



On the benefits of pension plan consolidation: Understanding the impact of full plan mergers

Jean-François Bégin¹, Barbara Sanders² and Wenyan Zhou

Department of Statistics and Actuarial Science, Simon Fraser University, Burnaby, BC, Canada

Corresponding author: Jean-François Bégin; Email: jbegin@sfu.ca.

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Abstract

This study investigates the benefits and drawbacks of pension plan consolidation by quantifying the impact of mergers of heterogeneous plans on different stakeholders in a unique Canadian implementation of defined benefit plans. Using a comprehensive framework that combines a realistic economic scenario generator, a stochastic mortality model that captures differences among subpopulations, a cost model with economies of scale, and a dynamic asset allocation methodology, we evaluate the combined effect of asset- and liability-side changes on three groups of measures: plan-related risk measures assessing profits from an economic capital perspective, consumption-based metrics to understand the impact on members, and contribution risk measures capturing the risk from the employer's viewpoint. We apply the framework to a hypothetical and empirically relevant merger and find that consolidation is favorable under most circumstances: the positive impacts of better diversification and economies of scale continue to outweigh the negative effects of heterogeneity even when the merging plans have different mortality expectations, different maturity levels, or modest differences in initial funded ratios.

Keywords: Retirement; defined benefit pension schemes; economies of scale; simulation models; economic capital

1. Introduction

The occupational pension system is in a state of transition. On the defined benefit (DB) side, plan sponsors continue to adjust their plan's risk exposure. Some sponsors turn to investment solutions that enhance the plan's risk and reward profile, while others seek to transfer risk to insurers via annuity buy-ins and buy-outs or choose to adjust plan design elements.¹ An additional emerging solution is to adjust plans' fiscal and demographic profiles by expanding their membership and merging with other plans.² For instance, at least two large Canadian public-sector DB plans – the Colleges of Applied Arts and Technology Pension Plan (CAAT) and OPTrust – opened their doors to new, unrelated employee groups in recent years. In addition, three small-sized Canadian university plans decided to merge and formed the University Pension Plan Ontario (UPP) in 2021. UPP now has 16 participating employers.

¹In Canada, jointly sponsored pension plans, shared risk plans, and target benefit plans are examples of design innovations aimed at restructuring the balance of costs and risks in traditional DB plans.

²Pension plan mergers are not new; historically, however, they were primarily tied to mergers and acquisitions of employers (see, e.g., Bailin *et al.*, 1989). In the present study, we consider mergers of pension plans only – unrelated to mergers and acquisitions of employers.

The trend appears to be catching on: a recent survey of 50 medium and large Canadian pension funds found that 38% have already consolidated assets and liabilities by merging with other plans, and 44% are considering doing so in the future (CIBC Mellon, 2021).³

This trend toward plan consolidation is not unique to Canada. In the Netherlands, the number of occupational pension plans declined from more than 1,000 to just under 250 over the last 25 years, with about 50 very large plans dominating the landscape today (Wolzack, 2018). Further consolidation is desired by some plans but is slowed down by regulations which require that funded ratios not be adversely affected by plan mergers. In response, new plan structures that facilitate the achievement of economies of scale in the investment, administration, and governance functions without combining the liabilities of the plans coming together have emerged (Preesman, 2015). Similar considerations support the recent growth of master trusts in the UK.

In the US, plan mergers over the last decade mainly occurred in the context of corporate merger and acquisition activity; however, there is growing interest in multi-employer pension solutions. This interest is primarily focused on defined contribution (DC) plans now, but it may spill over to the DB sphere in the future, especially in the public sector.

In Canada, most pension plan merger activity is happening in the realm of jointly sponsored pension plans or JSPPs – a unique implementation of DB plans. As the name indicates, in a JSPP, plan members and participating employers share the responsibility that normally belongs to the DB plan sponsor, including all aspects of governance and operations, from design choices through funding to administration. They also share equally the plan risks, bearing equal burden for funding shortfalls and benefiting equally from surpluses.⁴

In light of this trend, a key question is: under what circumstances are pension plan mergers beneficial for different stakeholders, including the members of each merging plan, its employers, and the pension plan as an entity in its own right? The present article answers this question. Existing research (and common sense) suggests that consolidation of homogeneous plans should bring, in theory, many asset-side benefits to all parties involved, including greater efficiency in the administration and investment functions and better access to alternative assets. However, the actual benefit of plan mergers also depends on the specific features of the plans or groups to be combined (e.g., type of benefit, fund size, funded status, and demographic profile), which could be different. Our study discusses the merits and limitations of plan mergers and consolidation in the context of heterogeneous groups.

