CORRESPONDENCE.

ON THE EMPLOYMENT OF THE "INSTITUTE OF ACTUARIES' LIFE TABLES" IN FINDING THE VALUE OF AN ANNUITY ON THE LAST SURVIVOR OF THREE LIVES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—It may be well to point out that notwithstanding the terms of "Simpson's formula" are so conveniently tabulated in the Institute Tables, the employment of these tables in the mode recently suggested to you involves the introduction of an error from which the ordinary formula is free. The formulas may be written as follows.

Simpson's:—

$$a_{\overline{xyz}}=(a_x+a_y-a_{xy})+(a_x+a_z-a_{xz})-(a_x+a_w-a_{xw}).$$

The ordinary formula:—

$$a_{\overline{xyz}}=(a_x+a_y+a_z)-(a_{xy}+a_{xz}+a_{yz})+a_{xw}.$$

Both are equally tainted with the error of the assumption (expressed at the close of each), that because $a_w=a_{yz}$, therefore $a_{xw}=a_{xyz}$; but the difference in the notation of these two formulas will be found (upon cancelling whatever is common to both) to be that Simpson's third term has a_w instead of the a_{yz} in the second term of the common formula. These are theoretically equal, but upon converting Simpson's third term in gross into figures by adopting for $a_{\overline{xw}}$ a value tabulated in the Institute Tables, a new error is here introduced through the incorporation of a value for a_w which is only a more or less close approximation to the true and known a_{yz} instead of being exactly equal to it as contemplated in the formula itself; and to this circumstance the difference ·0935 between the results of the two calculations on p. 267 is traceable. Thus, in extracting 20·8569 or $a_{\overline{30,56}}$ (that is, $a_{30}+a_{56}-a_{30,56}$) the value 11·7242 of a_{56} replaces the known true value 11·8177 of $a_{40,50}$; and their difference is the ·0935 just mentioned. To guard against the introduction of this difference, which will produce sometimes an excess and sometimes a deficiency, the notation of Simpson's formula, if for use with the Institute Tables, may be altered to

$$a_{\overline{xyz}}=a_{\overline{xy}}+a_{\overline{xz}}+a_{xw}-(a_x+a_{yz})$$

Working in this manner with the given example, we have

$a_{\overline{30,40}}$		22·2274
$a_{\overline{30,50}}$		21·2947
$a_{30,56}$		10·7347
		54·2568
a_{30}	19·8674	
$a_{40,50}$	11·8177	
		31·6851
As by the common formula p. 267		22·5717

The facilities afforded by the last-survivor values are indeed thus reduced, but the avoidance of the new error is a matter of greater importance.

This error is, it will have been seen, quite extra to the well-known error involved in the acceptance of a_{xw} for a_{xyz}, and which, as already pointed out, equally affects Simpson's formula and the common one. For the lessening of this error, Milne (Article 529) has given estimates. He would have taken for w neither the fractional age at which a single life annuity exactly equals an annuity upon two joint lives aged y and z; nor the nearest integral age; nor yet (in this example) the next higher integral age; but he would have put for w, 55·8. Thus,

a_{55}	12·0938	a_{55}	12·0938
a_{56}	11·7242	$a_{40,50}$	11·8177
	·3696)		·2761(·7

Milne would increase 55·7 to 55·8 for w.

$$a_{30,55}=11{\cdot}0378$$
$$a_{30,56}=10{\cdot}7347$$
$$\cdot 3031$$
$$\cdot 8$$
$$\cdot 2425$$
$$a_{30,55}=11{\cdot}0378$$

Milne would put $a_{30,55\cdot 8}$ or 10·7953 for $a_{30,40,50}$,
instead of the $a_{30,56}$ or 10·7347 put for ,, at p. 267.

So he would, in effect, have added ·0606
to the result on p. 267 22·5717

and have given 22·6323 for $a_{\overline{30,40,50}}$.

This is very close to the value, 22·616, which is the mean of those you have determined by Mr. Woolhouse's formula.

I remain, Sir,
Your obedient servant,

EDWARD SMYTH.

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