

THEORETICAL PROBLEMS OF DISCRETE RADIO SOURCES

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The range of theoretical problems connected with the discrete sources is very large. It is convenient to distinguish between the normal and abnormal radio-frequency emitters that have quite different energy outputs per unit volume. Since the estimates by Minkowski and Greenstein [1] there has been considerable progress in the identification of these objects and in the provision of quantitative data about them. Table 1 includes revised estimates of the total luminosity, L , the emitting volume, V , and the luminosity per cubic parsec, J . Here L has been obtained by multiplying the observed power at 100 Mc./s. by an effective band-width of 500 Mc./s. and using the newly estimated distances. J is expressed in units of the total energy output of the sun ($= 3.82 \times 10^{26}$ watt) per cubic parsec. The figure for the Galaxy and M 31 is considerably higher than that given by Baldwin in this symposium, since I have not revised the estimate of the emitting volume from the original paper by Minkowski and Greenstein. If most of the emission comes from a larger galactic halo the specific luminosity, J , for the Galaxy and M 31 should be considerably reduced. For certain of the extra-galactic sources it is not certain whether the volume of the whole

Table 1. *Observed radio power of discrete sources*

Object	r (psc.)	$\log L$ (watts)	$\log V$ (psc. ³)	J (L_{\odot} /psc. ³)	Remarks
Galaxy	—	30.38	11.67	1.3×10^{-8}	
M 31	5×10^5	30.08	11.38	1.2×10^{-8}	
NGC 1068	6×10^6	31.63	5.54	0.31	Nucleus
NGC 5128	10^6	32.00	8.11	1.9×10^{-8}	Present collision region
			12.42	9.6×10^{-8}	Whole nebula
M 87	6×10^6	33.38	6.34	2.7	Jet
			11.55	1.7×10^{-5}	Whole nebula
NGC 1275	3×10^7	33.88	8.00	0.19	Present collision region
			8.47	0.06	Present and past collision regions
Cyg A	9×10^7	36.80	9.78	2.6	Nucleus
			11.67	0.033	Whole nebula
Crab	10^3	26.00	0.21	0.16	
Cas A	540	26.48	-0.59	2.9	

nebula, or only of the parts now in collision (or peculiar in nature, like the jet in M87) should be used in computing the specific emissivity, J . In Table 1, two values are then given.

The interesting feature, for abnormal radio-frequency emitters, is that the emission per cubic parsec is often of the order of or greater than one solar unit. Since in our own Galaxy the total optical emission in the galactic plane is very much less than the luminosity of the sun, it is clear that the radio-frequency emission process must be fundamentally of very short life, and must tap other energy than that of the stars.

There are two main subjects in which some definite theory is now possible, (1) the *thermal emission* from ionized regions, and (2) the possible sources of relativistic electrons that are needed if the abnormal radio emission is to be explained by means of the *synchrotron mechanism*. Before discussing some aspects of these two problems, I would like to state that (3) *plasma-type theories* should not be overlooked completely. Some recent theoretical work by Field (Princeton) and by Gould (California Institute of Technology) is suggestive in providing mechanisms for the escape of the plasma radiation, i.e. for the selective conversion of thermal energy into radio-frequency energy, with moderately high efficiency. Gould's mechanism involves the conversion of the longitudinal plasma oscillations into transverse waves by the normal thermal density fluctuations. And it need not be excluded that magnetized shock waves may provide high enough compression to yield plasma radiation at correct frequencies in discrete sources as well as in the sun.

I. THERMAL EMISSION FROM H II REGIONS

The possibility of detecting free-free emission of galactic emission nebulae was predicted by Reber and Greenstein in 1947 [2]; for the galactic plane, along spiral arms, it was predicted in 1949 [3]. With the detection by Haddock, Mayer and Sloanaker [4] of many diffuse nebulae at 3000 and 9500 Mc./s., it is apparent that a powerful new method of studying the ionized hydrogen has become available. I will first list some of the advantages of the radio-frequency measurements:

- (1) Freedom from absorption by dust.
- (2) Integration over the variable gas density.
- (3) Detection at very large distances.

