

ARTICLE

Nonseparability of credit card services within Divisia monetary aggregates

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Abstract

We use the New Keynesian Dynamic Stochastic General Equilibrium (DSGE) framework and Vector Autoregressive (VAR) to investigate the usefulness and relevancy of monetary services, augmented to include credit card transaction services. We use the new credit-card-augmented Divisia monetary aggregates in the models to further the existing research on their usefulness and relevancy. In this research, we compare three different monetary aggregates within the New Keynesian framework: (1) the aggregation-theoretic “true” monetary aggregate, (2) the credit-card-augmented Divisia monetary aggregate, and (3) the simple sum monetary aggregate.

We acquire the following primary results. (1) The credit-card-augmented Divisia monetary aggregate tracks the theoretical (true) monetary aggregate, while simple sum does not. Although this result would be expected from the theory in classical economic models, the result is not an immediate implication of the theory in New Keynesian models and therefore needs empirical confirmation. (2) Under the recursive VAR framework, the credit-card-augmented Divisia monetary aggregate serves as a preferable monetary policy indicator compared to the traditional federal funds rate. (3) On theoretical grounds, we find that the separability condition for existence of a monetary aggregator function could fail, if credit card deferred payment services were excluded from the monetary services block, unless all markets are perfect.

Keywords: credit-card-augmented Divisia monetary aggregates; New Keynesian DSGE; credit card services; VAR

JEL Classification: E12; E41; E51; E52; E58

1. Introduction

Does credit matter in recent economic analysis? The credit sector has become widely viewed as one of the most important driving forces of economic fluctuation. The credit sector and credit-related variables have recently been found to be both causative factors in the occurrence of economic events and one of the variables most affected by the consequences of economic events. For example, the prevailing view of the Great Recession is that the failure in credit management was associated with the financial crisis. More recently, an unusual surge in credit supply occurred during the COVID-19 pandemic outbreak.

According to the Federal Reserve Board Payment Study (2019), the number of credit card payments and the amount of those credit card payments, both general purpose and private label, are increasing rapidly.¹ The number of credit card payments and the amount of credit card payments were 26.8 billion and \$2.55 trillion in 2012, 33.7 billion and \$3.05 trillion in 2015, and 44.7 billion and \$3.98 trillion in 2018, respectively. The annual growth rate of the number of credit card payments and the amount of credit card payments were 7.9% and 6.2% from 2012 to 2015 and 9.9% and 9.3% from 2015 to 2018, respectively. According to the St. Louis Federal Reserve Bank’s FRED

database, the value of outstanding credit card debt in the US has recently surpassed 1 trillion dollars. The increasing credit utilization has resulted in increasing research on its role in the economy, expanding rapidly since 2016 on early research beginning with Brito and Hartley (1995).

But the conventional monetary aggregates do not include the deferred payment transaction services of credit cards. We hypothesize that the credit-card-augmented Divisia monetary aggregates (hereafter “credit-augmented Divisia”), introduced by Barnett and Su (2016, 2018) and Barnett et al. (2023), would have more explanatory power than the traditional monetary aggregates, both empirically and theoretically. Simple sum monetary aggregates cannot include credit card services, since credit card balances are liabilities while monetary balances are assets, and accounting conventions do not permit adding liabilities to assets. But Divisia aggregation is based on economic aggregation theory and index number theory, which aggregate over service flows, regardless of whether produced from assets or liabilities.

In fact, in a New Keynesian framework with imperfect markets, we find theoretical reason to doubt the existence of an aggregator function over monetary services without credit card deferred payment services included. Without perfect markets, we find the need for an interaction between monetary services and credit card deferred payment services in utility functions. That interaction would violate the blockwise weak separability condition needed to exclude credit card services from a monetary services aggregator function.

Existing empirical research using credit-augmented Divisia includes Barnett and Su (2016, 2018), Barnett and Liu (2019), Barnett and Park (2024), Barnett and Park (2023), Liu and Serletis (2020, 2023), and Liu et al. (2020). We continue in that new tradition with further empirical and theoretical evidence using the aggregation-theoretic credit-augmented Divisia, available from the Center for Financial Stability (CFS) in NY City. There also is an even newer credit-card-augmented Divisia monetary aggregate that is Gross Domestic Product (GDP) optimized using nowcasting methodology. That newer credit-augmented Divisia aggregate was first introduced by Barnett et al. (2023), but those data are not yet available to the public from the CFS and therefore are not used in our current study.

We use two approaches to investigate the usefulness of the credit-augmented Divisia. First, we expand upon the New Keynesian DSGE framework to include economic agent behavior in the credit sector. We do so including three different monetary aggregates: (1) the Constant Elasticity of Substitution (CES) aggregator function, defining the microeconomic theoretic (true) monetary aggregate, (2) the credit-card-augmented Divisia monetary aggregate, and (3) the simple sum monetary aggregate. By comparing the behavior of impulse responses to various shocks imposed on two differently defined monetary aggregates, credit-augmented Divisia versus simple sum, we determine which behaves most similarly to the shocked theoretical aggregator function. Since the macroeconomic model does not have classical structure with economic agents being price takers in perfect markets, the known approximation properties of the Divisia index in neoclassical microeconomic theory cannot assure us of the results of this analysis in advance. We do not consider the usual Divisia monetary aggregate not containing credit card services, since one of our needed New Keynesian assumptions (equation 18) violates the weak separability condition for existence of a monetary aggregator function not including credit card services.

Second, we compare the empirical implications of credit-augmented Divisia before and after the Great Recession. At the zero-lower bound (ZLB), the federal funds rate provides an inappropriate monetary policy tool. Following the econometric approach in Christiano et al. (1999) and Keating et al. (2019), we investigate whether credit-augmented Divisia can provide a more informative role as a monetary policy indicator than the traditional federal funds rate. We use two different data sets, each containing the federal funds rate and the credit-augmented Divisia M2 monetary aggregate as candidates for monetary policy indicators. By using recursive VAR, we track the impulse responses with the two data sets to monetary policy shocks. We determine which candidate is consistent with the economic mechanism. We thereby determine which one, federal funds rate or credit-augmented Divisia, can serve as the most informative policy indicator.

As Bernanke and Blinder (1988) emphasized, the critical view of the dichotomy between money and credit was reconfirmed by Liu et al. (2020), who criticize the New Keynesian model for ignoring the importance of the deferred payment liquidity services provided by credit card transactions. Pesaran and Xu (2016) track the impulse responses to credit (supply) shocks in a business cycle model. They consider the effect of banking sector credit shocks on the probability of default of the firm. A positive credit shock, defined as an increase in the loan to deposit ratio, increases output, consumption, hours of work, and productivity. The paper's results emphasize the importance of credit in the New Keynesian model. Barnett and Park (2023) found that including the credit sector within a New Keynesian model reveals a greater cost of inflation than implied by the model without credit.

Since credit-augmented Divisia were introduced, many empirical applications and theoretical extensions have appeared. The results share the conclusion that credit-augmented Divisia track the theoretically exact liquidity services better than the alternatives and perform better than the alternatives for forecasting economic activity. For example, by using the multivariate state-space model, Barnett et al. (2016, 2023) found that nowcasting with the traditional Divisia monetary aggregates produces smaller mean square forecasting error than with the simple sum monetary aggregates, while nowcasting with the new credit-augmented Divisia aggregates yields a substantially smaller mean squared error than with the unaugmented original Divisia monetary aggregates. Dery and Serletis (2019) have shown that the inference ability of the Divisia monetary aggregates is strongest when used with the broadest Divisia monetary aggregates. Liu and Serletis (2020) investigate the cyclical properties of credit-augmented Divisia using Hamilton's filter and Granger causality tests. They showed that both the narrow and broad credit-augmented Divisia monetary aggregates are superior to the traditional Divisia monetary aggregates. Performance in forecasting GDP using credit-augmented Divisia is clearly better than using the simple sum or the original unaugmented Divisia monetary aggregate.

Bayesian analysis reflecting economic beliefs also confirms the better performance of the credit-augmented Divisia monetary aggregates. By using sign-restricted VAR, Barnett and Park (2024) find a strong co-movement between the cumulative effect of credit supply shocks measured by lending rates and the historical behavior of the credit-augmented Divisia.

The structure of this paper is as follows: Section 2 discusses the DSGE New Keynesian framework including monetary aggregation. Section 3 investigates the model empirically using recursive VAR. We conclude in Section 4.

2. DSGE New Keynesian framework

We use an extension of the traditional New Keynesian framework to include a credit sector as a liquidity service provider consistent with microeconomic foundations. Throughout Section 2, we extend Belongia and Ireland's (2014) New Keynesian DSGE structure, which supports use of the original Divisia monetary aggregates as true money. We do so using the structure in Barnett and Park (2023) as the benchmark model, except for the utility function and the monetary policy equations. The household's and the bank's behavior are changed substantially. The representative household can choose three options for liquidity service provision: the household can pay with cash, interest-bearing deposit, or credit card deferred payment transaction services. The bank is the sole financial intermediary and owns the credit cards sector.

For the credit sector, we make three assumptions. First, in accordance with Barnett and Su (2016, 2018) and Barnett et al. (2023), we do not include rotating balances used for transactions in prior periods as transaction services in the current period to avoid double counting of transaction services. Second, we assume the use of credit by the representative household always requires interest payment at rate r_t^{CC} with "no grace period" (Brito and Hartley (1995)). Since the model is of a representative household aggregated over all households, the assumption does not rule out

the possibility that some credit users do not pay interest, such as credit card holders who always pay before the end of the month. Finally, debit cards, store cards, reward programs, and cashback programs will be ignored, since they are not consistent with some of our assumptions. Inclusion of rotating balances in the model would have no effect, since we do not include them in our definition of credit card deferred payment services. See, for example, Barnett and Su (2016, 2018) and Barnett et al. (2023).

Meanwhile, the key characterizations of the usual New Keynesian framework remain valid. Money is included in the utility function in accordance with a shopping time model (Goodfriend and McCallum (1988)) in which shopping time for consumption goods is a disutility. The intermediate goods-producing firms are monopolistically competitive with the quadratic cost of price adjustment (Rotemberg (1982)).

2.1. Modeling

2.1.1. The representative household

The infinitely lived representative household's utility function follows the shopping time model:

$$U(C_t, h_t, h_t^s) = [\ln(C_t) - \eta(h_t + h_t^s)], \quad (1)$$

where C_t is consumption, η is the weight on leisure relative to consumption, h_t is labor, and h_t^s is shopping time for consumption goods, defined as:

$$h_t^s = \left(\frac{1}{\chi}\right) \left(\frac{v_t P_t C_t}{M_t^A}\right)^\chi, \quad (2)$$

where χ manages the rate at which increased effort is needed to purchase goods and services as the household save money asset holdings. v_t is the money demand shock, following an AR(1) process:

$$\ln(v_t) = (1 - \rho_v) \ln(v) + \rho_v \ln(v_{t-1}) + \varepsilon_{vt}, \quad \varepsilon_{vt} \sim (0, \sigma_v). \quad (3)$$

Here $v > 0$ is the steady-state level of real monetary services demanded relative to consumption, and $0 \leq \rho_v < 1$ is a persistence parameter subject to calibration.

