Vostok (Antarctica) climate record time-scale deduced from the analysis of a borehole-temperature profile

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ABSTRACT. Several sets of temperature measurements were carried out in 1972– 88 in the Vostok boreholes. They have provided the icc-sheet temperature profile down to a depth of 2000 m. The accuracy of the profile is sufficient to analyze perturbations induced by the surface-temperature variations over the last climatic cycle. The mathematical model developed for the ice-temperature computation is applied to solve an inverse problem. The amplitudes and phase lags of the main harmonic components in the surface-temperature variations are reconstructed on the basis of fitting the calculated ice-temperature profile to the experimental one with the assumption that Milankovich's cycles (100, 41, 23 and 19 kyear) are dominant in the climate oscillations. The paleotemperature record simulated with the inverse procedure is revealed to be insensitive to the model parameters varied within the range of their uncertainty. Minimal standard deviation between calculated and measured temperature profiles is found of the same order as the reproducibility of the temperature measurements $(0.005-0.01^{\circ}C)$. Although the simulated temperaturetime curve obtained in this study does not contain short-term variations, all the main climate events predicted from the ice-core isotope analysis can be recognized. Thus, the age of the events can be verified independently of the ice-sheet dynamics dating. The resultant time-scale for the Vostok record appears to be in good agreement with the dating of climate events recorded in deep-sea sediments.

INTRODUCTION

Special studies and issues (Dahl-Jensen and Johnsen, 1986; MacAycal and others, 1991; Palaeogeography, Palaeoclimatology, Palaeoecology, 1992) have been devoted to the inverse problems and methods aimed at reconstruction of past climatic changes from temperature measurements in deep boreholes. As a rule, this approach is extremely limited in application if no additional information about the form of an inferred paleotemperature time curve is taken into account. On the other hand, there is abundant evidence that Milankovich astronomic cycles probably were dominant in the Pleistocene climate oscillations (c.g. Jouzel and others, 1987; Martinson and others, 1987).

Estimations based on theoretical approaches (Budd, 1969; Muravyev and Salamatin, 1990) and numerical predictions (Ritz, 1987) have shown that in the central parts of Antarctica the last 20–100 kyear periods of the Milankovich harmonic components of surface-temperature variations penetrate through the ice sheet down to

its bottom, fading in amplitude not more than 5 times while passing 1-3 km, respectively. Short-term temperature variations with periods less than 3-5 kyear with comparatively small amplitudes $(\leq 1-2^{\circ}C)$ are almost completely filtered out in the upper part of the ice sheet (above a depth of 200-300 m) and cannot be observed in the deeper strata, while long-term variations with periods, exceeding 300 kyear do not induce a detectable waveshape signal in the vertical temperature profile because of insufficient thickness of the ice. The expected perturbations of the temperature field in the ice sheet, induced by the 20 100 kyear long paleoclimate oscillations (Lorius and others, 1985; Jouzel and others, 1987), appear to be not less than 0.2-1.0°C. Since the temperature measurements made in the 2000 m deep borehole at Vostok Station have an absolute error of 0.05°C, they provide a certain registration of such a record.

From the above discussion, it seems reasonable for our purpose to imagine the surface-temperature oscillations as a sum of harmonics with the fixed Milankovich periods of 100, 41, 23 and 19 kyear. Hence, only their amplitudes

and phase lags are to be found by minimizing a standard deviation between the theoretical temperature distribution and the experimental temperature profile measured in the Vostok borehole. Then a comparison of the inferred paleotemperature-time curve with the ice-core isotope record would provide a possibility of verifying the age of the main climatic events independently of the ice-sheet dynamics dating. For the implementation of this plan, a sufficient mathematical model providing simulation of non-stationary temperature fields in the central part of the Antarctic ice sheet taking account of inhomogeneity and compressibility of the near-surface strata (Salamatin, 1991) has been developed by Salamatin and others (in press).

Thus, the dating approach introduced in this paper combines the inverse procedure with an orbital tuning concept: a "metronomic record" destined for adjusting the isotope record time-scale is derived here from the borehole-temperature profile using the common assumption for orbital tuning about the linear climate response to variations in the Earth's orbital parameters. Although there is evidence of non-linear effects associated with icesheet waxing and waning (Imbrie and Imbrie, 1980) as well as with the amplifying role of greenhouse gases (Genthon and others, 1987), it has already been shown that they mainly result in the shape of climatic events, while their timing does not change more than 3 kyear (Martinson and others, 1987). This is the basic belief of the authors which ensures the validity of further considerations. Indeed, the detailed temperature history older than 50-100 kyear does not noticeably affect the measured temperature depth profile and the borehole thermometry alone could not resolve the surfacetemperature variations at greater times.