Indeed, the existence of economies of scale in pension administration and investment is well established in the literature, with the first scholarly studies on this topic dating back to the early 1980s. Using US data, Mitchell & Andrews (1981) proposed a model based on the Cobb–Douglas cost function to understand plan expenses and demonstrated the economies of scale – going as far as encouraging plan consolidation. More recently, Bikker & de Dreu (2009) found that administrative and investment costs are explained primarily by pension fund size: small-sized funds tend to have higher costs (per member) and *vice versa*. Larger funds have more negotiation power in investment and can spread their fixed costs across a larger number of members (Bikker *et al.*, 2012); they also benefit from internal investment management – shown to be three times less expensive than external management. This conclusion is consistent with the work of Dyck & Pomorski (2011): larger plans outperform smaller ones. In fact, the authors found that

³There may be some overlap in these statistics as some funds that have already merged may plan to do so again in the future.

⁴This joint-sponsorship model is often supported by a special bicameral governance structure with a sponsor committee and a formal board of trustees working together, each with its own responsibilities in managing the plan, and each having both member and employer representation. This robust structure helps avoid conflicts of interest by clearly delineating where members and employers can play partisan roles (i.e., on the sponsor committee) and where they ought to stay neutral and fulfill a fiduciary role (i.e., on the board of trustees). This structure is particularly well suited to managing the interests of multiple (potentially disparate) employee and employer groups within the plan, supporting the trend of mergers among unrelated parties.

the abnormal returns of larger plans are about 50 basis points (bps) per annum higher than those of smaller plans.^{5,6}

Fund size also affects the investment universe; that is, the range of assets available to the plan. de Dreu & Bikker (2012) showed that smaller plans tend to be less sophisticated, and less sophisticated pension funds allocate nothing to complex asset classes and international investments. This conclusion is aligned with Gorter & Bikker (2013): larger investors are expected to have more diversification opportunities and a better risk profile, providing large pension plans with more risk-bearing capacity.

All of these results point to the potential advantages of plan consolidation. However, as is evident from the Dutch and UK cases, many of these benefits can be achieved by simply pooling the plan assets without merging the plan liabilities. By contrast, full plan mergers may have additional solvency and welfare impacts arising from changes in the demographic profiles of the plans and other liability-side factors. For example, bringing together relatively homogenous groups could have a positive impact on all parties due to better diversification of liability-side risks (e.g., idiosyncratic mortality risks). However, heterogeneity in the funding, investment, or demographic profiles of the merging plans may reduce (or possibly negate) the beneficial impacts of a merger.

From a practical perspective then, it is imperative for pension stakeholders to understand both the asset- and liability-side impacts of mergers; however, this question has not received any attention in the literature to date. Instead, studies of pension mergers have almost exclusively focused on economies of scale and the impact of fund size on performance.⁷ This study fills the gap by developing a comprehensive framework that, for the first time, takes all of the relevant elements of pension plan operation into account to quantify a merger's total solvency and welfare impacts. This framework includes (1) a full-featured economic scenario generator (ESG), (2) a stochastic model for mortality and longevity risk that considers multiple subpopulations, (3) an administrative cost model that reflects economies of scale, and (4) a dynamic asset allocation method using a reference-dependent utility function focused on the funded ratio that mimics the forward-looking process used by pension plans to decide on an optimal asset allocation strategy.

The framework is estimated on Canadian economic, financial, and mortality data.⁸ It is then applied to measure the solvency and welfare impacts of hypothetical but empirically relevant mergers that fit within the Canadian context. Inspired by CAAT's example, we consider a merger between a small Ontario corporate plan of about 1,000 members and a large Ontario JSPP of 50,000 members in this study.⁹

Overall, we provide well-grounded evidence of the benefits of merging when the plans are homogeneous on the liability side. In the cases considered in this study, we report improvements from the plan's, the members', and the employer's perspectives. As expected, these gains are primarily explained by the reduction of the administrative and investment costs, better – more optimal – asset allocation strategies, an improved investment universe that allows for superior diversification, and better mortality pooling as the number of members increases post-merger.

This result in the homogeneous case should not come as a surprise; more importantly, we also assess the robustness of our conclusions by adding heterogeneity on the liability side as this

⁵Economies of scale is still a topical subject; see, for instance, Agostini *et al.* (2014), Alserda *et al.* (2018), and Bikker & Meringa (2022) for recent contributions on the topic.