During each period, the representative household has M_{t-1} units of currency, carried over from the previous period, B_{t-1} units of bonds, and $s_{t-1}(i)$ ownership shares of each intermediate goods-producing firm, $i \in [0, 1]$. At the beginning of each period, the household receives Tr_t lump-sum transfer units from the government. The household has B_{t-1} units of bonds and buys B_t units of new bonds at the price of $1/r_t$ dollars per bond, where r_t represents the gross nominal interest rate between the period t and $t + 1$, and $s_t(i)$ shares in each intermediate goods-producing firm, $i \in [0, 1]$, at the price of $Q_t(i)$ dollars per share. The household holds N_t dollars of currency and borrows L_t dollars from the bank. The household utilizes credit card transaction services, CC_t , in dollars from the bank. The total nominal value of its deposits is

$$D_t = M_{t-1} + Tr_t + B_{t-1} + \int_0^1 Q_t(i) s_{t-1}(i) di - \frac{B_t}{r_t} - \int_0^1 Q_t(i) s_t(i) di - N_t + L_t + CC_t. \quad (4)$$

At the end of the period, the household owes the bank $r_t^L L_t$ and $r_t^{CC} CC_t$, where r_t^L and r_t^{CC} represent the gross nominal interest rate on loans and credit cards transaction services, respectively. That is, we assume the interest on credit card usage always accompanies every transaction service provided by credit cards (i.e., no grace period). The bank owes the household $r_t^D D_t$ dollars, where r_t^D represents the gross nominal interest rate on deposits. The household receives a nominal dividend payment of $F_t(i)$ for each share that it owns of each intermediate goods-producing firm. Finally, the household carries M_t units of currency into the next period, defined as:

$$M_t = N_t + W_t h_t + \int_0^1 F_t(i) s_t(i) di + r_t^D D_t - P_t C_t - r_t^L L_t - r_t^{CC} CC_t. \quad (5)$$

The representative household's maximization problem is

$$\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t [a_t \{\ln(C_t) - \eta(h_t + h_t^s)\}], \quad (6)$$

subject to the constraints (4) and (5), where $0 < \beta < 1$ and $\eta > 0$, and where a_t is a preference shock following the AR(1) process:

$$a_t = \rho_a \ln(a_{t-1}) + \varepsilon_{at}, \quad \varepsilon_{at} \sim (0, \sigma_a), \quad (7)$$

where $0 \leq \rho_a < 1$.

We define M_t^A as a true aggregate of monetary services provided from currency N_t , deposits D_t , and credit cards transactions CC_t , according to

$$M_t^A = \left[\nu^{\frac{1}{\omega}} N_t^{\frac{\omega-1}{\omega}} + \xi^{\frac{1}{\omega}} D_t^{\frac{\omega-1}{\omega}} + (1 - \nu - \xi)^{\frac{1}{\omega}} CC_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \quad (8)$$

which is in CES form. Following Lucas's (2000) argument, the true monetary aggregate has the CES specification for its aggregator function, since CES makes the model consistent with balanced growth. The parameters ν and ξ stand for the steady-state ratio of currency and deposits to the total amount of monetary services, respectively. The parameter ω determines the elasticity of substitution among the monetary service components. Appendix A.1 explains the optimization problem for the representative household.

2.1.2. The representative final goods-producing firm

The final goods market is competitive, and during each period t the firm utilizes $Y_t(i)$ units of each intermediate good, $i \in [0, 1]$, to produce final goods quantity Y_t . The production function for final goods is assumed to be

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (9)$$

where $\theta > 1$ represents the elasticity of substitution between the various intermediate goods in producing the final good. The firm's profit maximization problem is given by:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di, \quad (10)$$

for all $t = 0, 1, 2, \dots$. The first-order conditions for this decision are

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t, \quad (11)$$

for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$. Since the final goods market is competitive, the final goods-producing firm earns zero profit. As a result, it follows that

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad (12)$$

in equilibrium for all $t = 0, 1, 2, \dots$.

2.1.3. The representative intermediate goods-producing firm

At each period t , the representative intermediate goods-producing firm utilizes $h_t(i)$ units of labor from the representative household to produce $Y_t(i)$ units of intermediate good i with the CRS technology:

$$Y_t(i) = Z_t h_t(i), \quad (13)$$

where Z_t represents the aggregate technology shock, assumed to follow a random walk with positive drift:

$$\ln Z_t = \ln(z) + \ln(Z_{t-1}) + \varepsilon_{Zt}, \quad \varepsilon_{Zt} \sim (0, \sigma_Z), \quad (14)$$

where $z > 1$.

In the New Keynesian framework, the intermediate goods-producing firms face monopolistic competition. At each period, the intermediate goods-producing firm sets the nominal price for its output, subject to the requirement that it satisfies the representative finished goods-producing firm's demand. Following Rotemberg (1982), the intermediate goods-producing firm is subject to a quadratic cost of adjusting its nominal price, measured in units of the finished good, and given by:

$$\frac{\phi}{2} \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right]^2 Y_t, \quad (15)$$

where $\phi \geq 0$ represents the magnitude of the price adjustment costs, and $\pi > 1$ represents the gross, steady-state inflation rate. Appendix A.2 explains the optimization problem for the intermediate goods-producing firm. To maximize real market value, the firm chooses the sequence for $P_t(i)$ for all $t = 0, 1, 2, \dots$.

2.1.4. The representative bank's credit sector

We assume the banking sector owns a credit cards business (Schuh et al. (2010)). The profit structure of the credit management business consists of (1) fees, such as penalty fees, annual fees, cash advance fees, and interchange fees, (2) interest, and (3) transaction fees from the business. As mentioned above, we focus only on the interest imposed on every credit card transaction for the aggregated representative household. Commercial bank interest payments on credit cards can be found in the St. Louis Federal Reserve's database, FRED.

During each period, the representative bank receives D_t dollars of deposits from the representative household, makes L_t dollars of loans, and approves CC_t dollars of credit card transaction volumes from the representative household.² At the end of the period, the household receives interest on deposits, r_t^D , pays interest on loans, r_t^L , and pays interest on credit card transaction service, r_t^{CC} . All interest rates are defined as gross rates.

Let τ_t defines the bank's reserve ratio, assumed to follows the AR(1) process:

$$\ln(\tau_t) = (1 - \rho_\tau) \ln(\tau) + \rho_\tau \ln(\tau_{t-1}) + \varepsilon_{\tau t}, \quad \varepsilon_{\tau t} \sim (0, \sigma_\tau), \quad (16)$$

where $0 < \tau < 1, 0 \leq \rho_\tau < 1$.

The relationship between deposits and loans are determined by the bank's chosen reserves ratio τ_t in accordance with the equation:

$$L_t = (1 - \tau_t) D_t. \quad (17)$$

We assume the amount of monetary service provided by credit cards transactions is constrained by the aggregate amount of deposits in accordance with the equation:

$$CC_t \leq \varpi_t D_t, \quad (18)$$

where ϖ_t refers to a shock on credit usage behavior. We assume that the credit:deposit ratio shock follows the AR(1) process:

$$\ln(\varpi_t) = (1 - \rho_\varpi) \ln(\varpi) + \rho_\varpi \ln(\varpi_{t-1}) + \varepsilon_{\varpi t}, \quad \varepsilon_{\varpi t} \sim (0, \sigma_\varpi), \quad (19)$$

where $0 < \varpi < 1, 0 \leq \rho_\varpi < 1$.

We shall assume that constraint (18) will always be binding. After substitution into (8), a constraint connecting credit card volumes to deposits can violate the existence condition for a monetary aggregator function excluding credit card services, since monetary services may no longer be weakly separable from credit card services. We provide a clear example in equation (18a) below. Also such constraints imply the existence of a New Keynesian market imperfection in the market for credit card services. The rationale for constraint (18) or for more general constraints, such as (18a) below, could be viewed as supply side, since the source of funds for bank lending, including credit card lending, are primarily bank deposits. Nevertheless, consumers are assumed to take that bank constraint into consideration in their own consumer preferences.³

The existence of separate aggregator functions for monetary assets and for credit card deferred payment services requires monetary assets and credit card deferred payment services to be in separate weakly separable blocks in the representative household's utility function. That would be true if two conditions were met: (a) the marginal rate of substitution between currency and deposits must be independent of credit card deferred payment services, and (b) the marginal rate of substitution among the transaction services of different credit cards must be independent of currency and deposit quantities. Imposition of a constraint linking deposits to credit card deferred payment transaction services can violate condition (a) for existence of an aggregator function over monetary assets with credit card deferred payment services omitted from the aggregator function.⁴

Equation (18) is a particularly simple way to connect deposits with credit card services. We chose that equation to simplify our empirical work. But the implications for separability of a connection between demand deposits and credit card services becomes especially clear with more general equations connecting deposits with credit card services. Consider, for example, the following habit formation version of equation (18):

$$CC_t - kCC_{t-1} = \varpi_t D_t, \quad (18a)$$

where k is a constant and $CC_t - kCC_{t-1}$ is “supernumerary” consumption of credit card services in excess of habit formed quantities, kCC_{t-1} , of those services. Equation (18a) can be solved for CC_t and substituted into equation (8). Then it becomes clear that an aggregator function containing currency and demand deposits and not lagged credit card services does not exist. Factorability of a monetary aggregator function without credit card services does not exist. The weak separability condition for existence of a monetary aggregator function without credit card services is violated.

The reason for introducing equation (18) or more sophisticated versions, such as equation (18a), is interesting, perhaps surprising, and potentially important. Prior work on introducing credit card deferred payment services into Divisia monetary aggregates, including the seminal papers by Barnett and Su (2016, 2018) and Barnett et al. (2023), assumed perfect markets in a classical model. With complete decentralization to perfect markets, equations such as (18) or (18a) are not needed to complete the model. But without perfect markets in our current New Keynesian model, we found that there are not enough equations to solve the model without the introduction of a nonseparable interaction between credit card services and deposits. As a result, there is a very strong theoretical reason to include credit card services within Divisia monetary aggregates in New Keynesian models. Within the representative household's utility function, the existence condition for a monetary services aggregator function can fail without inclusion of credit card services within the weakly separable block containing monetary services.⁵

Following Belongia and Ireland (2014), the bank creates deposits with total real value D_t/P_t at each period. Under the bank's constant returns to scale (CRS) technology, $x_t(D_t/P_t)$ units of the finished goods are demanded. The bank's credit card transaction services production are also consistent with this cost structure. The bank creates a credit cards account with total real value CC_t/P_t , and requiring $x_t(CC_t/P_t)$ units of finished goods. We assume the bank earns profits from the interest on credit card transaction volumes. We assume the existence of a financial sector cost shock, x_t , following the AR(1) process:

$$\ln(x_t) = (1 - \rho_x) \ln(x) + \rho_x \ln(x_{t-1}) + \varepsilon_{xt}, \quad \varepsilon_{xt} \sim (0, \sigma_x), \quad (20)$$

where $x > 0$ and $0 \leq \rho_x < 1$.