EXPERIMENTAL DATA

Systematic temperature measurements in the deep boreholes drilled at Vostok Station were started in 1972 in the dry hole which reached a depth of 950 m. The results of geophysical studies that were carried out at the station in different boreholcs before 1983, including temperature measurements made down to 2040 m in deep borehole 3G, were summarized by Vostretsov and others (1984). The accuracy of these data below 900 m does not exceed 0.1°C because they were obtained soon after the drilling was completed. Then temperature logging of borehole 3G was repeated several times during 1983-88. The last and most precise set of data was obtained in 1988 when the temperature profile was measured three times during the year: in April, July and September, with four-five readings each time at each depth. This has allowed us to estimate the experimental reproducibility of the measurements which has been found to be 0.005-0.01°C, while absolute accuracy of the data is considered to be about 0.05°C. The average values of the temperature together with its standard deviations at each examined depth level are presented in Table 1. The accuracy achieved in this set of measurements is about one order greater than that of the joint results from 1972-82 (Vostretsov and others, 1984), which were used earlier by Ritz (1989) for

Table 1. Borehole temperature (experimental data)

Depth	Mean temperature	Standard deviation	Depth	Mean temperature	Standard deviation
m	°C	°C	m	°C	°C
100	50,000	0.000	1000	10.000	0.004
100	-56.630	0.008	1020	-48.939	0.004
120	-56.509	0.012	1040	-48.736	0.008
140	-56.359	0.021	1060	-48.525	0.008
100	-30.208	0.007	1080	-48.324	0.004
180	-56.080	0.012	1100	-48.110	0.000
200	-55.959	0.012	1120	-47.901	0.011
220	-55.802	0.005	1140	-47.088	0.007
240	-33.634	0.007	1100	-47.473	0.005
260	-33.307	0.007	1180	-47.233	0.005
280		0.004	1200	-47.039	0.007
300	-55.207	0.000	1220	-40.818	0.005
320	-55.052	0.007	1240	-40.391	0.005
260	-34.901	0.005	1200	-40.300	0.004
300	-34.743	0.007	1200	40,134	0.009
400	-34.309	0.000	1300	-45.908	0.000
490	-34.432	0.005	1340	-45.070	0.000
440	-54.114	0.000	1360	-45 915	0.005
460	-53 955	0.003	1380	-44 978	0.000
480	-53 795	0.004	1400	44 735	0.004
500	-53 634	0.002	1400	-44 491	0.000
520	-53.001	0.003	1440	44 251	0.001
540	-53 311	0.005	1460	-44 004	0.006
560	-53.143	0.002	1480	-43 754	0.010
580	-52.977	0.005	1500	-43.507	0.014
600	-52.811	0.004	1520	-43.271	0.007
620	-52.642	0.005	1540	43.017	0.003
640	-52.472	0.005	1560	-42.735	0.005
660	-52.301	0.001	1580	-42.493	0.004
680	52.127	0.004	1600	-42.213	0.003
700	-51.953	0.002	1620	-41.943	0.004
720	-51.774	0.004	1640	-41.669	0.010
740	-51.598	0.004	1660	-41.412	0.005
760	-51.421	0.005	1680	-41.131	0.001
780	-51.239	0.004	1700	-40.857	0.002
800	-51.054	0.005	1720	-40.581	0.004
820	-50.871	0.007	1740	-40.295	0.006
840	50.687	0.006	1760	-40.008	0.005
860	-50.498	0.006	1780	-39.714	0.007
880	-50.312	0.006	1800	-39.439	0.008
900	-50.121	0.006	1820	-39.134	0.007
920	-49.931	0.006	1840	38.858	0.003
940	49.734	0.005	1860	38.552	0.014
960	-49.537	0.007	1880	-38.257	0.007
980	-49.340	0.007	1900	-37.956	0.006
1000	-49.140	0.005	1920	37.660	0.022

verification of their agreement with the ice-core isotope analysis. Let us assume hereafter the temperature $T_{\rm h}^{\ 0} = -57.4^{\circ}{\rm C}$ to be a reference origin point close to

