

The preceding remarks give some indication of the traps that beset the reader. If he perseveres and is prepared to treat every statement with due caution he will find much to reward his efforts. To the reviewer the section (§9.8) on algebraic independence of arithmetic functions is one of the most interesting and owes much to the author's own work. Moreover, there is a wealth of bibliographical information at the end of each chapter to encourage the reader to further study.

R. A. RANKIN

MINC, H., *Permanents* (Encyclopedia of Mathematics and its Applications, vol. 6, Addison-Wesley Advanced Book Programme, 1978), xviii+205 pp., \$21.50.

In his preface, the author states: "Permanents made their first appearance in 1812 in the famous memoirs of Binet and Cauchy. Since then 155 other mathematicians contributed 301 publications to the subject, more than three quarters of which appeared in the last 19 years. The present monograph is the outcome of this remarkable re-awakening of interest in the permanent function." (In fact, 303 publications are quoted in the Bibliography.)

In an attempt to give a complete account of the theory of permanents, the author has traced their development from their inception until the present day (1978). Chapter 1 contains a historical survey in which only the classical results are discussed in detail, while Chapter 2 covers the basic elementary properties of permanents and Chapter 3 is devoted to the permanent of $(0, 1)$ matrices, including the classical theorem of Frobenius and König (Let A be an $n \times n$ matrix with non-negative entries. Then $\text{per}(A) = 0$ if and only if A contains an $s \times t$ zero submatrix such that $s + t = n + 1$.)

The next three chapters are given over entirely to inequalities involving permanents, either upper and lower bounds for permanents of (mainly) non-negative matrices, or, in the case of Chapter 5, the Van der Waerden conjecture. This states that if S is a doubly stochastic $n \times n$ matrix then $\text{per}(S) \geq n!/n^n$. The whole chapter is devoted to a discussion of this, giving the then most recent developments in the pursuit of a solution to this conjecture. In 1980 it was proved to be true by a Russian author, Egoritsjev, in Russian. However, J. H. Van Lint has produced an account of Egoritsjev's proof in *Linear Algebra and its Applications*, **39** (1981), pp. 1-8, entitled "Note on Egoritsjev's Proof of the Van der Waerden Conjecture."

Chapter 7 discusses methods of evaluating permanents and compares their efficiency, while the final chapter deals with a variety of applications of permanents, e.g. to the estimation of the number of latin squares of a given order, the number of non-isomorphic Steiner triple systems of a given order and to the n -dimensional dimer problem. A list of conjectures and unsolved problems completes this final chapter.

It has to be said that this book is well-written and beautifully produced, with few misprints. To anyone working with, or requiring knowledge of, permanents, it should be regarded as essential. It is likely to become the standard reference on permanents.

E. SPENCE

ASIMOW, L. and ELLIS, A. J., *Convexity Theory and its Applications in Functional Analysis* (London Mathematical Society Monograph 16, Academic Press, 1980), x+226 pp. £23.20.

Convexity theory is a beautiful subject, combining geometry with algebra and analysis, and providing a unified approach to classical and modern results in areas such as potential theory, ergodic theory and operator algebras. It tells us that it is always possible to decompose points in a compact convex set into suitable combinations of extreme points (vertices), and when it is possible to do so uniquely. In finite dimensions this is a classical theorem of Carathéodory; an infinite-dimensional example is Bochner's Theorem on positive-definite functions.

After the discovery of Choquet's Theorem in 1956 and the Bishop-de Leeuw Theorem in 1960, there was much activity in general convexity theory for about 10 years, before a change of emphasis occurred in the 1970s. Alfsen's book "Compact convex sets and boundary integrals" (Springer, 1971) therefore appeared at a propitious time, and it has become a standard reference