The bank's nominal profits during the period t are

$$\Pi_t^b = (r_t^L - 1) L_t + (r_t^{CC} - 1) CC_t - (r_t^D - 1) D_t - P_t x_t (D_t/P_t) - P_t x_t (CC_t/P_t). \quad (21)$$

Substituting (17) and (18) into (21), we acquire the optimization problem with respect to D_t . Finally, we obtain the equation that explains the relationship among the interest rates:

$$r_t^D = 1 + (r_t^L - 1) (1 - \tau_t) + \varpi_t (r_t^{CC} - 1) - x_t (1 + \varpi_t). \quad (22)$$

This relationship holds in equilibrium at each period.

2.1.5. Efficient allocations and output gap

We consider a central bank using a modified Taylor rule. A social planner allocates $h^*(i)$ units of the representative household's labor to produce $Y_t^*(i)$ units of intermediate good $i \in [0, 1]$, while using those intermediate goods to produce Y_t^* units of the finished good according to the economy's technology. The first-order condition for the social planner's problem can be expressed by the equation,

$$Y_t^* = (1/\eta) Z_t, \quad (23)$$

based on Kydland and Prescott (1982). Appendix A.3 derives (23).

Since technology shocks are a main driving force to efficient business cycle fluctuations, policy stabilization should not target those fluctuations. By definition, the output gap is expressed by:

$$g_t^* = Y_t/Y_t^* = \eta(Y_t/Z_t). \quad (24)$$

2.1.6. The central bank

The monetary base consists of currency in circulation, N_t , and reserves, N_t^v ,

$$M_t = N_t + N_t^v, \quad (25)$$

where $N_t^v = \tau_t D_t$. Denote the inflation rate and output growth rate as below:

$$\pi_t = P_t/P_{t-1}, \quad (26)$$

$$g_t^y = Y_t/Y_{t-1}. \quad (27)$$

We assume the central bank imposes monetary policy based on the modified and simplified Taylor rule:

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + \rho_\pi (\pi_t/\pi) + \varepsilon_{rt}, \quad \varepsilon_{rt} \sim (0, \sigma_r), \quad (28)$$

where ε_{rt} can be defined as a monetary policy shock.

2.1.7. Monetary aggregation

The traditional simple sum monetary aggregate, published by most central banks, is

$$M_t^S = N_t + D_t, \quad (29)$$

which can track the true aggregation-theoretic monetary service flow, if and only if the two components are perfectly substitutable. However, this assumption has been unrealistic since the 1950s. Therefore, the necessity of introducing monetary aggregates derived from aggregation theory has emerged. See, for example, Barnett (1978), Barnett (1980), and Barnett et al. (2023) regarding Divisia monetary aggregation.

The monetary aggregate M_t^A in equation (8) includes monetary services provided by credit card transactions, as in Barnett et al. (2023), along with Barnett and Liu (2019), Barnett and Park (2024), Barnett and Park (2023), Barnett and Su (2016, 2018), Liu and Serletis (2020, 2022), and Liu, et al. (2020). The opportunity cost of holding monetary assets instead of bonds is given by equation (30), which can be viewed as the price dual of the CES quantity aggregator function (8):

$$r_t - r_t^A = \left[\nu (r_t - 1)^{1-\omega} + \xi (r_t - r_t^D)^{1-\omega} + (1 - \nu - \xi) (r_t^{CC} - r_t)^{1-\omega} \right]^{\frac{1}{1-\omega}}. \quad (30)$$

This result can be viewed as an extension of Belongia and Ireland (2014) to include credit card services.

Below are the formulas for the user costs of the true monetary aggregate, of currency, of deposits, and of credit card transactions, respectively, as originated by Barnett (1978), Barnett (1980), Barnett and Su (2016, 2018), and Barnett et al. (2023):

$$u_t^A = (r_t - r_t^A)/r_t, \quad (31)$$

$$u_t^N = (r_t - 1)/r_t, \quad (32)$$

$$u_t^D = (r_t - r_t^D)/r_t, \quad (33)$$

$$u_t^{CC} = (r_t^{CC} - r_t)/r_t. \quad (34)$$

Total expenditure on monetary services is

$$E_t = u_t^N N_t + u_t^D D_t + u_t^{CC} CC_t, \quad (35)$$

which can be expressed by:

$$E_t = u_t^A M_t^A. \quad (36)$$

The expenditure shares of currency, deposits, and credit card transaction services, respectively, are

$$s_t^N = u_t^N N_t / E_t, \quad (37)$$

$$s_t^D = u_t^D D_t / E_t, \quad (38)$$

$$s_t^{CC} = u_t^{CC} CC_t / E_t. \quad (39)$$

The growth rate of the credit-augmented Divisia quantity index of monetary services, based upon the Törnqvist-Theil discrete time approximation to the continuous time Divisia index, can be written as:

$$\mu_t^Q = (\mu_t^N)^{(s_t^N + s_{t-1}^N)/2} (\mu_t^D)^{(s_t^D + s_{t-1}^D)/2} (\mu_t^{CC})^{(s_t^{CC} + s_{t-1}^{CC})/2}, \quad (40)$$

where the μ terms are as defined below for currency, deposits, and credit card transaction services, respectively:

$$\mu_t^N = N_t / N_{t-1}, \quad (41)$$

$$\mu_t^D = D_t / D_{t-1}, \quad (42)$$

$$\mu_t^{CC} = CC_t / CC_{t-1}. \quad (43)$$

The growth rate formula corresponding to (41), (42), and (43) for the Divisia price aggregate of monetary services is

$$\pi_t^P = \left(\frac{u_t^N}{u_{t-1}^N} \right)^{\frac{(s_t^N + s_{t-1}^N)}{2}} \left(\frac{u_t^D}{u_{t-1}^D} \right)^{\frac{(s_t^D + s_{t-1}^D)}{2}} \left(\frac{u_t^{CC}}{u_{t-1}^{CC}} \right)^{\frac{(s_t^{CC} + s_{t-1}^{CC})}{2}}. \quad (44)$$

2.2. Calibration

The setting of the parameters under the New Keynesian framework follows the benchmark studies of Kydland and Prescott (1982) and Belongia and Ireland (2014). The discount factor of the representative household, $\beta = 0.99$, is comparable with a long-run annual interest rate of 4%. Setting $\eta = 2.5$ is consistent with a steady-state value of hours worked by the representative agent of 8 hours per day. The setting of $\theta = 6$ makes the steady-state markup of price over marginal cost to be 0.2 with monopolistically competitive markets for the differentiated intermediate goods. For the speed of price adjustment in this model, setting $\phi = 50$ with quadratic price adjustment costs closely corresponds with Calvo (1983) pricing when individual goods prices are adjusted every 3.75 quarters on average (Ireland (2004)). Rotemberg's quadratic adjustment cost and Calvo's pricing are distinct; the long-run relationship between inflation and output is positive in the Rotemberg model and negative in the Calvo model. Ireland (2004) mentions that when utility is nonseparable, the nominal price rigidity follows Calvo's staggering specification instead of the quadratic adjustment cost specification. The shopping time form is quadratic by setting $\chi = 2$. Substitutability among monetary assets in the CES aggregator function is determined by the setting $\omega = 1.5$. The value $z = 1.005$ supports 2% of the annual growth of real variables in the model. The target value of $\pi = 1.005$ implies 2% steady-state annual inflation rate.

A setting of $\nu = 0.252$ makes the steady-state ratio of aggregate monetary service provision to nominal consumption expenditures in the model equal to 2.68. This setting is comparable with a value of 2.61 for the ratio of monetary services provided by currency, deposits, and credit card transactions to quarterly personal consumption expenditure in the U.S. from 2000 to 2018. The 2019 Federal Reserve Payments Study provides the trend in non-cash payments, including credit cards payments. For the composition of monetary service provider components, settings of $\nu = 0.19$ and $\xi = 0.60$ induces the steady-state ratio of currency N_t , deposits D_t , and credit cards transactions CC_t to be 10.7, 87.2, and 1.98, respectively. Data from the CFS show that the average ratio among them is 10.3, 87.7, and 2.0, respectively, using monetary data from July 2006 to May 2021.

Setting $\tau = 0.05$ is consistent with the average ratio of total reserves to the deposit component of M2 (excluding currency), as calculated from the FRED database data from January 1959 to December 2019. Considering the Fed has decided to provide interest on reserves since 2008, the value has been skyrocketing from 0.05 to 0.1493 (Section 3.1.1 narrows the data period from July 2006 to December 2019 and calibrates the τ value of 0.1493). The setting $x = 0.0115$ implies that 2.61% of the steady-state total aggregate output can be explained by banking activity, with deposit costs measured in units of the finished good. To support this setting, the annual data from FRED for 2001 to 2016 show that the ratio to GDP of Federal Reserve banks, credit intermediation, and related services for U.S. metropolitan portion is 2.79. Additionally, the data from the BEA indicate that the ratio of Federal Reserve banks, credit intermediation, and related activities to GDP is 2.72 from 1997 to 2019.

Equation (18) explains the relationship between credit card transaction volumes and deposits. The setting $\varpi = 0.0228$ supports the ratio of credit card transaction services to deposits, with that ratio calculated from monthly CFS data for July 2006 to May 2021. Meanwhile, the 2019 Federal Reserve Payments Study supports the fact that the ratio of credit card payments to total non-cash payments equals 0.029 on annual average from 2000 to 2018.

For the value of the benchmark parameters, we set $\rho_r = 0.75$ and $\rho_\pi = 0.3$ to obtain the observed smoothness of interest rates, while $\rho_v = 0.95$ and $\rho_a = 0.9$ are consistent with the money demand and preference shocks being highly persistent. The parameter settings of $\rho_x = 0.5$ and $\rho_\tau = 0.5$ are consistent with the effects of financial cost shocks and reserve shocks decaying quickly compared to other shocks. Finally, the setting $\rho_\varpi = 0.8$ implies that the shock of credit usage behavior is persistent compared to the reserve shock and financial cost shock.

The standard deviation of the innovations to each shock are given by different scales, where $\sigma_v = 0.01$, $\sigma_a = 0.01$, and $\sigma_z = 0.01$ explain the general cases of innovation. To simulate the financial crisis, we set the extreme values of $\sigma_\tau = 1$, $\sigma_x = 0.25$, and $\sigma_\varpi = 1$. By doing so, we can track what happens to the impulse responses to reserve shocks, when the bank's demand for reserves doubles. The setting $\sigma_r = 0.0025$ is consistent with how the monetary policy shock affects the short-term nominal interest rate.

2.3. Results

Figures 1 and 2 provide justification for using credit-augmented Divisia instead of simple sum. Figures 1 and 2 show the impulse responses of the growth rates of three different monetary aggregates: the theoretical monetary aggregate (8), credit-augmented Divisia (40), and the simple sum monetary aggregate (29), to seven different exogenous shocks. The results in these figures can answer the question, does measurement still matter in a New Keynesian world? In short, the answer to the question in this framework is “yes,” especially if we expand the scope of the monetary services to include deferred payment credit card transactions. The first and second columns of Figures 1 and 2 demonstrate that the impulse responses of credit-augmented Divisia perfectly track those of the true monetary aggregate for all types of shocks, even in a New Keynesian model. We can trace the fact that their values are the same to the fourth decimal place.