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the mean contemporary temperature observed on the ice-sheet surface at Vostok Station. The present-day accumulation rate w_h^0 expressed in ice equivalent is supposed to be within the range of 2.0–2.7 cm ycar⁻¹ (Lorius and others, 1985; Jouzel and others, 1993) and the glacier thickness h_0 is estimated as 3725 m, according to direct measurements by Kapitsa and Sorohtin (1965) and temperature simulations by Barkov and others (1989). The ice-sediment density (ρ) variation versus depth (h) caused by the firn and ice densification (Salamatin and others, 1985) can be approximated by the following relation:

$$\rho = \rho_0 (1 - c_{\rm h} \mathrm{e}^{-\gamma h}) \tag{1}$$

where ρ_0 is the density of pure ice (0.92 Mg m^{-3}) , $\gamma = 0.021 \text{ m}^{-1}$, $c_h = 0.69$.

The corresponding ice-sheet thickness in ice equivalent h'_0 is 3692 m and the fusion temperature $T_{\rm f}$ at the bottom (pressure-melting point) is about -2.4° C. Additionally, the researches by Shumskii (1969) and Vostretsov and others (1984) have shown that the thermal conductivity coefficient of firn and ice with air inclusions differs from the thermal conductivity λ of pure ice. Their ratio A depends on the relative density $\rho^* = \rho/\rho_0$ following Maxwell's formula (Shumskii, 1969; Muravyev and Salamatin, 1990):

$$A = 2\rho^* / (3 - \rho^*).$$
 (2)

MODEL DESCRIPTION

The following studies preceded the final formulation of the mathematical model for temperature simulation in the ice sheet at Vostok. First, the general equations describing the glacier flow along a fixed flowline and taking account of the inhomogeneity and compressibility of near-surface ice strata were derived by Salamatin (1991). Then, the latter were investigated to estimate the possible influence of their different terms and factors on the temperature predictions (Salamatin and others, in press). In particular, the analysis of these equations shows that the convective-heat transfer in the vertical direction (provided that the profile of the density ρ is invariant in time) is governed by the effective rate of the ice-mass transfer, which can be expressed in the dimensionless form:

$$W^{*} = -W_{\rm h} + (1 - Z^{*}) \left[W_{\rm h} - W_{0} + \sigma \left(W_{\rm h} - W_{0} - \frac{\partial \Delta}{\partial t} \right) + \frac{1 - (1 - Z^{*})^{\alpha + 1}}{\alpha + 1} \right], \qquad Z^{*} = \frac{Z}{\Delta}.$$
 (3)

Here, $W_{\rm h}$ and W_0 are the relative rates of the surface accumulation and the basal ice melting normalized by $w_{\rm h}^{0}$; Z and Δ are the distance from the bed and the icesheet thickness in ice equivalent related to h'_0 , respectively; σ is the imitation switch parameter, i.e. the ratio between the ice-flow rate which corresponds to the glacier body plastic deformation and the total flow rate which includes ice sliding at the bottom: $0 \le \sigma \le 1$ ($\sigma = 0$, when ice sliding takes place in the basal layer and the shear strains in the glacier body are zero; $\sigma = 1$, when there is no ice sliding at the bed); α is the creep index in Glen's rheological law modified for non-isothermal conditions in accordance with the theory of Lliboutry (1979); $t = \tau w_{\rm h}^{0}/h'_{0}$ is the dimensionless form of the time τ .

To make it clear, it must be mentioned that Equation (3) was deduced on the basis of conventional assumptions of the thin-layer approximation theory. However, W^* is not exactly the vertical velocity of the ice. Actually, this value describes the ice movement in the normalized Z^* - coordinate system and, thus, takes into account temporal and longitudinal variations of ice-sheet thickness.

The following assumptions were also discussed and justified for the Vostok region by Salamatin and others (in press):

- (a) The strain heating in the near-bottom ice layer in the vicinity of Vostok Station remains practically invariant in time. Its influence on the temperature distribution in the upper 2000 m of ice strata can be taken into account by appropriate correction of the geothermal flux q_0 at the ice-rock interface.
- (b) The longitudinal convective-heat transfer does not render noticcable influence on the formation of temperature perturbations in the ice sheet linked with changes in climatic conditions, and the corresponding effect on the stationary component of the temperature field is negligibly small in comparison with the existent uncertainty of the model parameters.
- (c) The simplified model (Ritz, 1989)

$$\partial \Delta / \partial t = W_{\rm h} - \langle W_{\rm h} \rangle , \qquad (4)$$

where $\langle \cdot \rangle$ denotes long-history time averaging, can be accepted to estimate possible maximal iccthickness oscillations in the vicinity of Vostok Station and their effect on the temperature profile in the ice sheet.