⁶Larger plans may also be prone to higher costs. For example, the fund may be too large in relation to the number of high-quality investment opportunities available, which negatively impacts returns. Larger plans may also experience severe price impacts when large volumes of assets are traded. Nonetheless, most studies conclude that economies of scale dominate diseconomies (see, e.g., Bikker & de Dreu, 2009; Dyck & Pomorski, 2011).

⁷One notable exception is Sutcliffe (2006) who investigated the criteria for pension plan mergers: he argued that mergers should not generate any profits or losses and should not dilute the funded ratio.

⁸We focus on Canadian data as the motivation for this study came from recent, highly publicized mergers of Canadian pension plans. We expect that using data from different countries would lead to qualitatively similar results.

⁹We consider a second case (i.e., merger involving three small-sized Ontario universities of about 2,000 members each) in a robustness test presented in the supplementary material. This case is inspired by the experience of UPP.

could drastically change our conclusion. Specifically, we investigate the impact of modifying one plan's demographic profile and retirement age, among others. We also explore differences in initial funded ratios, and we modify the relative sizes of the plans. We find that consolidation can still be beneficial in most cases, even when the plans merging are heterogeneous. This assessment in the context of heterogeneous plans is the first contribution of our study.

Our second contribution is the modeling framework itself, which we use to quantify the risks and rewards of pension mergers. The framework is realistic and builds on key stylized facts of pension plan operation. It is also flexible and could be used to answer other questions, unrelated to mergers.

The remainder of this article is organized as follows. Section 2 describes the ESG, the mortality model, and the cost model with economies of scale. The pension plan operation is explained in Section 3 along with the dynamic asset allocation procedure. Section 4 presents the various solvency and welfare measures used in the study; we introduce measures relevant from different perspectives – the plan, the members, and the employer. Section 5 studies a merger in which a small and large plan merge. Some modeling assumptions relating to asset allocation and costs are assessed in Section 6. Section 7 concludes.

2. Economic scenario generator, mortality modeling, and administrative and investment costs

The evolution of a given pension plan and its solvency depend on future economic and mortality conditions, among others. It is therefore paramount to model the underlying changes in these two dimensions to understand the impact of potential mergers. This section briefly introduces some of the models needed to simulate future economic and mortality scenarios. We also explain the rationale behind the administrative and investment costs borne by the plan and introduce models and assumptions to account for those.

2.1 Economic and financial framework

Over the last 40 years, various frameworks have been proposed to model economic and financial variables relevant to actuaries. These frameworks – called economic scenario generators or ESGs – are comprehensive models that allow actuaries and risk managers to grasp the long-term uncertainty underlying financial market values and economic variables. The main end-users of these frameworks are pension, life insurance, and banking practitioners, which use them for various purposes: financial planning, asset and liability management, investment strategy, and regulatory compliance, among others (see, e.g., Pedersen *et al.*, 2016, for more details).

Wilkie proposed the first cascade model in a pioneering 1986 paper. His framework, which relies on the Box–Jenkins approach, is based on four connected models: an inflation model, a long interest rate model, a dividend yield model, and a stock index return model. In a follow-up article, Wilkie (1995) generalized his model by allowing for an earnings index, short-term interest rates, and property prices.

Modern technological and methodological advances have paved the way for more sophisticated frameworks. Specifically, in 2005, Ahlgrim, D'Arcy, and Gorratt proposed a model similar to Wilkie's but allowed for regime-switching dynamics for stock index returns to capture bull and bear markets. More recently, Bégin (2021) introduced a new cascade-type ESG based on the monetary policy, which is modeled via observable regime-switching dynamics. The model also considers the changing nature of the variance via generalized autoregressive conditional heteroskedasticity (GARCH) models. In the present article, we rely on a modified version of Bégin's (2021) model. Specifically:

- We use a homoskedastic autoregressive model for price inflation, wage inflation, and the dividend yield with (monetary) regime-dependent long-run levels, as supported by the conclusions of Bégin (2021, 2023).
- We rely on sophisticated models for the short rate, the stock index and alternative asset returns, as well as the investment grade corporate bond yields. Specifically, we use GARCH models to capture heteroskedasticity and a regime-switching component – based on the monetary policy – to capture changes in the level, as in Bégin (2021).
- The risk-free term structure is constructed based on observable factors: the level (proxied by the short rate), the slope, and the curvature of the term structure.¹⁰ The slope and the curvature are modeled by a two-dimensional autoregressive model.
- Investment grade bond yields are used to obtain bond portfolio returns. The bond returns are proxied using a portfolio of ten zero-coupon bonds with maturities increasing between one and ten years.¹¹

We use monthly Canadian economic and financial data to estimate the ESG parameters. Our sample period extends from 1998 to 2020. The parameters are obtained by Markov chain Monte Carlo methods similar to those used in Bégin (2021). Section SM.A of the Supplementary Material gives additional details about the ESG, the data used, and some of the estimation results.