However, considering the third column, the impulse responses of simple sum are substantially different from those of true money. We can directly grasp the difference of responses to preference shocks (Figure 1, row 1), credit-deposit shocks (Figure 2, row 2), and monetary policy shocks (Figure 2, row 4). In the last row of Figure 2, the decline of the growth rates of the true and credit-augmented Divisia are greater than that of the simple sum monetary aggregate. The results in these figures support the importance of the monetary measurement tools as Belongia and Ireland (2014) first asserted in a New Keynesian context.

Figures 3 and 4 provide the impulse responses of macroeconomic variables to seven shocks. Our objective in these figures is to investigate whether the behaviors of impulse responses are

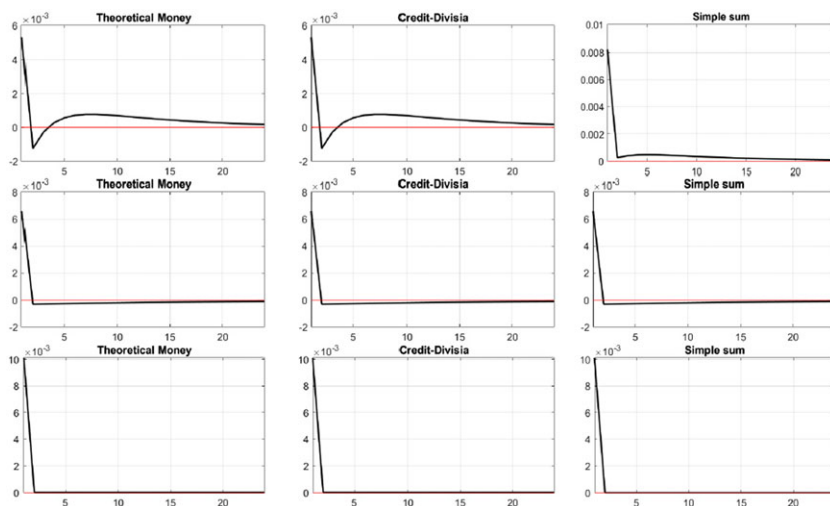


Figure 1. Impulse responses for nominal money growth. First row: Preference shock. Second row: Money demand shock. Third row: Technology shock.

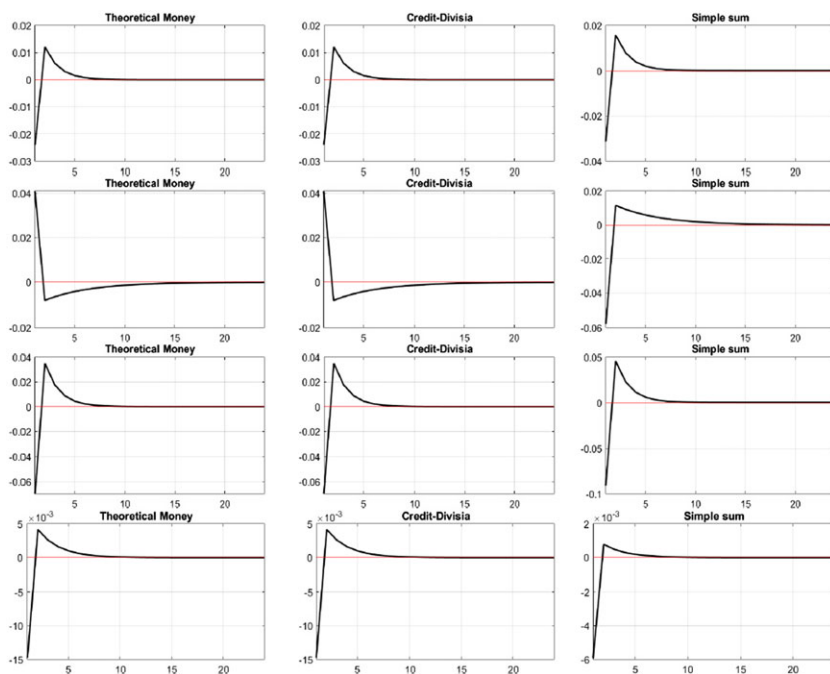


Figure 2. Impulse responses for nominal money growth. First row: Reserve ratio shock. Second row: Credit-deposit shock. Third row: Financial cost shock. Fourth row: Monetary policy shock.

consistent with the basic logic of the New Keynesian framework, when we introduce credit card services. We focus on the behavior of output, inflation rate, and interest rate.

The first row of Figure 3 provides the impulse responses to preference shocks inducing increases of output and inflation rate. The parameter settings of the modified Taylor rule, $\rho_r = 0.75$ and $\rho_\pi = 0.3$, require continuous monetary tightening to offset economic overheating. The

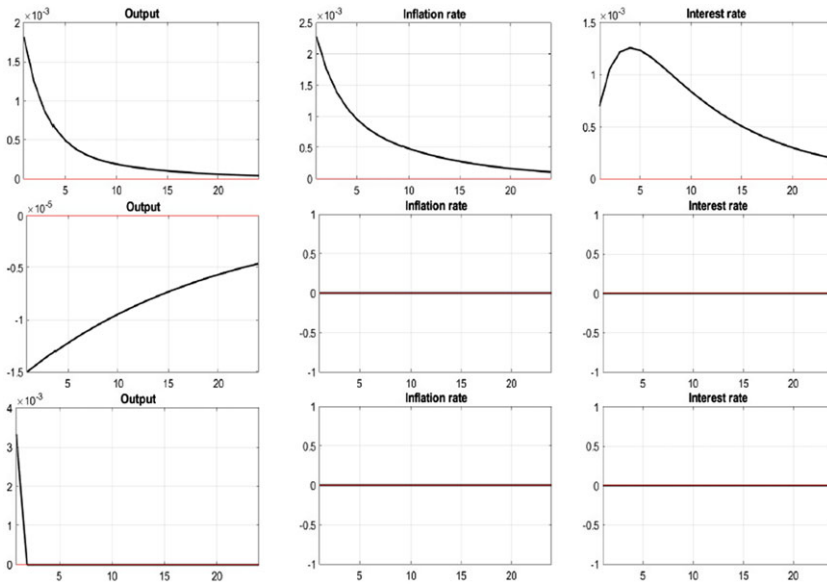


Figure 3. Impulse responses for macroeconomic variables; 1st row: Preference shock, 2nd row: Money demand shock, 3rd row: Technology shock.

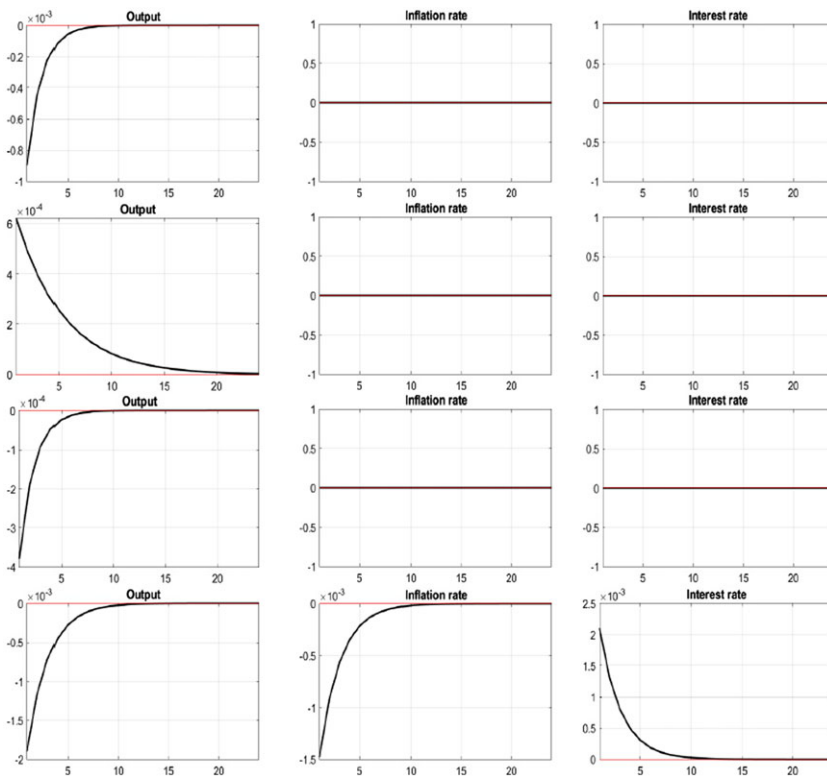


Figure 4. Impulse responses for macroeconomic variables; 1st row: Reserve ratio shock, 2nd row: Credit-deposit shock, 3rd row: Financial cost shock, 4th row: Monetary policy shock.

result implies that the preemptive policy for inflation would stabilize the economy successfully. The second row of Figure 3 shows the impulse responses to the money demand shocks. By maintaining the nominal interest rate at a fixed level, the modified Taylor rule requires the monetary authority to implement the policy that increases the money supply to produce stable output and inflation. The third row of Figure 3 provides the impulse response to the technology shock. Since the technology shock follows a random walk with drift, we would expect a permanent increase in productivity.

However, the result shows that the effect of the technology shock to output converges to zero after the initial shock. The reason appears to be our adoption of a stationary system. As a result, the 11 real variables, including output, the 9 money stock variables, and the 2 price variables are normalized by the technology shock, Z_t . The resulting full stationary system is provided in Appendix B.1.

In Figure 4, the first and third rows display the response to reserve ratio shocks and financial cost shocks, respectively. When the shocks are imposed, the inflation rate remains unchanged, while the output decreases. Again, under the modified Taylor rule, these two types of shocks put pressure on the central bank to accommodate increased demand for money to stabilize the price level. Observing the large decrease in the theoretical monetary aggregate and credit-augmented Divisia in Figure 2, we conclude that the increase in the monetary base implemented by the monetary policy is not adequate to cover the change in the monetary base caused by the impact of the financial sector shock. The decreasing liquidity induces a decrease in output, as shown in Figure 4 again.

The second row of Figure 4 provides the response to the credit usage behavior shock. Since credit card transactions contribute to liquidity services in the form of deferred payment services, a positive shock causes an increase in credit-augmented Divisia in Figure 2. Finally, the shock induces an increase in output. The last row of Figure 4 explains the effect of monetary policy shocks. Following monetary tightening, output and inflation rate decrease. The amount of the inflation rate decrease is smaller than that of output. As a result, nominal price rigidity induces monetary non-neutrality in the short run. Compared to the impulse response of the theoretical monetary aggregate and of output, we can track the liquidity effect clearly, since its fluctuation is larger than that of output.⁶

3. Empirical approach

The purpose of this section is to study the empirical performance of the credit-augmented Divisia monetary aggregate. By adopting credit-augmented Divisia as a monetary variable in the recursive VAR model, we investigate whether the empirical results with VAR are consistent with those of the calibrated DSGE New Keynesian model in the previous section. We expect that the empirical usefulness of credit-augmented Divisia would be confirmed, if common behaviors are observed by comparing the impulse response of the main variables in the DSGE structure with those in the VAR. Since the CFS publishes credit-augmented Divisia data monthly from July 2006, we focus only on the period from July 2006 to December 2019 for both approaches.⁷ The period includes the Great Recession and the ZLB period. However, we do not include the very abnormal COVID-19 pandemic era. We track the behavior of the impulse response to the monetary policy shock induced by the modified Taylor rule.

