RELATIVISTIC EFFECTS IN GEODETIC VLBI MEASUREMENTS

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ABSTRACT: The various relativistic effects occuring in VLBI measurements are discussed. A concrete example showing the influence of the gravitational delay due to the Sun upon the delay residuals from fit in a Mark III geodetic VLBI experiment is given. It is argued that regular geodetic VLBI observations might provide the best test of Einstein's theory of gravity on the post-Newtonian level in the near future.

Rapid advance in VLBI techniques (e.g. the general use of the Mark III system) has opened the possibility to determine astronomical angular separations with an accuracy of about 0.5 milliarcsec. At this level of accuracy, especially when typical geodetic target quantities like crustal dynamics etc. are to be determined, it will become increasingly important to formulate the whole process of VLBI measurement in a relativistic framework essentially following the light rays from the (extragalactic radio) sources through our clumpy universe and solar system to the radio antennas (Fig. 1). The cosmological optics part would not be of interest in this respect if there were no proper motions on the celestial sphere. However, geodetic VLBI measurements will tend to focus on certain quasar substructures (hot spots etc.) that easily might attain intrinsic velocities comparable with the speed of light. Even worse, the apparent proper motion might be enhanced by projection or gravitational lensing effects, so that apparent (superluminal) proper motions of some mas/y might result.
The more local relativistic effects upon group delay on a 2-element interferometer such as aberration, gravitational time delay and relativistic clock rates are summarized in Tab. 1 (see Finkelstein et al. 1983). Here, $\tau_0 = 1/c\, \vec{b}\cdot\vec{k}$ indicates the "Newtonian" value for the group delay, $\vec{b}$ is the geocentric baseline vector and $\vec{k}$ the heliocentric unit vector towards the radio source. Since we define the baseline at signal arrival time at antenna 1 the geocentric velocity $\vec{v}_2$ of antenna 2

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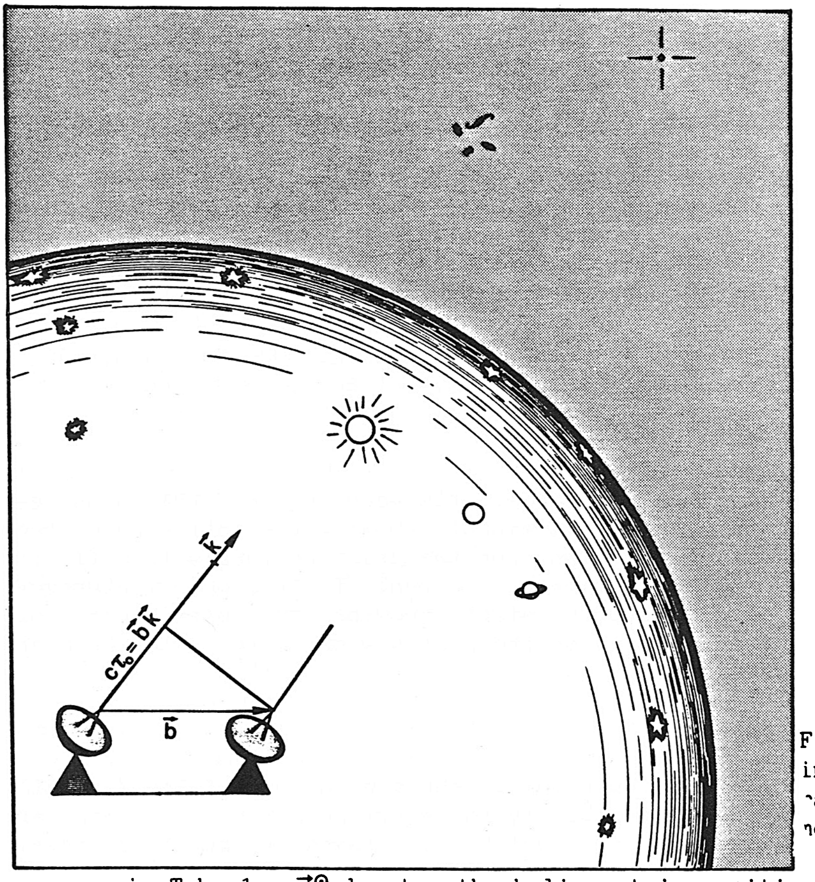

Fig.1 Geometry in a two-element radiointerferometer system

appears in Tab. 1. $\vec{x}_i^{\odot}$ denotes the heliocentric position vector of antenna i, $\vec{v}_{\oplus}$ the heliocentric velocity vector of the Earth, E_i the elevation angle of source at antenna i etc.

