# Study of centering CCD image of faint satellites near a bright primary object<sup>†</sup>

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**Abstract.** The polynomial-fit method is applied to remove the uneven background of a satellite when it is near a bright primary object. Detailed analysis of this method is given. Some useful conclusions are drawn from the results of simulated data.

**Keywords.** astrometry, solar system: general, techniques: image processing, planets and satellites: general

## 1. Introduction

When observing a faint satellite close to a bright primary object, the satellite is embedded in the primary's halo light, which has a gradient across the satellite's image. Usual methods, such as 2-dimension Gaussian with a constant background fitting, cannot provide a correct center of the satellite.

Many authors presented various methods to remove the systematic effects on centering the position of a satellite's image. But each method is only suitable for a special case. Here we attempt to find a general approach to tackle this problem. The "polynomial-fit' method has been mentioned and employed previously for removing uneven background without any detailed discussions or argumented conclusions given. The goal of this work is to analyse which order of the polynomial is suitable for most cases.

#### 2. Results from simulated data

First, the simulated images of a satellite near a bright primary were created under various assumptions, such as different distances between the satellite and the primary, different intensity ratios between the satellite and the primary, and so on. Second, different orders of the polynomial-fit method are used to fit the background when calculating the center of a satellite. Third, comparisons between the calculated and simulated centers are made.

Normally the images of both the satellite and the primary object can be represented by a two-dimension Gaussian model, with respectively different parameters of peak intensities  $I_s$  and  $I_p$ , center positions  $(x_s, y_s)$  and  $(x_p, y_p)$ , and Gaussian radius parameters  $R_s$  and  $R_p$ . The combined intensity of a satellite and a primary can be simulated as:

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$$I(x,y) = I_p EXP(-\frac{(x-x_p)^2 + (y-y_p)^2}{2R_p^2}) + I_s EXP(-\frac{(x-x_s)^2 + (y-y_s)^2}{2R_s^2}) + N \quad (2.1)$$

where N is the random error. To calculate the center of the satellite from the simulated data, a square area around the center of the faint satellite image is selected first. Then the following model is used to fit the center of the satellite.

$$I(x,y) = I_0 EXP(-\frac{(x-x_0)^2 + (y-y_0)^2}{2R_0^2}) + \sum_{i=0}^n \sum_{j=0}^n a_{ij}(x-x_0)^i (y-y_0)^j$$
(2.2)

where  $I_0$  is the fitted intensity of the image of the satellite,  $(x_0, y_0)$  is fitted center of the satellite, and n is the order of the polynomial (n=1,...5). Here  $I_0$ ,  $R_0$ ,  $(x_0, y_0)$  and  $a_{ij}$  are fitted by least squares.

Since the correct positions of the satellite are known when simulated under different orders of polynomial n(n = 1, 2, 3, 4, 5), the mean difference between the simulated and calculated center positions (under different random errors) can be used to judge the validity of different orders of the polynomial. Table 1, 2 and 3 respectively show the simulation results for various values of  $I_p/I_s$ ,  $r_{ps}$  and  $R_p$ .

**Table 1.** Results of five models with different  $(I_p/I_s)$  under different  $r_{ps}/R_p$  when  $R_p=32$  pixel. Note: "×" means the result is not convergent with this model, " $\triangle$ " means the residual is very large with this model, and " $\bigcirc$ " means good results for this model.

model	$  I_{p}   (12.5 \sim 25)$	$ I_s (r_{ps}/R_p; (25 \sim 37.5)) $	=2) (37.5~250)	$  I_p / (12.5 \sim 25)  $	$ I_s \ (r_{ps}/R_p \\   (25 \sim 100) $	=3)   (100~250)	$ \begin{vmatrix} I_p / I_s & (r_{ps} / R_p = 4) \\ (1.0 \sim 250) \end{vmatrix} $
1-deg 2-deg 3-deg 4-deg 5-deg	× < 000	× 4 4 00	× △ △ ○	40000	40400	× × OO	00000

**Table 2.** Results of five models with different  $r_{ps}/R_p$  under different  $R_p$  (unit: pixel).

model	$\begin{vmatrix} r_{ps}/2\\ (1\sim4) \end{vmatrix}$	$\begin{array}{c} R_p \ (R_p \\   (4 \sim 5) \end{array}$	=8)   (>5)	$r_{ps}/l$ (1~3)	$\begin{array}{c} R_p & (R_p = \\   (3 \sim 4) \end{array}$	=16)   (>4)	$r_{ps}/I$ (1~2)	$\begin{array}{c} R_p & (R_p = \\   (2 \sim 3) \end{array} \end{array}$	=32)   (>3)
1-deg 2-deg 3-deg 4-deg 5-deg	$  \begin{array}{c} \times \\ \times \\ \bigtriangleup \\$	× × △ ○ ○	00000	× × 000		00000	×	× 0000	00000

### 3. Discussion

Synthetically produced star images have been used to investigate the properties of the polynomial-fit method. Five polynomial models have been tested under a wide range of

Table 3. Results of five models with different  $R_p$  (unit:pixel) under different  $r_{ps}$  (unit: pixel). Here  $I_p/I_s = 25/8$ 

model	$\begin{vmatrix} R_p \\ (10 \sim 12) \end{vmatrix}$	when $r_{ps} = (12 \sim 16)$	=30)   (16 $\sim$ 34)	$\begin{vmatrix} R_p \\ (10>12) \end{vmatrix}$	when $r_{ps}$ : (12~14)	$=50)$   (14 $\sim$ 34)	$ R_p \text{ (when } r_{ps} = 80) \ (10 \sim 34)$
1-deg 2-deg 3-deg 4-deg 5-deg	× × 0 0		× × O O	00000	04000		00000

observational conditions. The numerical calculations show that the polynomial-fit method is useful for correcting the effects of the strong halo light while centering the image of a faint satellite. In addition, the applicable range of the 1st-order polynomial model is limited, while the 5th-order polynomial model produces the highest precision. Usually, the 2nd, 3rd and 4th-degree polynomials can also satisfy the requirements of accuracy level for actual observations. After obtaining the approximate values of  $I_p/I_s$ ,  $r_{ps}/R_p$ , and  $R_p$ , we can determine which model is suitable with the help of Tables 1-3.

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