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BOOK REVIEWS

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YANG, Y. *Solitons in field theory and nonlinear analysis* (Springer, 2001), xxiv+553 pp., 3 387 95242 X (hardback), £59.50.

Solitons date back to the nineteenth century when the Scottish engineer John Scott Russell observed a 'solitary wave' while he was riding by the side of the Edinburgh–Glasgow canal. He watched a boat stop suddenly and in doing so release a 'well defined heap of water' which propagated along the canal. What caught Russell's attention was the fact that the wave propagated preserving its original shape. This characteristic feature of solitons arises through a balancing between dispersion and nonlinearity. Russell's solitary wave was shown to be a solution of the first soliton equation, the Korteweg–de Vries (KdV) equation. Serious advances in the subject did not start until the late 1960s when Zabusky and Kruskal recognized the importance of some earlier work by Fermi, Pasta and Ulam on the dynamics of crystal chains. Zabusky and Kruskal observed from numerical simulations that the KdV equation exhibited wave-pulse-like solutions that had a very robust structure. These objects preserved their shape when they passed through other such structures and they came to be known as solitons to indicate their particle-like properties.

Although many soliton equations have their origins in fluid mechanics, some, including important equations such as Yang–Mills, have their origins in particle physics, nonlinear optics and field theory.

Field theory is a challenging area of mathematical physics, the underlying desire behind it being to present a theory which unifies all the fundamental forces of nature. It is the study of the equations that govern these fundamental forces. It draws upon many parts of mathematics, in particular topology and geometry, and by its very nature involves advanced and diverse mathematics. In addition to the mathematics many different aspects of physics are involved, both classical and quantum.

The study of solitons within field theory is rather different from in classical soliton theory. The classical theory is quite restrictive in as much as finding exact solutions can be a tricky procedure, often requiring techniques such as inverse scattering. Such techniques can vary considerably from one equation to another. In field theory the study encompasses a wider perspective, maybe with less emphasis on solutions, and calls upon tools from modern analysis.

Solitons in field theory and nonlinear analysis presents a thorough analysis of many different parts of field theory. The introductory chapter, called 'Primer of field theory', is just that. It is a whistle-stop tour of preliminary features such as classical mechanics, quantum mechanics, electromagnetism and relativity. Material is presented at a breathtaking pace and I feel you would not get far here if you did not have some knowledge of these areas already. At the end of the chapter, however, several books are recommended for a more 'in-depth study' of the topics treated in the chapter.

After this introductory chapter the book delves into some of the many aspects of field theory: asking questions of existence and uniqueness of solutions, asymptotic estimates, solutions on compact surfaces, to name just a few. As the title of the book suggests, the emphasis is mostly on the analysis aspect of the theory, but not exclusively so. For example, in the chapter on Chern–Simons systems there is quite a lot of work on Lie algebras. For the enthusiastic, there are some open problems to think about at the end of each chapter.

Solitons in field theory and nonlinear analysis is a solid research monograph with a lot of information in it and should certainly stimulate those who already have some background in field theory and analysis. Although it has a hefty 553 pages, I feel that for a full understanding one would need to delve into some of the recommended books as well. Reading this book is quite challenging and is not for the faint hearted.

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DE LA HARPE, P. *Topics in geometric group theory* (University of Chicago Press, 2000), vi+310 pp., 0 226 31721 8 (paperback), £13.00, 0 226 31719 6 (hardback), £28.50.

Geometric group theory is a relatively new subject with old roots. The deep connections between groups and geometry have been recognized since the nineteenth century, with Klein's Erlangen Program highlighting the importance of the symmetry group of a geometry, and Poincaré's work on topological spaces, manifolds and fundamental groups.

The use of topological and geometric methods as a tool to prove algebraic theorems about groups can be traced back to the early twentieth century work of Dehn on the fundamental groups of surfaces. This led to the development of the subject known for much of the twentieth century as *combinatorial group theory*, often defined as the study of groups expressed in terms of generators and defining relations (*presentations*).

For those of a topological bent, a presentation of a group G is (essentially) a two-dimensional cell complex whose fundamental group is isomorphic to G. Given such an object, one can apply the machinery of topology (covering spaces, continuous maps) to prove things about G. For the more algebraically minded, one can instead work directly with the presentation, using combinatorial operations on words in the generators in place of topological tricks. Combinatorial group theory appears in both guises (and a spectrum of compromise forms) throughout its history. However (at least in the reviewer's opinion), the topology is always present, either explicitly or implicitly.

The topological version of combinatorial group theory is for the most part purely topological rather than geometric, in the sense that no underlying metric plays any role in the arguments (although the spaces involved are all metrizable as nicely embedded subspaces of Euclidean space). The notable exception to this is what is known as *small cancellation theory*. Again, this can trace its roots back to the work of Dehn on surface groups. In modern parlance, Dehn produced an algorithm to solve the word problem for the fundamental group of an orientable surface of genus at least 2. His arguments effectively use the negative curvature of such surfaces, and the resulting algorithm works in linear time (as a function of the length of the input word). Small cancellation theory is a combinatorial way of applying curvature arguments to more-general group presentations.

Geometric group theory emerged in the 1980s, in work of Gromov. The distinction from combinatorial group theory is that an explicit metric is always present: namely, the word metric on the group G itself with respect to a given (finite) generating set for G. This metric is of course dependent on the choice of generating set, but many of the coarse properties of the metric turn out to be independent of this choice, which makes it a useful tool. It is the idea of viewing a group as a metric space that is inherently new here, and leads to further new concepts such as the 'boundary' of a group. Gromov initially used this idea in his classification of groups of