = \pi/2$. Also when $b^2 = 8a^2$ these two coalesce giving a masked point of inflexion on the axis of y. See figures iv., v., vi.

It is interesting to notice that as a approaches c two disconnected equal portions of the loops become the circle, and the inverse of the conic; so that each loop degenerates into a semi-circle, and half of the inverse of the conic.

(iii.) $a > c$.

In this case $\cos\theta$ is limited, its maximum value being c/a.

If $b^2 > (a+c)^3/c$, we find that the values of $\cos^2\theta$ in equation (1) are one positive and less than c^2/a^2 (as may be at once verified by substituting c/a for $\cos\theta$ in the expression in the left which is then positive, while it is negative if $\cos\theta$ is put equal to zero), and the other negative. This gives us one minimum value of y^2, and its two positions are on the lower part of the upper loop and *vice versa*. See figure vii.

It is noteworthy that to pass from the case $a = c$, to the case $a > c$, we may regard the *upper* half of the circle as combined with the *upper* half of the inverse curve, whereas in passing to the case $a < c$, as seen above, the upper half of the circle is combined with the lower half of the inverse curve.

If $b^2 = (a+c)^3/c$ the equation for $\cos^2\theta$ becomes

$$8a^2\cos^4\theta + (b^2 - 10c^2 - 6a^2)\cos^2\theta = 0,$$

giving $$\cos^2\theta = 0 \text{ or } (a^2 + 3c^2)(3c - a)/8a^2c,$$

the last value is always less than c^2/a^2, and we thus have, in this case, an additional zero value of $\cos^2\theta$, and the tangent at the vertex meets the curve in four coincident points. If $3c > a$ there are two other symmetrical minima on each loop (fig. viii.), if $3c = a$ these also move up to the vertex where the tangent now meets the curve at six coincident points (fig. ix.), and if $3c > a$, these minima disappear (fig. x.).

The equation to the curve of fig. ix. is

$$x^6 + 3x^4(y^2 - 48c^2) + 3x^2(y^2 - 48c^2)(y^2 - 36c^2) + y^2(y^2 - 48c^2)(y^2 - 60c^2) = 0$$

from which it appears that the lines $y^2 = 48c^2$ each meet the curve in six coincident points, and the lower part of the curve approaches very closely to a straight line.

If $b^2 < (a+c)^3/c$ clearly no other maxima or minima exist unless $b^2 < 10c^2 + 6a^2$ (fig. xi.); if $b^2 < 10c^2 + 6a^2$, we have to divide the cases into, from equation (2), (i.) $b^2 > 8(a^2 - c^2)$ in which case both values if $\cos^2\theta$ are real and admissible giving four)symmetrical) maxima or minima heights on each loop (fig. xii.); (ii.) $b^2 = 8(a^2 - c^2)$ in which case these coalesce two and two giving a point of inflexion with a horizontal tangent (fig. xiii.); (iii.) $b^2 < 8(a^2 - c^2)$, in which case the points of inflexion are left but the horizontal tangent becomes oblique (fig. xiv.).

II. $b = a + c$.

This case naturally divides like the last into

(i.) $a < c$, (ii.) $a = c$, (iii.) $a > c$;

and in each case

$$r^2 = 2(a^2\cos^2\theta + ac \pm a\sin\theta\,\sqrt{c^2 - a^2\cos^2\theta}),$$

or $$r = \sqrt{a(1+\cos\theta)(c + a\cos\theta)} \pm \sqrt{a(1-\cos\theta)(c - a\cos\theta)}.$$

The equation to obtain the maximum height is now, besides $\cos\theta = 0$, $16a^2\cos^2\theta = 9c^2 - 2ac + 5a^2 \pm (3c - a)\sqrt{(c+a)(9c-7a)}$.

Case (i.) $a < c$.

Here the upper sign gives a value of $\cos^2\theta$ greater than unity, and the lower sign gives a positive root less than unity, as may be seen from the more general case I. (i.), or by comparing $(9c^2 - 2ac + 5a^2)^2$, and $(9c^2 - 2ac - 11a^2)^2$ with $(3c - a)^2(9c^2 + 2ac - 7a^2)$.

Hence in this case there is one maximum height besides that on the axis (fig. xv.). The two minima heights on the axis here become zero, the two loops meet one another at the origin with a common vertical tangent there, so that there is a choice of path at the central point.

Case (ii.) $a = c$.

Here the pairs of tangents at the points of intersection on the axis of x, which were inclined to that axis, become coincident, and we simply get three circles (fig. xvi.).

Case (iii.) $a > c$,

In this case we have, as in I. (iii.), the condition $\cos^2\theta \not> c^2/a^2$; subject to this both values of r^2 are real. For the maximum height we have the same equation as in case (i.), namely, $16a^2\cos^2\theta = 9c^2 - 2ac + 5a^2 \pm (3c - a)\sqrt{(c+a)(9c-7a)}$, and the values of $\cos^2\theta$ are real if $9c$ is equal to or greater than $7a$, or if $3c = a$. Now if $7a < 9c$, it is easily seen that both values of $\cos^2\theta$ are admissible, and we have (fig. xvii.) four (symmetrical) maxima and minima: if $7a = 9c$, these coincide pair and pair, giving a point of inflexion with a horizontal tangent (fig. xviii.).