Detailed spectroscopy is of course not possible; methods for distance determination may yet be developed; brightness distributions may be determined interferometrically.

The effect of absorption by dust in altering the apparent shape, size and brightness of the H_{α} emission regions is quite large. An approximate ratio of the absorption at H_{α} to the photo-electric colour excess on the E_1 scale can be obtained from the $1/\lambda$ law, which gives the ratio as 6. Using Whitford's [5] value of the infra-red absorption, the lower limit to the ratio is 3.8 (assuming absorption at 1.1μ as zero); an approximate value may also be derived from Morgan, Harris and Johnson [6] as 4.9. I will adopt 4.3. In that case the apparent emission measure

$$EM = EM' \times 10^{-1.72E_1},$$

where EM' is the true value of $\int n_i n_e dl$. As usual, EM and EM' will be expressed in parsec \times cm.⁻⁶. I have made a rough estimate of the colour excess of the exciting stars or objects in the region of some H II radio sources. The colour excesses range from 0^m.11 for NGC 7000 to 0^m.40 for M 17, NGC 6604. Consequently, nearby H II regions can be dimmed by factors of five, and these behind denser dark clouds by much larger amounts.

An inspection of 48-inch Palomar Schmidt plates suggests that the apparent size and shape of emission nebulae is often largely controlled by overlying dark lanes. For example, the Orion nebula suffers heavy obscuration just north of the Trapezium; objects like M 16 and especially M 8 and M 20 are located in extremely dense absorbing regions and show sharp dark markings superposed. Behind the central absorption lanes of the Milky Way even the brightest emission nebulae will disappear. I have made rough estimates of the required correction to the apparent size of some of the radio sources, for extinction in the dark lanes; in several cases the factor is greater than two. Thus the H_{α} surface brightness times the apparent area may be reduced by factors of the order of ten.

The variation of gas density (and possibly of excitation) produces large effects on the apparent surface brightness in H_{α} . For example, NGC 6604 has a diameter about 50' at low surface brightness, and has a core of strong H_{α} emission about 5' in diameter. Without isophotes it is difficult to predict the total H_{α} magnitude, and therefore the radio frequency emission. Fundamentally, it is even difficult to *define* a galactic nebula. From the rough moduli of the stars involved, from their closeness in the sky, similarity of surface brightness, and involvement with dark lanes, it is not certain that M 8 and M 20 are two distinct objects, although separated by 90', which is about the long dimension of M 8. Sharpless [7] suggests that NGC 6604, M 16 and M 17 may be a physically connected group. Radio isophotes at intermediate declinations would be decisive.

Some of the features of the radio emission may be best understood if the optical nebulae are brighter and relatively absorption-free condensations in larger units. The radio isophotes integrated over the larger areas then will give valuable information on the mean gas density in these giant H II complexes.

The observations of H II regions by Haddock, Mayer and Sloanaker [8] permit derivation of the true EM' from the observed power received. From the theory of free-free emission in an optically thin nebula (all are thin at these high frequencies), the power P_f at frequency f is approximately

$$P_f \approx 5 \times 10^{-23} EM' \theta^2 T_e^{-1/2} \text{ w.m.}^{-2} (\text{c./s.})^{-1},$$

where θ is the apparent size in radians. An approximate constant value of g , the logarithmic term in the absorption coefficient, has been used and $T_e \approx 10^4$ °K. The size must be corrected from the Schmidt plate measurements. Table 2 gives the resultant values of EM' , i.e. essentially the predicted H_α surface brightness; EM is the value after reduction for interstellar absorption. The areas are given in square millimetres on the Schmidt prints, scale 1'2/mm.; F is the factor by which areas are multiplied to allow for the obscuration.