(d) If the ice sheet is assumed to be composed of pure ice throughout its thickness, one must write the special boundary conditions on its surface not to violate the heat balance and temperature distribution below the upper heterogeneous layer:

$$-\frac{\partial\theta}{\partial Z^*}\Big|_{Z^*=1} = \left[\int_0^1 \left(\frac{1}{\rho^* \Lambda} - 1\right) \mathrm{d}Z^*\right]^{-1} (\theta|_{Z^*=1} - \theta_\mathrm{h}) \quad (5)$$

where θ and $\theta_{\rm h}$ are the dimensionless temperatures in the ice sheet and on its surface, respectively, $\theta = (T - T_{\rm h}^0)/(T_{\rm f} - T_{\rm h}^0)$; T is the temperature in °C.

The computational experiments performed by Barkov and others (1989) and by Ritz (1989) have made it evident that the basal ice in the vicinity of Vostok Station is at its melting point. Hence, using Equations (1) and (2) to evaluate the integral in relation (5), we finally arrived at the following quasi-one-dimensional heat-transfer model (Salamatin and others, in press):

$$c^{*} \operatorname{Pe} \Delta^{2} \left(\frac{\partial \theta}{\partial t} + \frac{W^{*}}{\Delta} \frac{\partial \theta}{\partial Z^{*}} \right) = \frac{\partial}{\partial Z^{*}} \left(\lambda^{*} \frac{\partial \theta}{\partial Z^{*}} \right);$$

$$- t_{0} < t < 0, \quad 0 < Z^{*} < 1;$$

$$(a) \ \theta_{|Z^{*}=0} = 1; \quad (b) \ -(\kappa/\Delta) (\partial \theta/\partial Z^{*})_{|Z^{*}=1} = \theta_{|Z^{*}=1} - \theta_{h}.$$

$$(6)$$

Here, t_0 is the whole time interval in the past taking into consideration ($t_0 > 0$); c^* and λ^* are the relative specific heat and thermal conductivity coefficients of ice normalized by their corresponding values c_0 and λ_0 at a temperature of -30° C; $Pe = c_0 \rho_0 w_h^0 h'_0 / \lambda_0$ is the Peclet number;

$$\kappa = ig[c_{
m h} - igl(rac{3}{2} igr) {
m ln} (1-c_{
m h}) igr] / (\gamma h_0') pprox 0.032$$
 .

In accordance with Budd (1969) and Vostretsov and others (1984):

$$c_{0} = 1.88 \,\text{kJ} \,\text{kg}^{-1} \,^{\circ}\text{C}^{-1}; \qquad \lambda_{0} = 2.55 \,\text{W} \,\text{m}^{-1} \,^{\circ}\text{C}^{-1}; \\c^{*} = 1 + \alpha^{*}{}_{c}(\theta - 0.5); \qquad \lambda = 1 - \alpha^{*}{}_{\lambda}(\theta - 0.5); \quad (7) \\\alpha^{*}{}_{c} \approx 0.23; \qquad \qquad \alpha^{*}{}_{\lambda} \approx 0.24 \,.$$

The relative ice-melting rate at the bed in Equation (3) is determined by equation:

$$W_0 = \left[K_{\rm q} + \lambda^* (\partial \theta / \partial Z^*)_{Z^* = 0} / \Delta \right] / K_{\rm f} \,. \tag{8}$$

Criteria K_q (dimensionless geothermal flux) and K_f (phase-change number) are given by formulas:

$$K_{\rm q} = q_0 h'_0 \left[\lambda_0 (T_{\rm f} - T_{\rm h}^{\ 0}) \right], K_{\rm f} = \rho_0 w_{\rm h}^{\ 0} h'_0 L / \left[\lambda_0 (T_{\rm f} - T_{\rm h}^{\ 0}) \right]$$

where $L = 333 \text{ kJ kg}^{-1}$ is the latent heat of ice fusion.