Next, we evaluate the effectiveness of credit-augmented Divisia as a monetary policy indicator by using a recursive VAR. Traditionally, the effective federal funds rate has played an important role as an indicator. However, since the behavior of the federal funds rate at the lower bound was not adequate to explain the economic phenomena, alternatives have been introduced, such as the shadow federal funds rate (Wu and Xia (2016)). We investigate whether credit-augmented Divisia can be a good alternative as a monetary policy indicator. To do so, we replace the federal funds

Table 1. Parameter calibration (July 2006 to December 2019)

Parameter	Definition	Value
z	Quarterly growth of real variables	1.00445
v	SS ratio of monetary service to consumption	0.2646
ν	Determinant weight of monetary component	0.15
ξ	Determinant weight of monetary component	0.60
τ	Average ratio of reserves to M2	0.1493
ϖ	Average ratio of CC to deposit	0.0228
χ	SS output explained by banking	0.0115
σ_a	Std. dev. of the innovation to preference shock	0.01
σ_v	Std. dev. of the innovation to money demand shock	0.2995
σ_z	Std. dev. of the innovation to technology shock	0.01497
σ_τ	Std. dev. of the innovation to reserve ratio shock	0.08
$\sigma_{\overline{m}}$	Std. dev. of the innovation to credit deposit ratio shock	0.0018
σ_r	Std. dev. of the innovation to monetary policy shock	0.4752
σ_x	Std. dev. of the innovation to financial cost shock	0.00115

rate data with monetary aggregate data in the VAR. This method has previously been suggested by Christiano et al. (1999) with simple sum M1 and M2 as policy indicators. Keating et al. (2019) expanded that study by introducing the broader Divisia monetary aggregate M4 (DM4) as a substitute for the federal funds rate. We will consider credit-augmented Divisia M2 as a candidate for monetary policy indicator. We do not consider credit-augmented Divisia M4, since properly consolidated simple sum M4 is not currently available for comparison from any sources.

3.1. DSGE New Keynesian model revisited

3.1.1. Calibration

First, we adopt the DSGE New Keynesian model from the previous section and apply a new calibration relevant to the limited time period. Table 1 displays the newly calibrated parameters, and Appendix C explains the choices in detail. While the setting for τ is increased to match the higher level of reserves relative to deposits, the model continues for simplicity to assume that reserves do not pay interest. In future work, it could be useful to extend the analysis from this paper to account for interest payment on reserves, perhaps along the same line as Ireland (2014).

We choose the impulse response of five key variables to monetary policy shocks: output, inflation, interest rate, credit-augmented Divisia, and monetary base. The results are displayed in the first column of Figure 5.

3.2. VAR estimation

The reduced form VAR model, as in Sims (1980), is:

$$\mathbf{Y}_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{Y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{Y}_{t-p} + \mathbf{u}_t, \quad (45)$$

where p is the number of lags, with \mathbf{u}_t assumed to be serially uncorrelated disturbances having covariance matrix, $\mathbb{E}(\mathbf{u}_t \mathbf{u}_t') = \mathbf{V}$. The reduced form VAR disturbances are related to the underlying structural economic shocks, ε_t , by

$$\mathbf{u}_t = \mathbf{B}_0 \varepsilon_t, \quad (46)$$

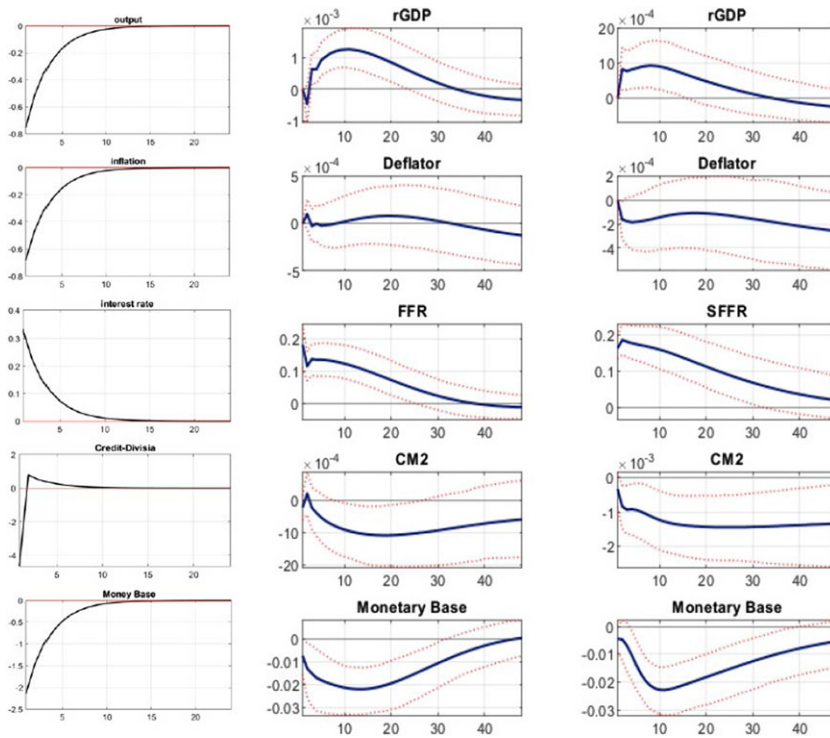


Figure 5. Impulse response for economic variables to monetary policy shock. First column: DSGE model; second column: VAR model with effective federal funds rate; third column: VAR model with shadow federal funds rate.

where, ε_t is serially uncorrelated white noise with a diagonal covariance matrix Σ_ε of full rank, and \mathbf{B}_0 is a lower triangular matrix. Substituting (46) into the VAR equation (45) and multiplying by \mathbf{B}_0^{-1} on both sides, we get

$$\mathbf{B}_0^{-1}\mathbf{Y}_t = \mathbf{B}_0^{-1}\mathbf{A}_0 + \mathbf{B}_0^{-1}\mathbf{A}_1\mathbf{Y}_{t-1} + \cdots + \mathbf{B}_0^{-1}\mathbf{A}_p\mathbf{Y}_{t-p} + \varepsilon_t \quad (47)$$

We determined the relationship $\mathbf{V} = \mathbf{B}_0\mathbf{B}_0'$ by using the Cholesky decomposition. The order of the variables becomes important, when we impose the zero restriction with the Cholesky decomposition. There is no fixed rule for the variable ordering, but conventionally the most exogenous variable is located in the first row of the vector \mathbf{Y}_t , and the least exogenous variable is located in the last row of the vector.

We introduce 6 variables with monthly frequency in the VAR model. The real GDP data are collected from IHS Markit.⁸ Their monthly real GDP calculations and aggregation methods are comparable to the official GDP data from the U.S. Bureau of Economic Analysis (BEA). Next, the GDP deflator and commodity price follow. Since IHS Markit provides nominal and real GDP in monthly frequency, we can calculate the monthly frequency GDP deflator. These three variables show lagged impulse response to the monetary policy shock. The alternative monetary policy instrument, credit-augmented Divisia M2, is in the following row. For the last row, the monetary base and the user cost of credit-augmented Divisia M2 are located. They are the least exogenous. Finally, we choose the VAR(2) model with constant, since that model produced the lowest AIC and BIC.

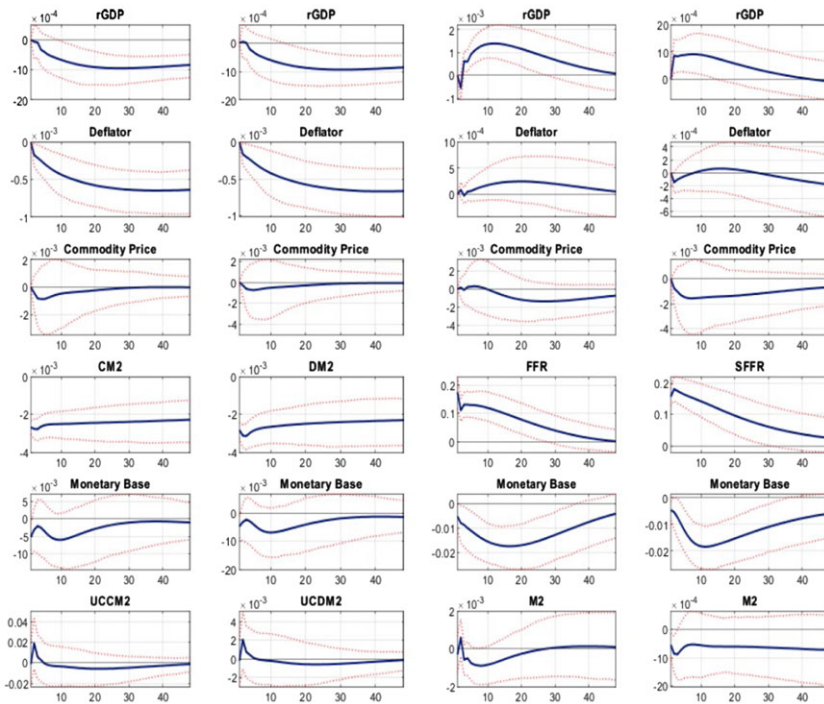


Figure 6. Impulse response to monetary policy shock (Keating et al. (2019) benchmark).

3.3. Results

The first column in Figure 5 shows the DSGE impulse responses to contractionary monetary policy shocks for the key economic variables. Output and inflation rate fall immediately and converge to zero. We could conjecture that the temporary drop in the inflation rate induces the price level to decrease and converge to the negative level permanently. Credit-augmented Divisia monetary quantity falls instantaneously, displaying a liquidity effect. The second and third columns display the impulse responses to the monetary policy shock of the key economic variables under the recursive VAR estimation. Each column uses the effective federal funds rate and the shadow federal funds rate as monetary policy indicators, respectively. Unfortunately, we cannot track the common behaviors between the DSGE and empirical results. The output and price puzzles are captured in both empirical results.

Considering credit-augmented Divisia as an alternative policy tool provides a robust result. We consider credit-augmented Divisia M2 as an indicator of a negative monetary policy shock. Figure 6 demonstrates the impulse responses to monetary policy shocks with 90% confidence intervals. Each column possesses various monetary policy indicators in the fourth row. The first and second columns in Figure 6 follow Figure 2 in the Keating et al. (2019) baseline VAR model, which uses Divisia M4 as a monetary policy indicator. The use of user cost in the last row is based on the fact that macroeconomic models typically include at least one interest rate (Keating et al. (2019)). The first column uses credit-augmented Divisia M2 as a policy indicator. The response of the output shows the typical behavior. It reaches the lowest point after 2 years and slightly increases toward the origin. Since the initial positive shock is not observed, the output puzzle problem does not exist in this model, and the behavior is statistically significant after 1 year.