Table 2. *Emission measures deduced from radio power*

Object	$10^{26}P_f$	Area	Area $\times F$	EM'	EM	Remarks
M20	11	100	150	1.2×10^5	4×10^4	Part of M8?
M8	20	1500	2000	1.6×10^4	8×10^3	
NGC 6604	10	{ 1800	2500	7×10^3	10^3	Total area
		{ 25	25	7×10^5	10^5	Bright core only
M16	16	{ 1600	1600	1.7×10^4	6×10^3	Total area
		{ 300	600	4.5×10^4	2×10^4	Bright core only
M17	68	{ 1200	1200	9×10^4	3×10^4	Total area
		{ 300	750	1.5×10^5	5×10^4	Bright core only
IC 4701	7	1000	1000	1.2×10^4	7×10^3	Part of M17?
NGC 7000	—	—	—	4×10^3	2×10^3	From T_A
NGC 1976	45	1600	2500	3×10^4	10^4	Total area
IC 443	6	1200	1200	8×10^3	3×10^3	Filamentary

Several of these EM' are so large that the nebulae would be optically thick at about 200 Mc./s., i.e. show energy decreasing with frequency. Interesting problems are raised by the EM in Table 2. The very large range of brightness in H_α results for certain nebulae in two determinations, dependent on which size is assumed for the nebula. Without radio and optical isophotes we can go no further. The values observed optically for low-surface-brightness nebulae run down to 500 (Sharpless-Osterbrock [9]); the weakest radio emissions here studied are still far brighter than the faint H II regions detected optically. Strömrgren's values

of EM for some of the fainter diffuse nebulae go up to 7000 (for IC 405). The brightest areas in NGC 6604, M 17 and M 20 run up to 40,000. It is known (although detailed optical measurements are lacking) that the nebulae in Sagittarius are about the brightest, with the exception of NGC 1976. Thus, the present sensitivity and resolution are not quite sufficient for detailed studies of the distribution of ionized hydrogen in the Galaxy. Whether M 20 and M 17 are really as high in EM as derived from radio data depends critically on the areas, and higher angular resolution would decide. Although optical data are lacking, we may conclude that the identifications of bright H II regions seem correct, and the radio emission is reasonably accordant with the optical.

It is interesting to note that IC 443, which may be a superthermal emitter at low frequencies, has a quite reasonable EM , so that its 300 Mc./s. radiation may be thermal. Observations of the details of its high-frequency spectral energy would be valuable.

One of the most remarkable entries in Table 1 is the predicted EM for NGC 1976, the Orion nebula. In spite of its rather high P_f , its large total area results in $EM \approx 10^4$, averaged over the nebula. Strömgren [10] found $EM = 8 \times 10^6$ from the surface brightness at the Trapezium; Greenstein [11], averaging over the inner part of the nebula from the spectra, gave $EM \approx 10^5$, and H. Johnson [12] gave 6×10^4 . There is undoubtedly an enormous gradient in H_α surface brightness which accounts for the disparity in these results, and the radio value of $EM = 10^4$ includes the outer extension. Osterbrock [13] has developed a new method using the ratio of $\lambda\lambda 3726-3728$ of (O II), which is pressure-sensitive, to determine the electron density in NGC 1976. At the Trapezium he gives $n_e \approx 4 \times 10^4$; at 4' distance $n_e \approx 10^3$; at 16', $n_e \approx 300$. This determination of $n_e(r)$ permits a numerical integration to give the effective mean EM averaged over the nebula. Assume it to be spherically symmetrical, of radius R . From a definition of the type:

$$\overline{EM} = \frac{N^2 \times \text{volume}}{\text{area}}$$

valid for a sphere of constant density N , we can formally define

$$\overline{EM} = \frac{4}{R^2} \int_0^R N^2(r) r^2 dr.$$

Even with Osterbrock's data the near-singularity of very high density at the centre of Orion makes integration difficult, the innermost 1' contributing nearly one-third the total. However, I find $\overline{EM} \approx 9 \times 10^5$, higher