The ESG yields monthly economic and financial variables that are aggregated to annual time intervals, matching the time step used in the pension plan operation. Section SM.A of the Supplementary Material also explains how the monthly quantities are used to obtain annual price and wage inflation, short and long rates, and the returns of various representative asset classes such as long-term risk-free government bonds (RF), investment grade bonds (IG), stock index (S), and alternative assets proxied by private equity (P).¹²

2.2 Mortality and longevity modeling

Merging pension plans changes the characteristics of the plan's membership, and this may have unexpected consequences. Generally speaking, adding more members to a plan should have a positive effect in terms of diversification when future lifetimes are independent and identically distributed. The diversification effect is less clear when populations with heterogeneous mortality and longevity are combined (Barrieu *et al.*, 2012). For example, if pension mergers occur between groups of members from different industries and geographic regions, new members' life expectancies may not match those of members already in the plan, and this may affect the overall benefit of the merger. Moreover, the future evolution of mortality improvements might be different for diverse groups of individuals. To study this effect, we use a stochastic mortality model for the Canadian population that accounts for mortality improvements in subpopulations that are not necessarily identically and independently distributed, producing heterogeneous membership in the merged plans.

A common approach when dealing with multipopulation modeling is to consider different age, period, and cohort effects for each subpopulation (e.g., Li & Lee, 2005; Cairns *et al.*, 2011; Dowd *et al.*, 2011; Villegas & Haberman, 2014, among others). This strategy works relatively well when the number of subpopulations considered is small. When the number of subpopulations under study is large, however, this is impractical. To reduce the number of parameters – age, period, and cohort effects – we aim our attention at subpopulation features instead of focusing on each subpopulation, and we stratify some of our parameters according to these features as done in

¹⁰These factors are consistent with the literature on interest rate term structure modeling: for instance, Litterman & Scheinkman (1991) showed that 99% of the yield curve's total variation can be explained by three fundamental shifts: a level component, a slope component, and a curvature component.

¹¹This proxy leads to reasonable returns for this asset class.

¹²The stock index returns take both capital gains and dividends into account to obtain the total stock index returns.

Bégin *et al.* (2023b). In the present study, we rely on three different features: sex, region, and socioeconomic status.

1. We first denote the set of sexes by $\mathcal{I}_1 \equiv \{\text{Male, Female}\}$.
2. Second, we divide the ten Canadian provinces into five main regions that share similar features: Atlantic provinces (Newfoundland and Labrador, New Brunswick, Nova Scotia, Prince Edward Island), Quebec, Ontario, Prairies (Alberta, Manitoba, Saskatchewan), and British Columbia; the latter regions are denoted by $\mathcal{I}_2$.¹³
3. Third, socioeconomic groups are constructed based on individuals' pension amounts at retirement under second-pillar earnings-related programs (i.e., Canada Pension Plan and Quebec Pension Plan), which is related to their salary at retirement. The set of socioeconomic groups is denoted by $\mathcal{I}_3 \equiv \{1, 2, \dots, 11, \text{Others}\}$, where low numbers refer to low socioeconomic statuses and high numbers to high statuses.¹⁴

Based on the insights of Wen *et al.* (2020) and Bégin *et al.* (2023b), we consider two period factors and no cohorts effects. Similar to Cairns *et al.* (2019), we begin our modeling by using a multipopulation gravity model of the two-factor CBD-X type (Hunt & Blake, 2020; Dowd *et al.* 2020):

$$\begin{aligned}
 m^{(i_1, i_2, i_3)}(x, t) &= \frac{D^{(i_1, i_2, i_3)}(x, t)}{E^{(i_1, i_2, i_3)}(x, t)} \\
 &= \exp\left(\beta_0^{(i_1)}(x) + \kappa_1^{(i_1, i_2, i_3)}(t) + \kappa_2^{(i_1, i_2, i_3)}(t) (x - \bar{x})\right) \\
 &= \exp\left(\beta_0^{(i_1)}(x) + \left(\kappa_1^{(i_1)}(t) + \kappa_1^{(i_2)}(t) + \kappa_1^{(i_3)}(t)\right) + \left(\kappa_2^{(i_1)}(t) + \kappa_2^{(i_2)}(t) + \kappa_2^{(i_3)}(t)\right) (x - \bar{x})\right), \quad (1)
 \end{aligned}$$