An appropriate relation between the climatic characteristics W_h and θ_h is to be formulated to complete the mathematical model for the simulation of non-stationary temperature processes in the ice sheet. The findings of Robin (1977) give us the basis for expressing the accumulation rate via the water-vapor pressure in the air. Using the empirical Magnus's formula and following Jouzel and others (1987) and Ritz (1989), for relatively small surface-temperature variations θ_h , one can obtain:

$$W_{\rm h} = \exp\left[\beta^*_{\rm w}(\theta_{\rm h} - \theta_{\rm h\,|t=0})\right],\tag{9}$$

with $\beta_{\rm w}^* \approx 4.12 \times 10^3 (T_{\rm f} - T_{\rm h}^{-0}) / (273.15 + 0.67 T_{\rm h}^{-0})^2 \approx 4.1$ at Vostok.

Finally, substitution of the latter expression into Equation (4) yields the model for the ice-thickness calculation (Salamatin and others, in press):

$$\partial \Delta / \partial t = \beta^*_{\rm w} (\theta_{\rm h} - \langle \theta_{\rm h} \rangle) \exp \left[\beta^*_{\rm w} (\langle \theta_{\rm h} \rangle - \theta_{\rm h}|_{t=0}) \right].$$
(10)

INVERSE PROCEDURE

As has already been mentioned, the high-frequency paleotemperature records (of periods less than 3-5 kycar) are filtered out and do not penetrate into the

Antarctic ice sheet deeper than 200 m, because of the low thermal conductivity of the upper firn-ice sediments. On the other hand, long-period temperature oscillations on the ice-sheet surface (with periods larger than 300 kyear) do not induce noticeable temperature fluctuations in the ice because of its limited thickness. At the same time, the temperature profile from the deep borehole drilled at Vostok can contain information about temperature variations with Milankovich astronomic periods ($\tau_1 =$ 100, $\tau_2 = 41$, $\tau_3 = 23$, $\tau_4 = 19$ kyear) over the last climatic cycle. These oscillations prevail in the known paleoclimatic records (e.g. Jouzel and others, 1987; Martinson and others, 1987). Hence, the inferred component of climate changes on the Antarctic ice-sheet surface in the vicinity of Vostok Station can be written as a sum of harmonics:

$$\theta_{\rm h}(t) = \langle \theta_{\rm h} \rangle + \sum_{i=1}^{4} \left\{ A_i \cos \left[\Omega_i(t+t_0) \right] + B_i \sin \left[\Omega_i(t+t_0) \right] \right\}$$
(11)

where $\Omega_i = 2\pi h'_0/(\tau_i w_h^0)$. Obviously, this sum does not take into account individual secondary details and, therefore, may differ from the real temperature variations in the magnitude of amplitudes and in the relative height of peaks. The values Ω_i are fixed and there are eight unknown parameters: A_i and $B_i(i = 1, ..., 4)$ in Equation (11).

Let us denote a sufficiently large interval of time in the past: $t_0 \gg t_1 = \tau_1 w_h^0 / h'_0$ and introduce an initial condition in the boundary value in problem (6) in the following form:

$$\theta_{|t=-t_0} = \theta_0(Z) \tag{12}$$

where θ_0 is the steady-state temperature distribution in the ice sheet with the surface temperature $\theta_h \equiv \langle \theta_h \rangle$.

Then, let θ_{ex} be the dimensionless temperature profile measured in the deep borehole at Vostok Station at points $Z = Z_k, \ k = 1, \ldots, N$. The mean square-deviation function can be defined on the basis of the mathematical model (3), (6) (12) and the experimental temperature profile:

$$S(A_1, \ldots, B_1, \ldots; \langle \theta_h \rangle, \text{Pe, } K_q, K_f, \sigma, \alpha) = \left\{ \sum_{k=1}^{N} \left[\theta_{\text{ex}}(Z_k) - \theta(Z_k, t=0) \right]^2 / N \right\}^{\frac{1}{2}}.$$
 (13)

Thus, the inverse procedure of inferring the climatic parameters, A_i , $B_i(i = 1 ..., 4)$ comes to the problem of minimizing the S-function at fixed plausible values of $\langle \theta_h \rangle$, Pe, K_q , K_f , σ , α .

The solution of the boundary-value problem (3), (6) (12) is computed, using the finite-difference method in accordance with the implicit time-scheme algorithm. The space step for the Z^* -coordinate is 1/40. The time-interval length t_0 is taken equal to $3t_1$ and the time step is $t_0/150$. The choice is based on the preliminary test runs of the model. This ensures that the accuracy of the calculations is not worse than the reproducibility of the field measurements.