than previous spectroscopic averages and about thirty times the EM deduced from the radio observations. It is probable that the outermost parts of the nebula were not included in the N.R.L. measurements (my area is about four times their beam-width). In conclusion, if there is any discrepancy between radio and optical emission, it is in the sense that the radio emission is too weak, so that we can exclude superthermal mechanisms at these high frequencies. Other nebulae with the high EM of NGC 1976 or M 17 could be detected at very great distances. M 17 is approximately 1000 parsecs from the sun; at 17,000 parsecs on the opposite side of the galactic centre it would have $P_f \approx 0.2 \times 10^{-25} \text{ w.m.}^{-2} (\text{c./s.})^{-1}$, which does not seem beyond the reach of high-frequency technique.

One statistical comment may be of interest. If we assume that radio and optical emission are in proportion, the ratio of radio power to observable H_α surface brightness increases because of dust absorption. Consider a uniform, random distribution of H II regions and dust. Let the emissivity in H_α be ϵ_α , in radio frequencies ϵ_R , and let k_α be the absorption by dust. Then the surface brightness will be in the ratio:

$$I_R/I_{H\alpha} = \frac{\int_0^{l_1} \epsilon_R dl}{\int_0^{l_1} \epsilon_\alpha e^{-k_\alpha l} dl},$$

$$I_R/I_{H\alpha} = \frac{\epsilon_R l_1}{\frac{\epsilon_\alpha}{k_\alpha} (1 - e^{-k_\alpha l_1})} = \frac{\epsilon_R \tau_1(\alpha)}{\epsilon_\alpha (1 - e^{-\tau_1(\alpha)})}.$$

Thus, as $\tau_1(\alpha)$, the optical absorption increases, the observed radio power compared with the H_α EM becomes of the order of $\tau_1(\alpha)$ times its value for an unobscured H II region. Gas along a spiral arm will probably become invisible optically if $\tau_1(\alpha) > 3$, i.e. if more distant than 3000 parsecs. Thus, along the direction of Cygnus, or through inner arms in Sagittarius, large numbers of invisible H II regions will be detectable at radio frequencies.

2. ENERGY CONSIDERATIONS IN GAS COLLISIONS

The strong radio-frequency emission of colliding gas masses suggests that some fundamental process becomes operative in almost any collision at sufficiently high velocities. Let us briefly consider the various energy contents of a gramme of hydrogen. The thermal energy is

$$\frac{kT}{m_H} \approx U_{th.},$$

the kinetic energy at collision velocity v is

$$\frac{v^2}{2} \approx U_{\text{kin.}},$$

and the nuclear energy (for conversion into helium) is

$$0.007c^2 \approx U_{\text{nuc.}}(H).$$

In general, the last expression gives too high a yield because the direct proton-proton interaction is very slow; in consequence the reactions of hydrogen with light nuclei are the only effective ones, and therefore it is only the deuterium reaction that should be included. With x_D the abundance of deuterium, which is about 1/7000,

$$0.007x_D c^2 = U'_{\text{nuc.}}$$

Let us compare these energies when collision velocities of 1000 km./sec. are considered:

$$U_{\text{th.}} \approx 8.3 \times 10^7 \text{ T ergs/gm.},$$

$$U_{\text{kin.}} \approx 5 \times 10^{15} \text{ ergs/gm.},$$

$$U'_{\text{nuc.}} \approx 9 \times 10^{14} \text{ ergs/gm.}$$

Ordinary gas temperatures are so low that $U_{\text{th.}} \ll U_{\text{kin.}}$, unless the shock-wave heating is included, when $U_{\text{th.}} \approx U_{\text{kin.}}$; even in the latter case the heating comes from the translational energy. Surprisingly, the easily realizable nuclear energy is somewhat less than the kinetic energy. In consequence, even if nuclear energy sources can be tapped, they will not greatly increase the available energy, and we must not expect direct effects of nuclear processes in the production of radio power. I believe, however, that high-velocity gas collisions could provide relativistic electrons for further acceleration by a Fermi or a betatron process in such objects as the Cassiopeia A source, or the Crab nebula. I compute below effects to be expected if gas clouds with a mass near one solar mass collide at velocities of 1000 km./sec. or higher. These computations could not apply to the Crab nebula if the mass is as low as Prof. Oort conjectures, they might apply to the Cassiopeia A source, to other supernova envelopes colliding with interstellar gas, or to the Cygnus A source with proper scaling factors.