Now, the total accuracy of the VLBI-technique presently is of the order of 0.1 nsec, so theoretically one might wish to keep terms with amplitude $\leqslant$ 10 psec. At this level of accuracy the gravitational time delay due to Jupiter and Saturn generally can be neglected, the corresponding term from the Earth, however, should be retained. The "relativistic" terms in the expression for the group delay therefore read:

$$\tau \approx \tau_0\left[1 - \frac{1}{c}(\vec{v}_{\oplus} + \vec{v}_2)\,\vec{k} + \frac{1}{c^2}\left\{(\vec{v}_{\oplus}\cdot\vec{k})^2 - (U_{\odot} + U_{\oplus} + \frac{1}{2}\vec{v}_{\oplus}^2)\right\}\right]$$

$$- \frac{1}{2c^3}(\vec{v}_{\oplus}\cdot\vec{k})(\vec{b}\cdot\vec{v}_{\oplus}) + \frac{\vec{b}\cdot\vec{v}_{\oplus}}{c^2} + \tau_g$$

where τ_g denotes the total gravitational time delay, U the gravitational potential at antenna 2 and the term $\vec{b}\cdot\vec{v}_{\oplus}/c^2$ achieves geocentric clock synchronisation (Soffel et al. 1984).

The effect of $\tau_g^{\odot}$ in a geodetic VLBI experiment is demonstrated in

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Effect	delay τ (baseline = 6000 km, $\tau_o \lesssim 20$ ms)	Magnitude in ns	
Aberration			
diurnal	$\tau_o \left[\frac{1}{c} \vec{v}_2 \cdot \vec{k} + \frac{1}{c^2} (\vec{v}_2 \cdot \vec{k})^2 \right]$	$40 + 8\ 10^{-5}$	
annual	$\tau_o \left[\frac{1}{c} \vec{v}_\oplus \cdot \vec{k} + \frac{1}{c^2} (\vec{v}_\oplus \cdot \vec{k})^2 \right]$	$2\ 10^3 + 0.2$	
differential	$\tau_o \left[\frac{\tau_o}{c} \dot{\vec{v}}_\oplus \cdot \vec{k} + \ldots \right]$	10^{-5}	
Grav. delay			
Sun		limb	170
		1°	45
		$\sim 180^\circ$	0.4
Jupiter	$\frac{r_s^{\odot,♃,♄}}{c} \ln \left[\frac{x_1^i + \vec{x}_1^i \cdot \vec{k}}{x_2^i + \vec{x}_2^i \cdot \vec{k}} \right]$	limb	1.7
		1°	0.01
Saturn		limb	0.7
		1°	0.0014
Earth	$\frac{r_s^\oplus}{c} \ln \left[\frac{1 + \sin E_1}{1 + \sin E_2} \right]$	max	0.021
Clock rates			
$t \leftrightarrow \tau$	$\frac{\tau_o}{c^2} \left[\left(\frac{GM_\odot}{R_\oplus^\odot} + \frac{1}{2} v_\oplus^2 \right) + \frac{GM_\oplus}{R_\oplus} \right]$	$0.3 + 0.014$	

Tab.1: Aberration, gravitational delay and relativistic clock rates in a 2-element VLBI interferometer