The minimization of target function (13) is performed

on the basis of the Newton method and the gradient method of steepest descent. A special interactive computer system for IBM-PC has been developed to perform the inverse procedure and the comparison of the reconstructed paleotemperature-time curve with clim-atic records from the Vostok ice core.

PALEOCLIMATIC RECONSTRUCTION

All computations on paleotemperature reconstructions considered in this section have been performed using the data obtained in 1988 (see Table 1). The total number of various sets of numerical experiments at different plausible values of the model parameters: $\langle \theta_{\rm h} \rangle$, Pe, $K_{\rm q}$, $K_{\rm f}$, σ and α has exceeded 100.

The starting tests have revealed the fact that the possible small variations of the mean surface temperature within the range $-0.09 < \langle \theta_h \rangle < -0.05$ (i.e. $-62.35 < \langle T_h \rangle < -60.15^{\circ}$ C) do not influence the accessible minimal level of deviation (13) and the inferred climatic signal. Therefore, this parameter should be fixed. According to the Vostok isotope record (Jouzel and others, 1987), the $\langle \theta_h \rangle$ value is taken hereafter equal to -0.07 (i.e. $\langle T_h \rangle = -61.25^{\circ}$ C).

Following Lliboutry (1979) and Ritz (1989), the nonisothermal conditions of the Antarctic ice-sheet shear deformation in the basal layer can be modeled assuming $\alpha \approx 10$. With this assumption, the two limiting situations are considered: (1) $\sigma = 1$, when the ice-sliding velocity on the bedrock is zero, and (2) $\sigma = 0$ (i.e. $\alpha \to \infty$) with no shear strain in the bottom ice. The minimum of the standard deviation *S* between the computed and observed temperature profiles is determined for different values of the geothermal flux (including strain heating) and the present-day accumulation rate within the intervals: $2.0 \le w_h^{-0} \le 2.7$ cm year ¹ and $0.05 \le q_0 \le 0.06$ W m ². In both cases, the best fits are found at $q_0 \approx 0.053$ W m ² and at definite values of w_h^{-0} :

(1) for $\sigma = 1$

(2) for $\sigma = 0$ $w_{\rm h}^{0} = 2.2 \,{\rm cm}\,{\rm ycar}^{-1}$ (Pc = 1.7, $K_{\rm q} = 1.4, K_{\rm f} = 5.6$) $A_1 = 0.0880, A_2 = -0.0566, A_3 = -0.0638, A_4 = 0.0142;$ $B_1 = 0.0614, B_2 = 0.0577, B_3 = -0.0345, B_4 = 0.0366;$ $S = 1.71 \times 10^{-4}$ (~ 0.0094°C).

The corresponding temperature-time curves are presented in Figure 1a and will be referred to hereafter as I and II, respectively. The discrepancies between the computed and observed temperature profiles are of the same order as a reproducibility of the experimental data (see Fig. 1b).

The minimization procedure is tested at various initial values of amplitudes $A_i, B_i, i = 1, ..., 4$. In a single case, for the condition with $\sigma = 1$ another minimum with standard deviation close to the best fit I has been found at $w_h^0 = 2.4$ cm year⁻¹. The corresponding paleotemper-



Fig. 1. a. Surface-temperature variations versus time inferred by inverse procedure from a borehole-temperature profile under the different assumptions: (1) when there is no sliding at the ice-rock interface ($\sigma = 1$); (11) basalsliding condition ($\sigma = 0$); (11) particular variant for a non-sliding condition ($\sigma = 1$). b. Mismatch between the computed and observed borehole-temperature profiles referred to variants 1, 11 and 111.

ature reconstruction (see curve III in Figure 1a) is very different in shape from the two main variants I and II among all tested, while no significant change in climatic event timing is observed. The shift of age for reconstructed temperature oscillations always remains less than 3 kyear.