3. NUCLEAR REACTIONS IN COLLIDING GAS MASSES

The very high observed velocities within different filaments of the Cassiopeia A source and the turbulent velocities in the outer filaments of the Taurus A source suggest the possibility of nuclear reactions. The hydro-

dynamics of shock fronts, with probable magnetic effects, are too complex for detailed treatment, but some rough considerations can be applied. Collisions may be internal, involve heating to very high temperatures and subsequent expansion into an appreciable volume where thermonuclear reactions occur, or nuclear collisions may occur only on the fronts themselves at the gas-collision velocities. The composition of the gas may be very abnormal (H deficient probably in Taurus A) or nearly normal, as in Cassiopeia A. The abundance of the most interesting nucleus, deuterium, might be expected to be low in an exhausted star, but on the other hand, D is produced abundantly in high energy and heavy element reactions. The most profitable situations for a first investigation are the (d, p) and the (d, d) thermonuclear reactions, assuming normal abundance.

A collision at relative velocity v results in a heating to

$$T \approx \frac{v^2}{2R} \frac{\gamma - 1}{4},$$

or 10^7 °K. for 1000 km./sec. relative velocity. (The compression is only by a factor of four.) At relative velocities of 5000 km./sec. such as are found within a diffuse filament of Cassiopeia A, $T \approx 250$ million degrees. Straightforward nuclear collisions at velocity 5000 km./sec. correspond to 0.13 MeV. for a proton (or 1.5×10^9 °K.). The latter energies correspond to relatively high Gamow barrier-penetration, and lead to appreciable reaction probabilities in spite of the very low densities.

The carbon-cycle reactions, however, are too slow even at these energies. $C^{12}(p, \gamma) N^{13}$ has a cross-section of 3×10^{-33} cm.² at 150 KeV., while the total cross-section for $D^2(d, p) T^3$ and $D^2(d, n) He^3$ is 3×10^{-26} . Thus, unless $X_D^2/X_C X_H$ is less than 10^{-7} , the carbon reaction can be neglected; the probable value of the abundance ratios is about 10^{-3} . (However, if the temperatures were ever appreciably higher, the carbon reactions would be important.)

The thermonuclear rates, with T in million degrees, are given for the D-D reaction as:

$$\begin{aligned} p_{DD} &= 4 \times 10^{10} \rho x_D T^{-\frac{3}{2}} e^{-42.6/T^{\frac{1}{2}}}, \\ p_{DD} &\approx 10^6 \rho x_D, \quad (250 \text{ million degrees}) \end{aligned}$$

per particle per second, while for the p-D reaction

$$\begin{aligned} p_{pD} &= 7 \times 10^4 \rho x_H T^{-\frac{3}{2}} e^{-37.2/T^{\frac{1}{2}}}, \\ p_{pD} &\approx 5 \rho x_H, \quad (250 \text{ million degrees}). \end{aligned}$$

The p-D reaction is slower by about 30, but represents a lower limit if D is exhausted in the source material, and still present in interstellar gas. The

energy output is obtained from Q , the yield per reaction and the mass M or volume V of the reacting colliding clouds:

$$L = \frac{pQx_D M}{m_H} = pQx_D V n_H.$$

The number of reactions is L/Q . The values of p are small, so there is no exhaustion of the nuclei.