The price level is decreasing and converges to a negative value in the long run. This is a desirable result. Commodity prices show sensitive behavior. They fall faster than the price level but converge to zero instantly. The monetary base and the user cost of credit-augmented Divisia M2 are the

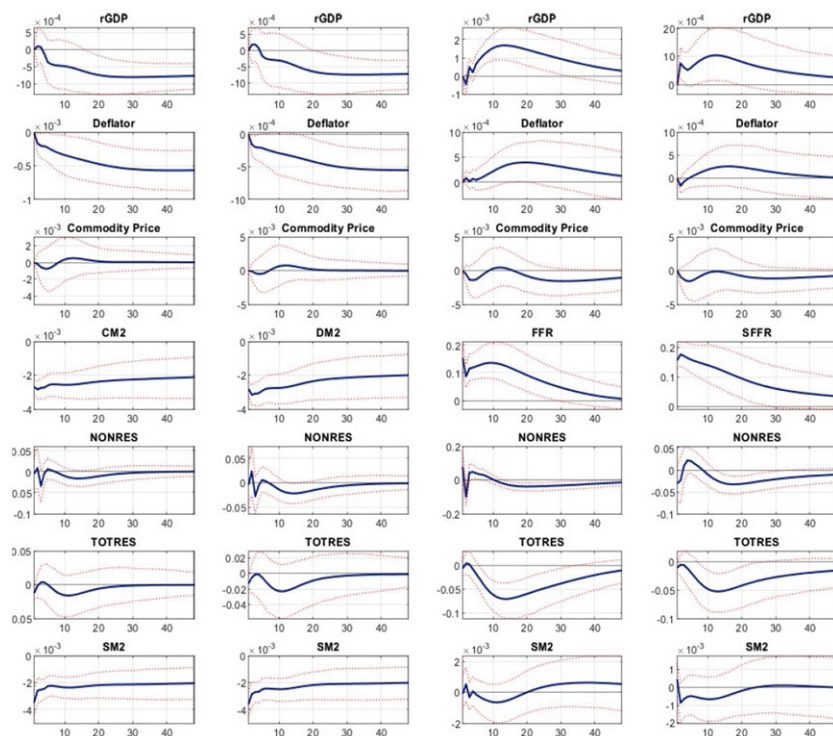


Figure 7. Impulse response to monetary policy shock (Christiano et al. (1999) benchmark).

least exogenous. The monetary base drops immediately and remains at its negative value with a statistically insignificant interval. In the last row, the user cost increases at the initial period and converges to the origin in the short run. The results in the first column are consistent with those of the theoretical model explained in the first column of Figure 5. Moreover, the model is not contaminated by the output, price, and liquidity puzzles.

For the third and fourth columns in Figure 6, we benchmark the additional VAR estimation of Keating et al. (2019), Figure 6, shadow rate VAR models. Keating et al. (2019) adopt the federal funds rate and the shadow federal funds rate as monetary policy indicators, and the simple sum monetary aggregate in the last row as proxy for user cost. The third and fourth columns show the impulse responses when the federal funds rate and shadow federal funds rate are considered as monetary policy tools, respectively. The most noticeable result is that the puzzle problems of output and price level last more than 2 years. Meanwhile, liquidity puzzle problems are not detected in both cases.

The use of a Divisia monetary aggregate as an alternative monetary policy indicator above is consistent with Keating et al. (2019), while the use of an M2-level monetary aggregate is consistent with Christiano et al. (1999). The results below, following Christiano et al. (1999), support our conclusion that considering credit-augmented Divisia M2 as an alternative policy indicator produces robust favorable results.

Figure 7 displays the impulse responses using various monetary policy indicators. Credit-augmented Divisia M2 is in the first column, while the federal funds rate and shadow federal funds rate are in the third and the last columns, respectively. We observe different dynamics for each column. In the first column, the output puzzle is observed in the short term, but the price puzzle is not observed. However, the responses for the other indicators are not free from either the output or price puzzle, and their positive responses last more than 3 years.

The fifth to seventh rows show the response of the variables related to liquidity. The behavior of non-borrowed reserves depicted in the first, third, fourth columns show initial fluctuations. After 18 months, the responses reach their lowest point, and later they converge to zero. For the response of total reserves, three cases commonly experience a positive spark at the initial period and touch their lowest point after 1 year. The response of the first column converges to zero relatively faster than their counterparts. The impulse response of M2 is consistently negative, when credit-augmented Divisia M2 is considered as an indicator. However, in the case of its counterparts, M2 shows initial fluctuations and converges to zero after experiencing a positive effect. That result is not free from the liquidity puzzle.

Based on the results in Figures 6 and 7, we conclude that credit-augmented Divisia M2 can be considered as a good alternative policy indicator and provides clearer economic implications than the alternatives we have considered. Furthermore, the empirical results are comparable to and consistent with the DSGE New Keynesian model, described in the first column of Figure 5.

4. Conclusion

Do credit card services matter? Does the Barnett Critique matter? Positive answers to both questions are consistent with the results of this paper. Since the original Divisia monetary aggregates were introduced, many researchers have shown the theoretical and empirical superiority of the Divisia-type monetary aggregates compared to the simple sum. Belongia's work (1996) further confirms this conclusion and emphasizes the importance of the correct data consistent with economic aggregation theory. Hence Belongia (1996) argues "Measurement Matters."

In this study, we introduce credit-augmented Divisia into a DSGE New Keynesian model having imperfect markets. The purpose of this research is to compare the credit-augmented Divisia with simple sum. The credit-augmented monetary aggregate incorporates the deferred payments services of credit cards into an aggregate also including services of monetary assets. But removing the credit card services from the aggregate in our New Keynesian model to produce a Divisia aggregate containing only monetary assets would not be consistent with economic aggregation theory, since we find that removing the credit card services would violate the weak separability condition necessary for existence of an aggregator function.

Prior research on inclusion of credit card services within a Divisia monetary aggregate in perfect markets has not precluded removal of the credit card services from within the aggregate. But we find that our New Keynesian imperfect market assumptions could preclude removal of the credit card services. Hence, we have not only provided empirical results supporting inclusion of credit card services into Divisia monetary aggregates, but also a New Keynesian theoretical reason emphasizing an interaction between credit card services and monetary deposit services not permitting removal of the credit card services without violation of the fundamental existence condition for quantity aggregation.

Our empirical results explore both the tracking ability of the credit-augmented Divisia monetary aggregate relative to the theoretical aggregator function and implications of impulse responses in a VAR model. We compare with other indicators, including Federal Funds rates and simple sum monetary aggregates. We emphasize M2, since that is the broadest monetary aggregate for which properly consolidate simple sum aggregates are currently available for comparison. In addition, the choice of M2 permits comparison with Christiano et al. (1999). However, we believe that credit-augmented Divisia M3 or M4 aggregates might perform even better than credit-augmented Divisia M2.

By interpreting impulse responses to various types of shocks, we demonstrate that similarity and consistency can be observed between the behavior of the aggregation-theoretic exact monetary aggregate and credit-augmented Divisia. Furthermore, the behavior of the impulse responses of economic variables retain the basic properties of New Keynesian framework. These results are

consistent with the benchmark paper, Belongia and Ireland (2014), which introduced original Divisia into the DSGE New Keynesian model.

Considering the credit-augmented Divisia as an alternative monetary policy instrument provides robust economic implications solving critical puzzles in the literature, including the output, price and liquidity puzzles. We also find consistency between the results with empirical VAR and those with our calibrated DSGE New Keynesian model. With the growing relevance of deferred payment credit card services in transactions and with the increasing importance New Keynesian DSGE models, we conclude that the new credit-card-augmented Divisia monetary aggregates provide an important step forward in methodology and potential applications.

Notes

- 1 <https://www.federalreserve.gov/newsevents/pressreleases/files/2019-payments-study-20191219.pdf>
- 2 Credit card transactions volumes are substantially less than credit card balances, since balances include unpaid rotating balances from previous periods used for transactions in previous periods. We do not include rotating balances, since that would produce double counting of transaction services.
- 3 As a result, equation (18) does not necessarily imply the existence of a corner solution produced by bank credit rationing but rather a voluntary understanding among banks and their customers affecting customer preferences. While this constraint certainly does not apply to every individual consumer, many of whom have more credit card transaction volumes than bank deposits, the constraint is consistent with the observed behavior of the aggregated representative household. This New Keynesian assumption is not consistent with the perfect markets assumptions in Barnett et al. (2023) but do not contradict the observed behavior modeled in that research or the conclusions of that research. In general, credit card transaction volumes positively correlate with bank deposits. The alternative of introducing credit rationing into the model would substantially complicate the aggregation theory.
- 4 The problem is similar to the reason that the CFS does not include store cards in its credit-augmented Divisia monetary aggregates. Store cards can only be used for purchases of consumer goods in that store. That constraint violates the condition for existence of a credit-augmented monetary aggregate, since consumer goods cannot be weakly separable from the components of the credit-augmented monetary aggregate.
- 5 We have not conducted a formal econometric test of blockwise weak separability of monetary services from credit card deferred payment services, but based on our theoretical results we would expect that hypothesis to be rejected unless all relevant markets in the banking industry are perfect.
- 6 The results of all figures are consistent with those of the benchmark paper, Belongia and Ireland (2014).
- 7 <https://centerforfinancialstability.org/>. While the construction of the CFS credit-card-augmented Divisia monetary aggregates does not impose our equation (18), that equation is not needed in the CFS computation, since the CFS aggregates are derived from classical economic theory with perfect markets. But any such linkage that exists in the economy from New Keynesian market imperfections are already embedded in the component data used by the CFS. In addition, the data are not inconsistent with that constraint in its form as an inequality. However, our computation of the true aggregator function from our New Keynesian model does impose the equation (18) linkage.
- 8 <https://ihsmarkit.com/products/us-monthly-gdp-index.html>

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Appendix

A. Optimization Problems

A.1. Household optimization

The household optimization problem with shopping time model starts from the utility function:

$$U(C_t, h_t, h_t^s) = \left[\ln(C_t) - \eta \left\{ h_t + \left(\frac{1}{\chi} \right) \left(\frac{v_t P_t C_t}{M_t^A} \right)^\chi \right\} \right], \quad (\text{A.1})$$

with three constraints (definition of deposit, true monetary aggregation, and currency) to be carried over to the next period, divided by the price, P_t , and with free disposal allowed:

$$\frac{D_t}{P_t} = \frac{M_{t-1}}{P_t} + \frac{Tr_t}{P_t} + \frac{B_{t-1}}{P_t} + \int_0^1 \frac{Q_t(i)}{P_t} [s_{t-1}(i) - s_t] di - \frac{B_t}{r_t P_t} - \frac{N_t}{P_t} + \frac{L_t}{P_t} + \frac{CC_t}{P_t}, \quad (\text{A.2})$$

$$\frac{M_t^A}{P_t} = \left[v^{\frac{1}{\omega}} \left(\frac{N_t}{P_t} \right)^{\frac{\omega-1}{\omega}} + \xi^{\frac{1}{\omega}} \left(\frac{D_t}{P_t} \right)^{\frac{\omega-1}{\omega}} + (1 - v - \xi)^{\frac{1}{\omega}} \left(\frac{CC_t}{P_t} \right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \quad (\text{A.3})$$

and

$$\frac{M_t}{P_t} = \frac{N_t}{P_t} + \frac{W_t h_t}{P_t} + \int_0^1 \frac{F_t(i)}{P_t} di + \frac{r_t^D D_t}{P_t} - C_t - \frac{r_t^L L_t}{P_t} - \frac{r_t^{CC} CC_t}{P_t}. \quad (\text{A.4})$$