Additional computational experiments have been performed to estimate the sensitivity of the deduced paleotemperature curve to the different boundary conditions and properties of ice. For instance, if the water_ produced by melting at the bottom of the ice sheet does not penetrate into the underlying rock and a subglacial lake is formed, the bottom melting rate W_0 tends to zero which can formally be taken into account by assuming the dimensionless number $K_{\rm f}$ in Equation (8) equal to infinity. In this case, the best fit has been determined at $w_{\rm h}^{0} = 2.3 \, {\rm cm \, year}^{-1} \, (\sigma = 0)$. The different thermophysical properties of ice: $c_0 = 1.9 \text{ kJ kg}^{-1} \text{ c}^{-1}$, $\lambda_0 = 2.66 \text{ W}$ $m^{-1} \circ C^{-1}$; $a_c^* = 0.193$; $a_{\lambda}^* = 0.308$ used by Ritz (1989) have also been considered. The optimal value of accumulation rate $w_{\rm h}^0$ is 2.25 cm year⁻¹ at $\sigma = 1$. In the two above cases q_0 is found to be equal to 0.053 W m⁻² and $S \approx 1.7 \times 10^{-4}$ (~0.0094°C). It is also remarkable that the predicted best-fit accumulation rate falls within the abovementioned range of experimental data for the recent precipitation at Vostok.

All the computational tests, as well as numerical experiments with various values of β^*_w in Equations (9) and (10), fully confirm the conclusion that in spite of the fact that computed paleotemperature oscillations can significantly differ from one variant to another by the amplitudes of their harmonic components, nevertheless, the principal events (peaks and troughs) in the simulated curve are always reproducible, easily recognizable and their ages are definitely predictable. The palcotemper-ature signal deduced from Equation (11) should be regarded only as a dominant "metronomic" part of the past surface-temperature variations. It cannot quantit-atively give the real ζ detailed) surface-temperature oscil-lations.

VOSTOK CLIMATE-RECORD DATING

Isotope, atmospheric gas and aerosol analysis of the Vostok ice core (Lorius and others, 1985; Jouzel and others, 1987; Petit and others, 1990; Jouzel and others, 1993) has provided information of fundamental importance, since the obtained time series quantitatively characterize the past climate changes at high resolution. One of the principal problems of interpretation of these data is the determination of the ice-core chronostrat-igraphy. Traditionally, the age of ice is computed on the basis of the ice-flow modeling (Lorius and others, 1985; Ritz, 1992), although this approach lacks the necessary information about the upstream surface conditions: accumulation rate, ice thickness, temperature gradient, etc. On the other hand, the paleotemperature reconstruction inferred from the deep borchole temperature measurements immediately refers climatic events to a certain place on the time-scale. As has been emphasized above, the age of the peaks in such a computed temperature-time curve depends only slightly on the model parameters. In this section, we consider these peaks as fixed points with known age (similar to stratigraphic markers) for correcting the isotope temperature-record preliminary time-scale.

First, we applied a mean square spline approximation to the original isotope temperature record from Jouzel and others (1987) which is shown in Figure 2a by dots. The smoothing aimed to conserve in the experimental record only the main temperature oscillations also reflected in the computed curve (see thick line in Figure 2b), thus all major climatic events (temperature maxima and minima) could be easily identified in both of them. Our smoothed curve is shown in Figure 2a together with a more detailed one obtained by Jouzel and others (1987) by spline smoothing of the 100 year equally spaced isotope values: thick and thin lines, respectively.

Secondly, we forced (stretched linearly between adjacent peaks and troughs) the smoothed isotope record versus depth to correlate with the temperature-time curve deduced from the borehole-temperature profile. The normalized surface-temperature oscillations (refer to variant I in Figure 1a) and the smoothed isotope-temperature record are plotted against time in Figure 2b by thick and thin lines, respectively. The correlation coefficient rbetween the two curves is 0.72, although amplitudes of some of their oscillations are significantly different. A



Fig. 2. The dating of the Vostok climate record on the basis of its fitting to the temperature-time curve derived from the analysis of the borehole-temperature profile. a. Isotope-temperature variations against depth in the Vostok ice core. The dots correspond to individual values; the thin line is a smoothed curve (from Jouzel and others, 1987). The thick (bold) line is a cubic-spline approximation of the isotope data adjusted to facilitate the comparison between the experimental and computed paleotemperature curves. The dashed vertical lines connect the identical peaks and troughs in the experimental and computed curves. b. Vostok normalized surface temperature-time curves. The thin line corresponds to the spline approximation of the Vostok isotope record which was correlated to the computed paleotemperature curve shown by the thick line (refers to variant I in Figure 1a).

similar value r = 0.70 is obtained for the best-fit curve II and r = 0.54 for solution III. It is relevant to emphasize here in the discussion that we did not expect better coincidence in amplitude size, because the in-formation about surface-temperature variations in the distant past (more than 70 100 kyear ago) had already been lost in the temperature profile in the Antarctic ice sheet. Thus, the Milankovich astronomic metronome is tuned in accordance with the comparatively recent climate history.