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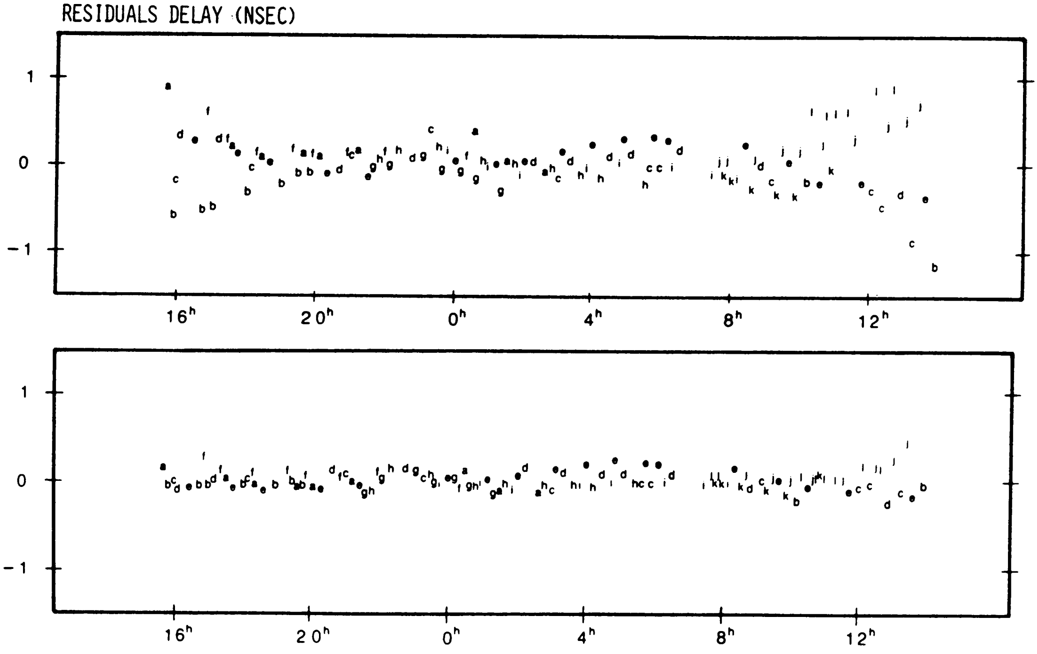
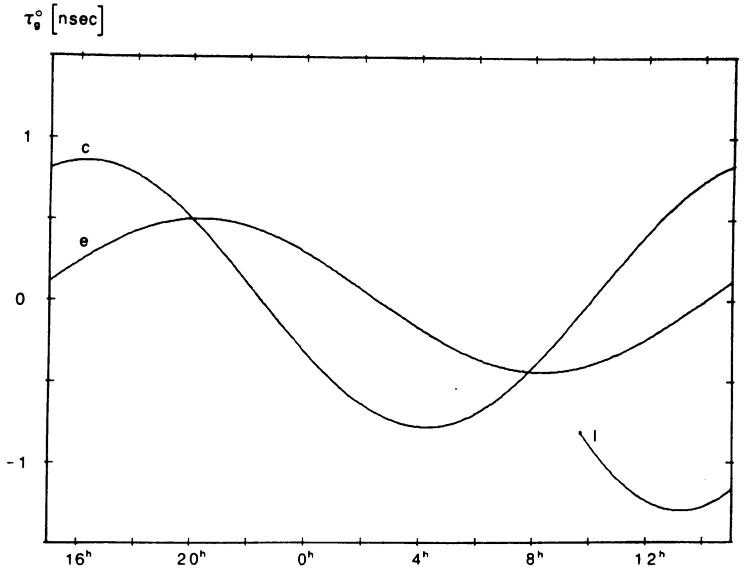


Fig.2 Delay residuals from fit for an Effelsberg - Haystack VLBI experiment from May 5-6 1983 without (a) and with (b) corrections for the gravitational solar time delay. The 12 radio sources observed are labelled by a - l.



RADIO SOURCES

a: 4C39.25
b: 0528+134
c: 0552+398
d: 0212+735
e: 1803+784
f: OJ287
g: 3C273B
h: OQ208
i: 3C345
j: 3C454.3
k: 2216-038
l: 0106+013

Fig.3 Temporal behaviour of gravitational solar time delay for three selected radio sources for the experiment of Fig.2.

