or since ${}^{n\alpha+\beta}P_{\alpha+\gamma}$ gives the unrestricted number of ways of filling up the places, the probability that no object is in a place correspondingly marked in a fortuitous distribution is

$$\sum_{k=0}^{\alpha} \frac{(-n)^k}{\underline{|k}} \cdot \frac{{}^{\alpha}C_k}{{}^{n\alpha+\beta}C_k}.$$

The particular result for the problem enunciated at the beginning is got by putting $n=1$, $\beta=\gamma=0$

$$i.e. \quad \sum_{k=0}^{\alpha} \frac{(-1)^k}{\underline{|k}}$$

or $\underline{|\alpha} \; \sum_{k=0}^{\alpha} \frac{(-1)^k}{\underline{|k}}$ according to the mode of statement of the problem.

WILLIAM MILLER.

Analytical Note on Lines Forming a Harmonic Pencil.

The following is a simple proof of the theorem that the concurrent lines whose equations are

$$a_1 x + b_1 y + c_1 = 0 \quad\text{.......................................(1)}$$
$$a_2 x + b_2 y + c_2 = 0 \quad\text{.......................................(2)}$$
$$a_1 x + b_1 y + c_1 = k\,(a_2 x + b_2 y + c_2) \quad\text{....................(3)}$$
$$a_1 x + b_1 y + c_1 = -k\,(a_2 x + b_2 y + c_2) \quad\text{.................(4)}$$

form a harmonic pencil.

Let a line through the origin parallel to the line (2) intersect (4) in $A\ (x_1, y_1)$, and (3) in $B\ (x_2, y_2)$.

The pencil is harmonic if the mid-point of AB, $C\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, lies on (1).

Since OAB is parallel to (2) we have

$$a_2 x_1 + b_2 y_1 = a_2 x_2 + b_2 y_2 = 0.$$

Hence since $A\ (x_1, y_1)$ lies on (4),

$$a_1 x_1 + b_1 y_1 + c_1 = -k\,c_2,$$

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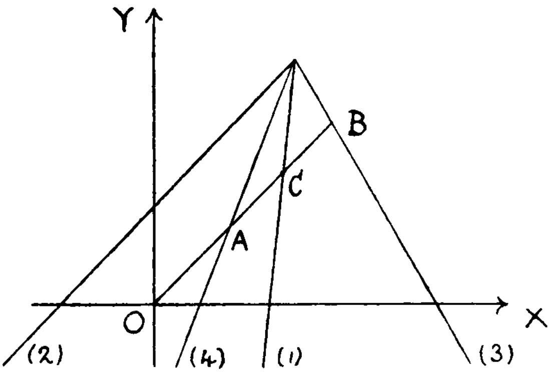
and since $B(x_2, y_2)$ lies on (3)

$$a_1 x_2 + b_1 y_2 + c_1 = +k c_2.$$

Adding, and dividing by 2,

$$a_1 \frac{x_1 + x_2}{2} + b_1 \frac{y_1 + y_2}{2} + c_1 = 0.$$

Therefore the mid-point of AB lies on (1).



N. M'Arthur.

Trigonometrical Ratios of the half-angles of a Triangle (Geometrical Proofs).

1. ABC is a triangle; bisect angle A by AE; produce AB; draw BDF and CEG perpendicular to AE; join FG.

$GCFB$ is a cyclic trapezium

$$\therefore \quad GC \cdot FB + CF \cdot BG = BC \cdot FG,$$

$$\therefore \quad 2\,EC \cdot 2\,DB + (b-c)^2 = a^2,$$

$$\therefore \quad 4\,EC \cdot DB = a^2 - (b-c)^2$$

$$= (a - b + c)(a + b - c)$$

$$= 4(s-b)(s-c),$$

$$\therefore \quad EC \cdot DB = (s-b)(s-c).$$

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