If $7a > 9c$ (fig. xix.) these points disappear, while if $a = 3c$, the values of $\cos^2\theta$ though real give imaginary values of r^2, and the figure resembles that just referred to.

III. $b < a + c$.

Writing our original equations in the form

$$r^2 = a^2 + b^2 - c^2 - 2a\sin\theta(a\sin\theta \pm \sqrt{c^2 - a^2\cos^2\theta}),$$

$$\sin\theta = \{c^2 - (a^2 + b^2) + r^2\}/2a\sqrt{b^2 - r^2},$$

we shall find a sub-division $c^2 > = < a^2 + b^2$ convenient; but we shall keep to the other division of $a > = < c$ as before.

Case (i.) $a < c$.

If $a^2 + b^2 < c^2$, $\sin\theta$ increases with r, and lies between $\{c^2 - a^2 - b^2\}/2ab$ and 1, and each value of $\sin\theta$ gives only one positive value of r^2: the least value of r^2 is zero, $\sin\theta = \{c^2 - a^2 - b^2\}/2ab$, and the greatest is $b^2 - (a-c)^2$, $\sin\theta = 1$: the curve consists of a simple figure of 8 (fig. xx.).

If $a^2 + b^2 = c^2$, the two loops touch with the axis of x for common tangent meeting the curve in six coincident points as in fig. xxi.: the equation in fact reduces to $4a^2y^2(b^2 - r^2) = r^6 = (x^2 + y^2)^6$.

If $a^2 + b^2 > c^2$, the values of r^2 are sometimes both positive, sometimes not: the limiting value of θ is clearly given by

$$a^2 + b^2 - c^2 \not< 2a\sin\theta(a\sin\theta + \sqrt{c^2 - a^2\cos^2\theta}),$$

or, in other words, $\sin\theta$ must not be greater than the value of θ that makes r^2 vanish: thus one available value of r^2 exists for all values of θ and two values form $\sin\theta = 0$ to $\sin\theta = (a^2 + b^2 - c^2)/2ab$.

The maximum height is found as before, and refers to the loop (fig. xxi.). In this case the curve is described by a single operation without any choice of direction at any point, or break of continuity.

Case (ii.) $a=c$.

Here we get at once the equations

$$r^2=b^2, \text{ and } r^2=b^2-4a^2\sin^2\theta,$$

which last, since $b<2a$, is the inverse of a hyperbola that becomes rectangular when $b^2=2a^2$ (fig. xxiii.).

Case (iii.) $a>c$.

In this case $\cos\theta<c/a, \sin\theta>\sqrt{1-c^2/a^2}$, and since we have

$$\sin\theta=\{(a^2-c^2)+(b^2-r^2)\}/2a\sqrt{b^2-r^2},$$

we see that $\sin\theta$ decreases with b^2-r^2 until, if possible, $b^2-r^2=a^2-c^2$, at which point it begins to increase again until $\sin\theta=1$.

When $r=0, \sin\theta=(a^2+b^2-c^2)/2ab$, which is always an admissible value: and between $\sin\theta=(a^2+b^2-c^2)/2ab$ and $\sin\theta=\sqrt{1-c^2/a^2}$ there are always two values of r^2, it only remains to see whether r^2 can ever become $b^2+c^2-a^2$: this of course depends on whether $a^2<=>b^2+c^2$.

If $a^2<(b^2+c^2)$ there are two real values of r between these limits, and one beyond, *i.e.*, from $\sin\theta=(a^2+b^2-c^2)/2ab$ to $\sin\theta=1$. On referring to the equations (1) and (2) we find that if $b^2>8(a^2-c^2)$ (and therefore $b^2+c^2>a^2$) there are four symmetrical maximum and minimum heights (fig. xxiv.); if $b^2=8(a^2-c^2)$ there are only two, through the coincidence of two pairs forming a point of inflexion with a horizontal tangent (fig. xxv.), while if $b^2<8(a^2-c^2)$ there are none (fig. xxvi.).

If $a^2=b^2+c^2$, the two limits coincide and we have a curve with a very approximately straight portion near the node: this, in fact, (fig. xxvii.) is Watts' parallel motion.

If $a^2>b^2+c^2$, the value of θ decreases till $r^2=0$, but cannot decrease further (fig. xxviii.) through the disappearance of the limit $\sin\theta=\sqrt{1-c^2/a^2}$.

In all the descriptions of these curves the upper half only has been referred to; and from the obvious symmetry it has only been necessary to discuss values of θ less than $\pi/2$. It is clear that we might from the beginning have proceeded under the heads $a<c$, $a=c$, $a>c$; but some consideration has led to the adoption of the process of this paper. There being two independent ratios, there are numerous cross connections between the curves, but these are best seen by the help of the diagrams.