By combining these four expressions, we can get the Lagrangian form:

$$L = \mathbb{E}_0 \sum_{j=0}^{\infty} \beta^{t+j} \left[\begin{aligned} & a_{t+j} \left\{ \ln(C_{t+j}) - \eta h_{t+j} - \eta \left(\frac{1}{\chi} \right) \left(\frac{v_{t+j} P_{t+j} C_{t+j}}{M_{t+j}^A} \right)^{\chi} \right\} \\ & + \Lambda_{t+j}^1 \left\{ \frac{M_{t-1+j}}{P_{t+j}} + \frac{Tr_{t+j}}{P_{t+j}} + \frac{B_{t-1+j}}{P_{t+j}} + \int_0^1 \frac{Q_{t+j}(i)}{P_{t+j}} [s_{t-1+j}(i) - s_{t+j}] di \right\} \\ & - \frac{B_{t+j}}{r_{t+j} P_{t+j}} - \frac{N_{t+j}}{P_{t+j}} + \frac{L_{t+j}}{P_{t+j}} + \frac{CC_{t+j}}{P_{t+j}} - \frac{D_{t+j}}{P_{t+j}} \\ & + \Lambda_{t+j}^2 \left\{ \left[v^{\frac{1}{\omega}} \left(\frac{N_{t+j}}{P_{t+j}} \right)^{\frac{\omega-1}{\omega}} + \xi^{\frac{1}{\omega}} \left(\frac{D_{t+j}}{P_{t+j}} \right)^{\frac{\omega-1}{\omega}} + (1-v-\xi)^{\frac{1}{\omega}} \left(\frac{CC_{t+j}}{P_{t+j}} \right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \right. \\ & \quad \left. - \frac{M_{t+j}^A}{P_{t+j}} \right\} \\ & + \Lambda_{t+j}^3 \left\{ \frac{N_{t+j}}{P_{t+j}} + \frac{W_{t+j} h_{t+j}}{P_{t+j}} + \int_0^1 \frac{F_{t+j}(i)}{P_{t+j}} s_{t+j}(i) di + \frac{r_{t+j}^D D_{t+j}}{P_{t+j}} \right. \\ & \quad \left. - C_{t+j} - \frac{r_{t+j}^L L_{t+j}}{P_{t+j}} - \frac{r_{t+j}^{CC} CC_{t+j}}{P_{t+j}} - \frac{M_{t+j}}{P_{t+j}} \right\} \end{aligned} \right]. \quad (\text{A.5})$$

The first-order conditions for the representative household's problem are as follows:

(a) First-order conditions with respect to B_t :

$$\frac{\Lambda_t^1}{r_t} = \beta \frac{\mathbb{E}_t(\Lambda_{t+1}^1) P_t}{P_{t+1}}. \quad (\text{A.6})$$

(b) First-order conditions with respect to s_t for all i :

$$\Lambda_t^1 \frac{Q_t(i)}{P_t} = \Lambda_t^3 \frac{F_t(i)}{P_t} + \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1 Q_{t+1}(i)}{P_{t+1}} \right]. \quad (\text{A.7})$$

(c) First-order conditions with respect to N_t :

$$\Lambda_t^1 - \Lambda_t^3 = \Lambda_t^2 \left[\frac{M_t^A}{P_t} \right]^{1/\omega} v^{1/\omega} \left[\frac{N_t}{P_t} \right]^{-1/\omega}. \quad (\text{A.8})$$

(d) First-order conditions with respect to D_t :

$$\Lambda_t^1 - r_t^D \Lambda_t^3 = \Lambda_t^2 \left[\frac{M_t^A}{P_t} \right]^{1/\omega} \xi^{1/\omega} \left[\frac{D_t}{P_t} \right]^{-1/\omega}. \quad (\text{A.9})$$

(e) First-order conditions with respect to CC_t :

$$-\Lambda_t^1 + r_t^{CC} \Lambda_t^3 = \Lambda_t^2 \left[\frac{M_t^A}{P_t} \right]^{1/\omega} (1-v-\xi)^{1/\omega} \left[\frac{CC_t}{P_t} \right]^{-1/\omega} \quad (\text{A.10})$$

(f) First-order conditions with respect to L_t ,

$$\Lambda_t^1 = r_t^L \Lambda_t^3. \quad (\text{A.11})$$

(g) First-order conditions with respect to h_t ,

$$\eta a_t = \Lambda_t^3 \frac{W_t}{P_t}. \quad (\text{A.12})$$

(h) First-order conditions with respect to C_t :

$$\frac{a_t}{C_t} \left[1 - \eta \left(\frac{v_t P_t C_t}{M_t^A} \right)^x \right] = \Lambda_t^3. \quad (\text{A.13})$$

(i) First-order conditions with respect to M_t^A :

$$\eta a_t \left(\frac{v_t P_t C_t}{M_t^A} \right)^x = \left(\frac{M_t^A}{P_t} \right) \Lambda_t^2. \quad (\text{A.14})$$

(j) First-order conditions with respect to M_t ,

$$\Lambda_t^3 = \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1 P_t}{P_{t+1}} \right]. \quad (\text{A.15})$$

A.2. Intermediate Goods-producing Firm optimization

Following Belongia and Ireland (2014), the representative intermediate goods-producing firm's total, real market value is proportional to

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \Lambda_t^3 \left[\frac{F_t(i)}{P_t} \right], \quad (\text{A.16})$$

where

$$\frac{F_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \left[\frac{P_t(i)}{P_t} \right]^{-\theta} \left[\frac{W_t Y_t}{P_t Z_t} \right] - \frac{\phi}{2} \left[\frac{P_t(i)}{\pi P_{t-1}(i)} \right]^2 Y_t. \quad (\text{A.17})$$

To maximize total market value, we substitute equation (B.2) into (B.1). The first-order condition with respect to $P_t(i)$ becomes

$$0 = \Lambda_t^3 \left[(1-\theta) \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t + \theta \left(\frac{P_t(i)}{P_t} \right)^{-\theta-1} \left(\frac{W_t Y_t}{P_t Z_t} \right) - \phi \left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right) \left(\frac{Y_t P_t}{\pi P_{t-1}(i)} \right) \right] \\ + \beta \phi \mathbb{E}_t \left[\Lambda_{t+1}^3 \left(\frac{P_{t+1}(i)}{\pi P_t(i)} - 1 \right) \left(\frac{Y_{t+1} P_{t+1}(i) P_t}{\pi (P_t(i))^2} \right) \right], \quad (\text{A.18})$$

for all $t = 0, 1, 2, \dots$

A.3. Efficient Allocation for Social planner

In accordance with their definitions, Y_t^* , P_t^* , and h_t^* are expressed by:

$$Y_t^* = \left[\int_0^1 Y_t^*(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (\text{A.19})$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad (\text{A.20})$$

$$h_t^* = \int_0^1 h_t^*(i) di. \quad (\text{A.21})$$

The optimization problem of the social planner is given by:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t a_t \left[\ln(Y_t^*) - \eta \left(\int_0^1 h_t^*(i) di \right) \right], \quad (\text{A.22})$$

subject to

$$Z_t \left(\int_0^1 h_t^*(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} = Y_t^*. \quad (\text{A.23})$$

The Lagrangian expression is

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t a_t \left[\ln(Y_t^*) - \eta \left(\int_0^1 h_t^*(i) di \right) \right] + \lambda \left[Z_t \left(\int_0^1 h_t^*(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} - Y_t^* \right]. \quad (\text{A.24})$$

The first-order conditions of Y_t^* and $h_t^*(i)$ are given by:

(a) First-order conditions with respect to Y_t^* are

$$\frac{\beta^t a_t}{Y_t^*} = \lambda. \quad (\text{A.25})$$

(b) First-order conditions with respect to $h_t^*(i)$ combined with the equation (A.25) are

$$\eta = \frac{Z_t}{Y_t^*} \left[\int_0^1 h_t^*(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{1}{\theta-1}} h_t^*(i)^{\frac{-1}{\theta}}. \quad (\text{A.26})$$

Substituting the equation (A.22) into the equation (A.26), we get

$$h_t^*(i)^{\frac{-1}{\theta}} = \eta \left[\int_0^1 h_t^*(i)^{\frac{\theta-1}{\theta}} di \right], \quad (\text{A.27})$$

which can be re-expressed by:

$$h_t^*(i)^{\frac{-1}{\theta}} = \eta h_t^*(i)^{\frac{\theta-1}{\theta}}. \quad (\text{A.28})$$

Then, we can get the relationship:

$$\eta^{-1} = h_t^*, \quad (\text{A.29})$$

and finally if we substitute (A.29) into the relationship, $Y_t^* = Z_t h_t^*$, the expression

$$Y_t^* = (1/\eta) Z_t \quad (\text{A.30})$$

is acquired.

A.4. Systematic Equilibrium

Under the systematic equilibrium environment, all intermediate goods-producing firms make the same decision, so we can set $Y_t(i) = Y_t$, $h_t(i) = h_t$, $P_t(i) = P_t$, $F_t(i) = F_t$, and $Q_t(i) = Q_t$ for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$. For the market clearing condition, we set $M_t = M_{t-1} + Tr_t$,

$B_t = B_{t-1} = 0$, and $s_t(i) = s_{t-1}(i) = 1$ for all $i \in [0, 1]$ and $t = 0, 1, 2, \dots$. Collecting equations with the conditions mentioned above, we have 46 variables and 46 system equations for equilibrium.

B. Stationary system

B.1. The Stationary system

Since the real variables in the system inherits a unit root process from the technology shock, and the nominal variables also hold a unit root process from the implementation of monetary policy represented by the Taylor rule, the variables in this system will be non-stationary. Therefore, we need to convert the system to be stationary along the model's balanced growth path. By redefining the variables, we can obtain a new stationary system.

- A. 11 Real variables: $c_t = C_t/Z_{t-1}$, $y_t = Y_t/Z_{t-1}$, $y_t^* = Y_t^*/Z_{t-1}$, $g_t^y, g_t^*, h_t, h_t^s, f_t = F_t/(P_t Z_{t-1})$, $\lambda_t^1 = Z_{t-1}\Lambda_t^1$, $\lambda_t^2 = Z_{t-1}\Lambda_t^2$, $\lambda_t^3 = Z_{t-1}\Lambda_t^3$.
- B. 9 Money stock variables: $m_t^A = M_t^A/(P_t Z_{t-1})$, $n_t = N_t/(P_t Z_{t-1})$, $d_t = D_t/(P_t Z_{t-1})$, $cc_t = CC_t/(P_t Z_{t-1})$, $m_t = M_t/(P_t Z_{t-1})$, $l_t = L_t/(P_t Z_{t-1})$, $n_t^v = N_t^v/(P_t Z_{t-1})$, $m_t^S = M_t^S/(P_t Z_{t-1})$, $e_t = E_t/(P_t Z_{t-1})$.
- C. 4 Money growth variables: $\mu_t^N, \mu_t^D, \mu_t^{CC}, \mu_t^Q$.
- D. 3 Price variables: $\pi_t = P_t/P_{t-1}$, $w_t = W_t/(P_t Z_{t-1})$, $q_t = Q_t/(P_t Z_{t-1})$.
- E. 5 Interest rates: $r_t, r_t^D, r_t^L, r_t^{CC}, r_t^A$.
- F. 8 User costs, expenditure shares, and price dual variables: $u_t^A, u_t^N, u_t^D, u_t^{CC}, s_t^N, s_t^D, s_t^{CC}, \pi_t^P$.
- G. 6 Shocks: $v_t, a_t, z_t = Z_t/Z_{t-1}, \tau_t, \varpi_t, x_t$.