$$L_{DD} = 8 \times 10^{33} n_H x_D^2 M / M_\odot,$$

which for one solar mass, a volume of 6×10^{54} cm.³ (i.e. $n_H = 200$) gives $L_{DD} \approx 3 \times 10^{28}$ ergs/sec., and the number of reactions as 8×10^{33} per sec. The p-D reaction gives

$$L_{pD} = 5 \times 10^{28} n_H x_D x_H M / M_\odot,$$

which results in $L_{pD} \approx 1.4 \times 10^{27}$ ergs/sec., and the number of reactions as 3×10^{32} per sec. Thus if we can conceive of as much as one solar mass in collision at 5000 km./sec. we will get something like 10^{34} reactions per sec. This number is not quite large enough for Oort's interpretation of the Crab nebula, as we shall see. The temperature coefficients of the reactions are small; $p_{DD} \propto T^{1.6}$, and $p_{pD} \approx T^{1.3}$, so that the collision velocities are not critical.

A larger yield is obtained if we do not allow the collisions to degrade into thermal motions, but imagine the same mass of gas impinging on a stationary target, and suffering reactions, a somewhat unrealistic picture, of course. Then $L/Q = \sigma v x_D^2 n_H^2$ for the DD reaction. The cross-section at 5000 km./sec., i.e. 130 KeV., is found in the laboratory to be

$$\sigma = 3 \times 10^{-26} \text{ cm.}^2.$$

Then L/Q is 7×10^{34} sec.⁻¹ for one solar mass, which seems to be about as large a yield as can be obtained reasonably. It refers probably more to Cassiopeia A than to Taurus A.

These reactions yield different products. The D-D yields ultimately, per reaction, in addition to kinetic energy, one-half of an 0.02 MeV. electron (tritium decay), one-half of a 0.8 MeV. electron (from neutron decay), and one-half of a 1.5 MeV. proton (which may produce some fast electrons). Each p-D reaction yields a 5.5 MeV. quantum. While the reactions do not seem important as a direct energy source they provide two important sources of relativistic electrons. Consider the 5.5 MeV. γ -ray; with a cross-section of about 0.10 of the Thomson scattering cross-section it can produce by Compton scattering an electron with average

energy of the order of 3 MeV. (and rising to 5 MeV.); the cross-section, $\sigma = 6 \times 10^{-26} \text{ cm.}^2$. Then if $n_H = 200$, thickness $l = 10^{18} \text{ cm.}$, the probability of absorption is 1.2×10^{-5} , and the p-D reaction produces 10^{-5} relativistic electrons per reaction. This is actually smaller than a nuclear process suggested privately by T. Lauritsen, called internal-pair-production, which is observed to occur in light nuclei. At about 5 MeV. a dipole γ -ray produces a positron and negaton within the nucleus, with probability about 2×10^{-3} ; thus 4×10^{-3} are produced per reaction; the life of the positron is long at these densities. In résumé, from one solar mass we can obtain up to 3×10^{32} relativistic particles ($\approx 3 \text{ MeV.}$) per sec. Since at 250 million degrees, kT is only 0.02 MeV., *the nuclear processes give a step-up in energy of over a hundred*, and may provide the fast electrons for a Fermi acceleration process. Their number is smaller than Oort has stated, in private communication, to be necessary for the probable rate of loss by synchrotron radiation (up to 10^{36} electrons sec.^{-1}). If regions of much higher density could exist, even if of smaller total mass, the output could be appreciably raised. No probable situation, at the present low gas density, could make the reactions significant as an energy source, but in the past the density would have varied inversely as the cube of the time.

In solar flares, for example, collision velocities are comparable, but densities are perhaps 10^{10} times higher. If a volume of 10^{27} cm.^3 is adopted, with $v = 5 \times 10^8 \text{ cm./sec.}$, $10^{-25} n_H^2 \text{ cm.}^{-2} \text{ sec.}^{-1}$, γ -rays would reach the top of the earth's atmosphere. If they originate at or near chromospheric densities such a flux of γ -rays might be detectable in balloon flights.