Obviously, the corresponding time-scales deduced for Vostok through the identified peak events are the compounds of straight lines. It should be emphasized here that irrespective of the degree of correlation between the inferred temperature time variations and the isotopetemperature record, all the chronologies co-incide with each other within a range of 2–3 kyear. The resulting Vostok climate record time-scale derived from the best-fit paleoreconstruction 1 is presented in Figure 3a. The peak age discrepancies in the different computational experiments are shown by error bars. The two other Vostok ice-core chronologies based on the iceflow modeling: (b) "old dating" (Lorius and others, 1985) and (c) "extended glaciological time-scale" (EGT) from Jouzel and others (1993) are also shown in Figure 3.



Fig. 3. Comparison of three Vostok ice-core chronologies: a. Time-scale deduced from the analysis of the boreholetemperature profile (the error bars show the age ranges in different computational variants); b. Depth-age relationship derived by Lorius and others (1985) on the basis of ice-flow modeling; c. Vostok ice core "extended glaciological time-scale" from Jouzel and others (1993).

Apart from the quality of the assumptions taken as a basis for our dating and uncertainties linked with model parameters (2 3 kyear), there is another source of error associated with success of positioning in the isotopetemperature record the temperature maxima and minima accepted as fixed points in the spline-smoothing proced-ure. To estimate the limits of this error, we examine the discrepancy between different time-scales in the upper part of the Vostok ice core, where the Holocene mid-peak positioning in the isotope record is quite uncertain and the datings derived from ice-flow modeling are most reliable. The difference between our time-scale and EGT for this part of the ice core does not exceed 3.5 kyear and can be considered hereafter as a limit of the peak position-ing error. The deviation of the depth-age relationship developed in this work from the EGT curve remains within the range of ± 5 kyear (i.e. within the general accuracy limit of dating) for most parts of the Vostok ice core (see Fig. 3). For example, our approach gives an age of 113.9 kyear for a depth of 1534 m instead of 110 \pm 3 kyear which was accepted by most investigators (Jouzel and others, 1993) and was used as a tuning point for the EGT chronology. The noticeable discrepancy can be observed for the large depths only, where the accumulation rate upstream of Vostok is determinant. This disagreement can be eliminated if one assumes an increase in the current precipitation rate in the ice-divide direction (Ritz, 1992). The latter seems to be realistic and has been partially confirmed by the information extracted from the ¹⁰Be data obtained for the Vostok ice core (Jouzel and others, 1993). A preliminary comparison of the reconstructed metronomic palcotemperature signal (see Fig. 1a) with the extended isotope-temperature record from Vostok down to a depth of 2546 m (Jouzel and others, 1993) also shows a good agreement between the two chronologies. According to our dating, the 2500 m depth ice is about 11 kycar younger than what is predicted by the EGT.

The new dating of the ice core puts the Vostok temperature record (see Fig. 2b, thin line) practically in phase (with an accuracy ± 2 kyear) with the global icevolume changes represented by the marine δ^{18} O SPECMAP (Martinson and others, 1987). Petit and others (1990) reached the same conclusion, using atmospheric dust as a stratigraphic marker to compare Vostok timing with other paleoclimate records.

CONCLUSION

The inverse procedure has been developed to reconstruct paleotemperature oscillations on the Antarctic ice-sheet surface from the temperature profile measured in the deep borehole at Vostok Station. The timing of the main events of the inferred climate changes appears to be insensitive to the possible variations of the model parameters within the range of their uncertainty. Thus, the correlation between the reconstructed temperature-time curve and the smoothed temperature record derived from the ice-core isotope analysis yields the age-depth relationship for Vostok paleorecords. Although our approach cannot present the detailed picture of paleotemperature variations, it provides another way of dating climate records. Actually, the results suggest that the Milankovich cycles have acted as a strong linear driver for the surface temperature at Vostok for a much longer time period than the time period over which we can detect their presence by borehole-temperature measurements. The dating technique developed here can be used in the future in combination with other methods (glaciological modeling, stratigraphic correlations, etc.) to ascertain the chronology of the deeper ice core.

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