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THE FUNDAMENTAL STATION WETTZELL RADIOTELESCOPE

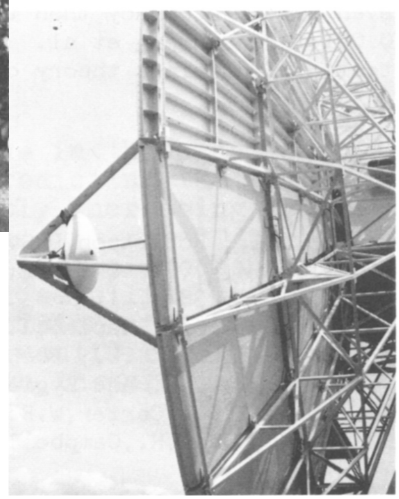
20 m dedicated geodetic VLBI radiotelescope

Antenna

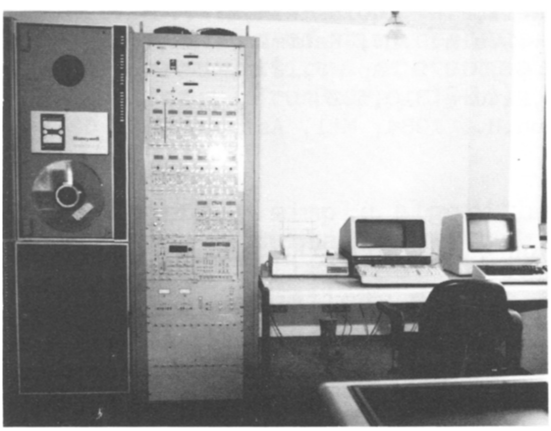
Type	on axis cassegrain
Main reflector diameter	20 m
Main reflector shape	true paraboloid
Main reflector focal length	9 m
Surface tolerance of main reflector – rms	0.4 mm
f/d ratio	0.45
Subreflector diameter	2.7 m
Subreflector shape	true hyperboloid
Surface tolerance of subreflector – rms	0.03 mm
Focus stability – rms	1 mm
Frequency capability	1.2 GHz – 25 GHz

Mount

Type	turning head
Az. range	± 270° from south
El. range	0° to 90°
Az. tracking velocity	0°.001/sec to 1°/sec
El. tracking velocity	0°.001/sec to 1°/sec
Az. acceleration	3°/sec²
El. acceleration	1°.5/sec²
Az. slewing velocity	3°/sec
El. slewing velocity	1°.5/sec
Pointing accuracy	better than 15″
Az.-axis verticality	± 3″
El.-axis orthogonality	5″
Control computer	pdp 11/23



Backstructure of 20 m radiotelescope



Operators bay with VLBI Mark III DAT magnetic tape unit and HP 1000 terminal

Receiver

S/X-band dualfrequency corrugated horn (JPL-design)
S/X-band receiver: 2.3 GHz and 8.4 GHz
Liquid helium cooled GaAs FET front end amplifier
Bandwidth: 400 MHz
Mark III VLBI data acquisition terminal
Magnetic tape: Honeywell 96
Control computer: HP 1000
Other planned frequencies: 5 GHz and G. P. S. frequencies

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Fig. 2. It shows residuals for the delay for an Effelsberg - Haystack VLBI experiment from May 5-6, 1983 without (Fig. 2a) and with (Fig. 2b) correction for the solar gravitational delay. The temporal behaviour of $\tau_g^{\odot}$ for this experiment is indicated in Fig. 3 for three selected radio sources.

If we speak about the german contribution to geodetic VLBI measurements we are mainly talking about the work done with the 20m radiotelescope in Wettzell in the Bavarian forest (Fig. 4). Observations in Wettzell are included in various international projects such as MERIT, POLARIS/IRIS with main interest in polar motion and length of the day. As a by-product of regular observations e.g. the PPN space-curvature parameter γ can be determined. A first analysis of VLBI data from these projects (Sept. 1980 - Jan. 1984) was performed by Robertson and Carter (1984). Although only 23 observing sessions out of a total of 163 contained sources within 10° from the Sun the result was very accurate:

$$\gamma = 1.008 \pm 0.005$$

In a recent analysis taking the data through October 1984 Carter et al. (1985) improved this value for γ to 1.000 with a formal standard error of 0.003. Incorporating a few more sources that lie close to the ecliptic into the regular schedules leads to the hope that geodetic VLBI measurements might determine the value for γ with comparable or even better accuracy than the VIKING time delay measurements ($\gamma = 1. \pm 0.001$; Reasenberg et al. 1979) and thus provide the most accurate test of Einstein's theory of relativity on the post-Newtonian level.

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DISCUSSION

<u>Cannon</u> : at the level of accuracy you quote, will the information on phase not become important ?

<u>Soffel</u> : yes, in the future one will have to refer to radio images to get an idea about proper motions, for instance of the quasar substructures.

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