4. ABSORPTION OF FAST NUCLEI

If the cross-section for absorption of γ -rays is small, it is probable that the gas is transparent for other energetic particles, although the mean-free path at thermal velocities may be small. With $n_H = 200$, $l = 10^{18}$ the probability of collision is $2 \times 10^{20} \sigma$, small, since for energetic particles $\sigma \approx 6 \times 10^{-26} A^{\frac{2}{3}} \text{ cm.}^2$, where A is the atomic weight. Then the collision of an accelerated particle with a stationary proton occurs in time

$$\tau \approx 2 \times 10^7 \text{ years} / A^{\frac{2}{3}} n_H^2.$$

Since even for $A = 100$, τ exceeds the age of the sources, the heaviest elements are substantially unaffected, and there is no important loss to the accelerating mechanism.

One possibility is that accelerated protons could destroy heavy stationary nuclei, and produce light elements by spallation. Cosmic ray

emulsions show that a spallation reaction in a heavy nucleus usually fragments it largely into isotopes of H and He. About 0.3 of the mass appears as D. For the collision rate we need the number of relativistic protons, N_H . Assume that for every electron accelerated there is also a proton; then in 10^3 years there will be 3×10^{46} protons, a density of $N_H \approx 5 \times 10^{-9} \text{ cm.}^{-3}$. The lifetime of a heavy nucleus is then 3×10^{14} years, so that the fraction 3×10^{-12} is destroyed, producing 3×10^{-11} D nuclei per original heavy particle. If the heavy particles formed 1/6000 of the original mass, the abundance of deuterium created becomes about 10^{-14} that of hydrogen—much too low to be significant. There has been no approach to a ‘steady state’ such as is imagined for galactic cosmic rays.

5. ABSORPTION OF LOW-ENERGY ELECTRONS

The nuclear reactions give relativistic electrons, with energies about 100 times the thermal. In a Maxwellian velocity distribution a completely negligible fraction have such high energies, but an additional important factor is the great stopping power of matter for low-energy electrons. A complete theory of the energy loss in an ionized gas is not available from low to relativistic electron energies. I use Bethe’s expressions for the energy loss per unit length in a neutral gas, $Z=1$ (hydrogen):

$$-\frac{dE}{dx} = \frac{4\pi e^4 N}{mv^2} \log mv^2/I; \quad (\text{non-relativistic})$$

$$-\frac{dE}{dx} = \frac{2\pi e^4 N}{mc^2} \log E^3/2mc^2I^2; \quad (\text{relativistic})$$

where N is the space density and I the mean excitation or ionization energy. For a neutral gas the logarithmic term is usually approximated by $I \approx 14$ volts; for an ionized gas the approximation may be used that I corresponds to the potential energy at the mean distance to the nearest neighbour; for a plasma more complex expressions could be used. Fortunately, even large errors in the logarithmic term produce little effect. The loss of energy per unit length is obtained from

$$\log E/E_0 = -\beta X,$$

as approximately:

$$\beta = \frac{2\pi e^4 N}{E^2} \log 2E/I,$$

or

$$\beta = \frac{2\pi e^4 N}{mc^2 E} \log E^3/2mc^2I^2.$$

As for the cut-off energy, I , if it can be taken as $\gtrsim e^2 N^{\frac{1}{2}}$, which is about 10^{-6} electron-volts it increases β by about 2.5 over the value derived for $I = 14$ volts. Values of β are:

$$\begin{array}{cccccc}
 E = & 10^3 & 10^4 & 10^5 & 10^6 & 10^7 & \text{eV.} \\
 \beta/N = \sigma = & 6 \times 10^{-18} & 8 \times 10^{-20} & 7 \times 10^{-22} & 3 \times 10^{-23} & 3 \times 10^{-24} & \text{cm.}^2
 \end{array}$$