INDEX TO DIAGRAMS.

	a	b	c	*No.*
I. $b > a + c$.				
(i.) $a < c$.				
All values of θ are here admissible with either sign; r never vanishes, and the curve has two separate loops cutting on the axis of x.				
$b^2 > (a+c)^3/c$, a maximum height at $\theta = \pi/2$ only	3	12	6	i.
$b^2 = (a+c)^3/c$, three coincident maxima there	$\frac{63}{16}$	12	$\frac{81}{16}$	ii.
$b^2 < (a+c)^3/c$, two maxima on each loop, besides those at $\theta = \pi/2$, ...	5	12	6	iii.
(ii.) $a = c$.				
The loops now touch at $\theta = 0$, and one half of each loop becomes a semi-circle; the other, half the inverse of a conic.				
$b^2 > 8a^2$, a special case of i., ...	3	12	3	iv
$b^2 = 8a^2$, do. ii., ...	$3\sqrt{2}$	12	$3\sqrt{2}$	v.
$b^2 > 8a^2$, do. iii., ...	$\frac{21}{4}$	12	$\frac{21}{4}$	vi.
(iii.) $a > c$.				
Here $a\cos\theta \not> c$, and the curve is bounded by two radii; we have two detailed loops.				
$b^2 < (a+c)^3/c$, the upper half of the circle in iv. joins the upper half of the inverse, and *vice versa*, ...	8	24	4·38	vii.
$b^2 = (a+c)^3/c$, the tangent at the lower vertex meets the curve at four points.				

	a	b	c	*No.*
$3c>a$, two other minima on each loop	6·84	18	4.5	viii.
$3c=a$, these move to the vertex, where a tangent meets the curve in six points, ...	$13\frac{1}{2}$	36	$4\frac{1}{2}$	ix.
$3c>a$, these minima disappear, ...	25·44	72	6	x.
$b^2<(a+c)^3/c$, we must consider in three cases, $b^2 \nless 10c^2+6a^2$, we have no other maxima, ...	25·5	72	6	xi.
$b^2<10c^2+6a^2, >8(a^2-c^2)$, there are four other maxima on each loop, ...	5	12	4	xii.
$b^2<10c^2+6a^2, =8(a^2-c^2)$, they coalesce giving two points of inflexion with a horizontal double tangent, ...	$8\frac{1}{4}$	18	$5\frac{1}{4}$	xiii.
$b^2<10c^2+6a^2, <8(a^2-c^2$, the points of inflexion are left, but the horizontal tangent goes, ...	9	18	6	xiv.
II. $b=a+c$.				
Here one branch of the curve has a zero value of r, all values of θ are possible if $a \not> c$.				
i. $a<c$.				
The whole curve may be continuously described, ...	$4\frac{1}{2}$	12	$7\frac{1}{2}$	xv.
ii. $a=c$.				
Three circles, ...	6	12	6	xvi
iii. $a>c$.				
In this case, $\cos\theta \not> c/a$,				
$7a<9c$, gives four other maxima, compare xii., ...	$6\frac{1}{4}$	12	$5\frac{3}{4}$	xvii.
$7a=9c$, these reduce to two as in xiii., ...	$6\frac{3}{4}$	12	$5\frac{1}{4}$	xviii.
$7a>9c$, and here disappear as in xiv., ...	8	12	4	xix.

https://doi.org/10.1017/S0013091500030455 Published online by Cambridge University Press

	a	b	c	*No.*
III. $b<a+c$.				
Here there is a zero value of r, but not every value of θ gives two admissible values of r^2.				
i. $a<c$.				
One value of r for each value of θ and *vice versa*.				
If $a^2+b^2<c^2$, we have a looped curve, and $\sin\theta$ must not be less than $\{c^2-(a^2+b^2)\}/2ab$, ...	6	12	15	xx.
If $a^2+b^2=c^2$, this angle becomes zero, and the loops touch instead of cutting, ...	6	12	$6\sqrt{5}$	xxi.
If $a^2+b^2>c^2$, some values of θ have two values of r; and two other nodes on the axis of x are developed, ...	6	12	9	xxii.
ii. $a=c$.				
Since $a^2+b^2>c^2$, this resembles xxii., except that the loops touch at the outer nodes, giving a double genesis, ...	$6\sqrt{2}$	12	$6\sqrt{2}$	xxiii.
iii. $a>c$.				
The curve is limited by the condition as regards $\cos\theta$; and we have if $a^2<b^2+c^2$ one value of r from $\sin\theta=(a^2+b^2-c^2)/2ab$ to $\theta=\pi/2$, and two from the latter limit to $\cos\theta=c/a$.				
$b^2>8(a^2-c^2)$, four maximum heights besides those on the axis, compare xii. and xvii., ...	7	12	6	xxiv.
$b^2=8(a^2-c^2)$, a horizontal inflexional tangent, compare xiii. and xviii., ...	$9\frac{1}{2}$	12	$8\frac{1}{2}$	xxv.
$b^2<8(a^2-c^2)$, the maxima vanish, compare xiv. and xix., ...	9	12	6	xxvi.
$a^2=b^2+c^2$ gives us Watt's motion, and the limits in the general case coincide, ...	15	9	12	xxvii.
$a^2>(b^2+c^2)$, the lesser limit disappears, and we have one value of r for each value of θ and *vice versa*, ...	20	12	12	xxviii.