become readily accustomed; at any one point we can draw an infinite number of mutually perpendicular lines. But as long as we stick to a finite number of directions, function space is just like the *n*-dimensional space we have been discussing earlier. Indeed, we realise with a shock that it is not merely just like-it is the same.

Let me bring out this point. Suppose we take just two functions, say,  $x^2$  and  $x^3$ , and do not let our minds wander outside the set of functions  $ax^2 + bx^3$ . where a and b are constants taking all values from minus infinity to plus That means that we are sticking to one plane in function space. infinity Then I assert that the geometry of that plane is absolutely identical in every respect with the familiar Euclidean geometry of the plane. The only difference lies in the interpretations of words such as point and straight line in terms of ideas which lie outside the geometry proper.

## The real and the complex.

That brings me to the end of what I have to say. But I shall add one remark about imaginary and complex coordinates. Every student of plane geometry is thrilled to learn that a circle passes through two imaginary points at infinity. It is an indecent and disloyal thrill of which he should be ashamed. The geometry of the plane with points having complex coordinates is not the geometry of two dimensions but of four, and it is only the bullying algebraist who holds the contrary. At the other end of the scale, the quantum theorist insists on having everything complex in his function space. What he succeeds in doing with this complex function space is indeed marvellous, but I hold that the concept of function space has in itself nothing to do with complex numbers, and that it should be explored, at first at least, in terms of real elements only As a last word, let me remark that the whole vast theory of functions of a complex variable could never have been constructed without the Argand diagram, in which the point has two real coordinates. Complex numbers, treated as entities and not as number-pairs, are a pain in the neck to the true geometer. J. L. S.

## CORRESPONDENCE.

## To the Editor of the Mathematical Gazette.

DEAR SIR,-In his review of my Theory and Application of Mathieu Functions, Mr. T. V. Davies says : "The reader, however, who expects the miscellaneous integrals to be of the same comprehensive and complex variety found in Watson's Bessel Functions, will be disappointed with Chap. XIV" I share the reviewer's disappointment, but am unaware that such integrals are extant. I contributed some 40 per cent. of new material in the text, and I think someone else might supply the missing integrals.

It would not have been difficult to increase the length of the book by 50 per cent. using existing material. But in these miserable days of almost astronomical printing costs and evanescent paper supply, an author must perforce be eclectic rather than exhaustive.

I take this opportunity of correcting some errors :

p. 17, in (6), for 1109 read 609.

in (8), (9), for 17 28000 read 27216 00000.

p. 310, l. 2, for  $k_1^2$  read  $k_1^4$ .

1. below (6), for  $\omega h/c$  read  $(\omega/c)^{\frac{1}{2}}h$ ,

and for  $\omega h^2/4c^2$  read  $\omega h^2/4c$ .

p. 313, in (1) and in the third line above (5), for  $\omega/c$  read  $(\omega/c)^{\frac{1}{2}}$ .

N. W. MCLACHLAN

263