Electrons of a few kilovolts, or less thermal energy, have ranges of about $10^{19}/N$ cm., i.e. smaller than the nebula if $N = 10^2$. Therefore, they will be re-oriented and share their energies in gas collisions, and not be able to enter the magnetic acceleration mechanism. However, electrons of a few MeV. (e.g. those from nuclear reactions) have free paths $10^{23}/N$ cm., large compared to the nebula. They will be acted on purely by the magnetic field, and need not lose energy to other electrons: While it must be admitted that the collision cross-sections are rough, thermal electrons should not move in straight trajectories, even in an expanding gas mass, and there is no doubt that the range of the MeV. electrons is large, except for the effects of the magnetic fields.

Others have discussed the Fermi acceleration mechanism for cosmic rays. In the case of the electrons in the Crab nebula, starting at MeV. energies, they will be raised to 10^6 MeV. after $14 c/v$ collisions with magnetized clouds of velocity v/c . For 1000 km./sec. as the velocities of clouds within the Crab nebula, 4×10^3 collisions are required, i.e. four per year. The present size of the nebula is of the order of a light year, the particles traverse it in a straight line in a year, so that the number of separate clouds within the nebula need only be four. Thus the structure required of the magnetic field need not be very fine, or small-scaled. The kinetic energy lost by the clouds is measured by the rate of decay of the relativistic electrons. If we take Oort's estimate of 10^{36} electrons, each of 10^{12} eV., we lose 10^{36} ergs/sec., or 10^{47} ergs in the life of the nebula. With one solar mass and 10^3 km./sec., the kinetic energy of expansion is 10^{49} ergs; it is apparent that a rate of loss much greater than 10^{36} ergs/sec. cannot be maintained. If Oort's mass estimate is used, much too little energy is available, and the relativistic electrons cannot be produced in the nebula, but must come from the star.

REFERENCES

- [1] Minkowski, R. and Greenstein, J. L. *Ap. J.* **119**, 238, 1954.
- [2] Reber, G. and Greenstein, J. L. *Observatory*, **68**, 15, 1947.
- [3] Greenstein, J. L. *A.J.* **54**, 121, 1949; *Sky and Telescope*, **8**, 149, 1949.
- [4] Haddock, F. T., Mayer, C. H., Sloanaker, R. M. *Ap. J.* **119**, 456, 1954.
- [5] Whitford, A. E. *Ap. J.* **107**, 102, 1948.

- [6] Morgan, W. W., Harris, D. L. and Johnson, H. L. *Ap. J.* **118**, 92, 1953.
- [7] Sharpless, S. *Ap. J.* **118**, 362, 1953.
- [8] Haddock, F. T., Mayer, C. H. and Sloanaker, R. M. *Nature*, **174**, 176, 1954.
- [9] Sharpless, S., Osterbrock, D. E. *Ap. J.* **115**, 89, 1952.
- [10] Strömngren, B. *Problems of Cosmical Aerodynamics* (Dayton, Ohio, C.A.D.O. 1951), ch. II.
- [11] Greenstein, J. L. *Ap. J.* **104**, 414, 1946.
- [12] Johnson, H. M. *Ap. J.* **118**, 370, 1953.
- [13] Osterbrock, D. E. *Ap. J.* **122**, 235, 1955.

Discussion

Gold: The generation of fast particles seems to be related to fast gas motions in the Sun, the Crab nebula, and probably also in other radio sources. One wonders which phenomenon in rapidly moving gases can be held responsible. Shock waves in magnetized tenuous gases are inadequately understood. They are places where energy would be available in a well-organized form for some subsidiary process, and where indeed most of the kinetic energy of the gas is finally dissipated.

Biermann: It seems that magnetic fields have only been mentioned in connexion with the Schwinger mechanism. I should like to draw attention, however, to the importance of magnetic fields for the emission by plasma oscillations: the ideal plasma oscillation does not radiate, but a superposed magnetic field gives coupling leading to emission. This is perhaps the most probable mechanism under astrophysical conditions.