B.2. The Stationary Equilibrium

$$h_t^s = \left(\frac{1}{\chi} \right) \left[\frac{v_t c_t}{m_t^A} \right]^\chi, \quad (\text{B.1})$$

$$\ln(v_t) = (1 - \rho_v) \ln(v) + \rho_v \ln(v_{t-1}) + \varepsilon_{vt}, \quad (\text{B.2})$$

$$a_t = \rho_a \ln(a_{t-1}) + \varepsilon_{at}, \quad (\text{B.3})$$

$$y_t = z_t h_t, \quad (\text{B.4})$$

$$\ln(z_t) = \ln(z) + \varepsilon_{zt}, \quad (\text{B.5})$$

$$\ln(\tau_t) = (1 - \rho_\tau) \ln(\tau) + \rho_\tau \ln(\tau_{t-1}) + \varepsilon_{\tau t}, \quad (\text{B.6})$$

$$l_t = (1 - \tau_t) d_t, \quad (\text{B.7})$$

$$cc_t = \varpi_t d_t, \quad (\text{B.8})$$

$$\ln(x_t) = (1 - \rho_x) \ln(x) + \rho_x \ln(x_{t-1}) + \varepsilon_{xt}, \quad (\text{B.9})$$

$$\ln(\varpi_t) = (1 - \rho_\varpi) \ln(\varpi) + \rho_\varpi \ln(\varpi_{t-1}) + \varepsilon_{\varpi t}, \quad (\text{B.10})$$

$$r_t^D = 1 + (r_t^L - 1)(1 - \tau_t) + \varpi_t(r_t^{CC} - 1) - x_t(1 + \varpi_t), \quad (\text{B.11})$$

$$y_t^* = \frac{1}{\eta} z_t, \quad (\text{B.12})$$

$$g_t^* = \eta \frac{y_t}{z_t}, \quad (\text{B.13})$$

$$m_t = n_t + n_t^\nu, \quad (\text{B.14})$$

$$\pi_t = \frac{P_t}{P_{t-1}}, \quad (\text{B.15})$$

$$g_t^y = \frac{y_t z_{t-1}}{y_{t-1}}, \quad (\text{B.16})$$

$$\ln\left[\frac{r_t}{r}\right] = \rho_r \ln\left[\frac{r_{t-1}}{r}\right] + \rho_\pi \ln\left[\frac{\pi_t}{\pi}\right] + \rho_{g^*} \ln\left[\frac{g_t^*}{g^*}\right] + \rho_{g^y} \ln\left[\frac{g_t^y}{g^y}\right] + \varepsilon_{rt}, \quad (\text{B.17})$$

$$r_t - r_t^A = [\nu(r_t - 1)^{1-\omega} + \xi(r_t - r_t^D)^{1-\omega} + (1 - \nu - \xi)(r_t^{CC} - r_t)^{1-\omega}]^{1/(1-\omega)}, \quad (\text{B.18})$$

$$u_t^A = \frac{r_t - r_t^A}{r_t}, \quad (\text{B.19})$$

$$m_t^s = n_t + d_t, \quad (\text{B.20})$$

$$u_t^N = \frac{r_t - 1}{r_t}, \quad (\text{B.21})$$

$$u_t^D = \frac{r_t - r_t^D}{r_t}, \quad (\text{B.22})$$

$$u_t^{CC} = \frac{r_t^{CC} - r_t}{r_t}, \quad (\text{B.23})$$

$$e_t = u_t^N n_t + u_t^D d_t + u_t^{CC} c_t, \quad (\text{B.24})$$

$$s_t^N = \frac{u_t^N n_t}{e_t}, \quad (\text{B.25})$$

$$s_t^D = \frac{u_t^D d_t}{e_t}, \quad (\text{B.26})$$

$$s_t^{CC} = \frac{u_t^{CC} c_t}{e_t}, \quad (\text{B.27})$$

$$\mu_t^Q = (\mu_t^N)^{(s_t^N + s_{t-1}^N)/2} (\mu_t^D)^{(s_t^D + s_{t-1}^D)/2} (\mu_t^{CC})^{(s_t^{CC} + s_{t-1}^{CC})/2}, \quad (\text{B.28})$$

$$\mu_t^N = \frac{n_t \pi_t z_{t-1}}{n_{t-1}}, \quad (\text{B.29})$$

$$\mu_t^D = \frac{d_t \pi_t z_{t-1}}{d_{t-1}}, \quad (\text{B.30})$$

$$\mu_t^{CC} = \frac{cc_t \pi_t z_{t-1}}{cc_{t-1}}, \quad (\text{B.31})$$

$$\pi_t^P = \left(\frac{u_t^N}{u_{t-1}^N} \right)^{(s_t^N + s_{t-1}^N)/2} \left(\frac{u_t^D}{u_{t-1}^D} \right)^{(s_t^D + s_{t-1}^D)/2} \left(\frac{u_t^{CC}}{u_{t-1}^{CC}} \right)^{(s_t^{CC} + s_{t-1}^{CC})/2}, \quad (\text{B.32})$$

$$d_t = m_t - n_t + l_t + cc_t, \quad (\text{B.33})$$

$$m_t^A = \left[v^{\frac{1}{\omega}} n_t^{\frac{\omega-1}{\omega}} + \xi^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} + (1 - v - \xi)^{\frac{1}{\omega}} cc_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \quad (\text{B.34})$$

$$c_t + r_t^L l_t + m_t + r_t^{CC} cc_t = n_t + w_t h_t + r_t^D d_t + f_t, \quad (\text{B.35})$$

$$\lambda_t^1 = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^1 r_t}{\pi_{t+1} z_t} \right), \quad (\text{B.36})$$

$$\lambda_t^1 q_t = \lambda_t^3 f_t + \beta \mathbb{E}_t (\lambda_{t+1}^1 q_{t+1}), \quad (\text{B.37})$$

$$\lambda_t^1 - \lambda_t^3 = \lambda_t^2 [v^{1/\omega} (m_t^A)^{1/\omega} n_t^{-1/\omega}], \quad (\text{B.38})$$

$$\lambda_t^1 - r_t^D \lambda_t^3 = \lambda_t^2 [\xi^{1/\omega} (m_t^A)^{1/\omega} d_t^{-1/\omega}], \quad (\text{B.39})$$

$$r_t^{CC} \lambda_t^3 - \lambda_t^1 = \lambda_t^2 [(1 - v - \xi)^{1/\omega} (m_t^A)^{1/\omega} cc_t^{-1/\omega}], \quad (\text{B.40})$$

$$\lambda_t^1 = r_t^L \lambda_t^3, \quad (\text{B.41})$$

$$\eta a_t = \lambda_t^3 w_t, \quad (\text{B.42})$$

$$\frac{a_t}{c_t} \left[1 - \eta \left(\frac{v_t c_t}{m_t^A} \right)^\chi \right] = \lambda_t^3, \quad (\text{B.43})$$

$$\eta a_t \left(\frac{v_t c_t}{m_t^A} \right)^\chi = \lambda_t^2 m_t^A, \quad (\text{B.44})$$

$$\lambda_t^3 = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^1}{z_t \pi_{t+1}} \right), \quad (\text{B.45})$$

$$f_t = y_t - \frac{w_t y_t}{z_t} - \frac{\phi}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 y_t \quad (\text{B.46})$$

$$0 = (1 - \theta) \lambda_t^3 y_t + \theta \lambda_t^3 \frac{w_t y_t}{z_t} - \phi \lambda_t^3 \left[\frac{\pi_t}{\pi} - 1 \right] \left[\frac{y_t \pi_t}{\pi} \right] + \beta \phi \mathbb{E}_t \left\{ \lambda_{t+1}^3 \left[\frac{\pi_{t+1}}{\pi} - 1 \right] \left[\frac{y_{t+1} \pi_{t+1}}{\pi} \right] \right\}. \quad (\text{B.47})$$

C. Parameter calibration

C.1. Section 3.1.1.

The parameters which characterize the general New Keynesian framework from chapter 2 are unchanged ($\beta, \eta, \chi, \theta, \phi$, and ω).

The setting $\pi = 1.005$ is still valid in this period. The value $z = 1.00445$ supports 1.79% of the annual growth of real variables in the model. The value $\sigma_z = 0.01497$ is calculated from the standard deviation of real, per capita growth in the U.S. data within the data period. The value of $\nu = 0.2646$ makes the steady-state ratio of the sum of monetary service provision to nominal consumption expenditures in the model equal to 2.74. This setting is comparable with the U.S. data, which equals 2.732 from 2006 to 2018. The setting $\sigma_\nu = 0.2995$ can be obtained from the standard deviation of the ratio of the monetary services provision to nominal consumption expenditures in the U.S. data within the limited period. For the composition of monetary service provider component, the settings $\nu = 0.15$ and $\xi = 0.60$ induce the steady-state ratio of currency, deposit, and credit cards transaction to be 10.8, 87.2, 2.0, which are consistent with the ratios given by the CFS data.

The most noticeable changes can be tracked from $\tau = 0.1493$, which explains the average ratio of total reserves to the deposit component from 2006 to 2019. The Federal Reserve began paying interest on reserves held by the banks in 2008. Since then, the reserve ratio has drastically surged, reaching 27.4% in August 2014. The value $\sigma_\tau = 0.08$ reflects the standard deviation of the reserve ratio within the data period. The setting $x = 0.0115$ implies that 2.63% of the steady-state total aggregate output can be explained by banking activity, because the deposit costs are measured in the units of the finished good. In the same data period, Federal Reserve and BEA data display 2.64% and 2.63%, respectively. The value $\sigma_x = 0.00115$ explains the standard deviation of banking activities normalized by the output within the data period. The setting $\varpi = 0.0228$ remains the same, because the calibration shares the same data period. However, the empirical data show the standard deviation $\sigma_\varpi = 0.0018$, which can be calculated from the empirical data.

We assume the value of the persistence parameters, $\rho_\nu, \rho_\tau, \rho_x$, and ρ_ϖ , still remain the same. For the monetary policy, equation (28), the parameters are calculated by the regressions using the federal funds rate, lagged federal funds rate, and inflation rate. The results report $\rho_r = 1.005$, $\rho_\pi = 0.2226$, and the standard deviation, $\sigma_r = 0.4752$.