where $m^{(i_1, i_2, i_3)}(x, t)$ is the time- t crude death rate for age x , sex $i_1 \in \mathcal{I}_1$, region $i_2 \in \mathcal{I}_2$, and socioeconomic group $i_3 \in \mathcal{I}_3$. Moreover, $\bar{x}$ denotes the average age. We assume that the baseline age effect is sex-specific but does not depend on other features. This choice allows our model to stay parsimonious by reducing the number of age effects. The first period effects $\kappa_1^{(i_1)}(t)$, $\kappa_1^{(i_2)}(t)$, and $\kappa_1^{(i_3)}(t)$ pick up the time changes in the mortality level, and the second period effects $\kappa_2^{(i_1)}(t)$, $\kappa_2^{(i_2)}(t)$, and $\kappa_2^{(i_3)}(t)$ capture changes in the slope of the log-mortality curve. Together, these two period effects capture the future evolution of death rates and potential for future longevity trends.

Following Dowd *et al.* (2011) and Cairns *et al.* (2019), we model the first stratum period effects via a common long-term trend with a short-term gravity effect. The second and third dimension's period effects are modeled through dynamics similar to those of the first stratum. More details on the dynamics of the period effects are available in Section SM.B of the Supplementary Material.

This model is estimated using a Kalman filter-based Markov chain Monte Carlo method. It uses Canadian data that span from 1980 to 2015 and ages from 65 to 109 years.¹⁵ Details about the datasets are also included in Section SM.B of the Supplementary Material.

From this mortality model, we generate mortality scenarios that are used to obtain death probabilities, annuity prices, and death counts. As is commonly done in actuarial science, we denote the time- t one-year death probability of an individual age x by $q_{x,t}$, which is obtained by calculating $1 - e^{-m(x,t)}$ based on the most current information in the model. Throughout the paper,

¹³We do not consider the three northern territories as they represent less than 0.4% of the country's total population.

¹⁴We use these groups as proxies for socioeconomic status; similar groups were recently used by Wen *et al.* (2020) and Bégin *et al.* (2023b), and more details about the dataset can be found in their studies.

¹⁵We assume that our life table starts at the age of 65 years, meaning that (active) members cannot die before the age of 65 years.

we remove the superscripts i_1 , i_2 , and i_3 denoting the individual features for convenience; the reader should remember that these death probabilities are member-specific and depend on their sex, region, and socioeconomic status.

The time- t annuity-due and deferred annuity prices for individuals age x are obtained by taking an expectation over possible mortality scenarios (and conditional to current conditions). Notice that future death probabilities and annuity prices are random and path-dependent as period effects change through time.¹⁶

2.3 Model for economies of scale and the cost of pension funds

Pension plans can be expensive to run. Typically, costs are separated into two different categories: administrative and investment costs. The former cost generally depends on the number of members, the service provided by the plan, the plan complexity, and other controls, whereas the latter cost is tied to the amount of assets under management.

In the present study, we rely on the administrative cost model estimated by Bikker *et al.* (2012). We focus on the parameters estimated based on Canadian pension plans for consistency. The administrative cost model is given by:

$$\log(\text{Administrative Cost}_t) = \alpha + \beta \log(\text{Members}_t) + \gamma \text{Service}_t + \theta \text{Complexity}_t + \sum_k \zeta_k \text{Controls}_{k,t} + \sum_{s=1}^t q_s, \quad (2)$$

where Members_t is the number of plan members, Service_t is a standardized service quality score, and Complexity_t is a standardized complexity score.¹⁷ The controls include the share of retired members, the share of deferred members, and multiple dummy variables related to the number of plans being offered, the pension fund type, and the type of employees in the plan. The authors also control for the effects of pension fund types.¹⁸ To Bikker *et al.*'s (2012) model, we add price inflation to account for the increase in the prices of goods and services, where q_t is the time- t annual inflation rate. Note that the parameters in Table 8 of Bikker *et al.* (2012) are estimated based on costs in euros expressed in the 2005 price level; we therefore adjust these costs for inflation and convert them to Canadian dollars.¹⁹ We report the adjusted parameters in Table 1.

Fig. 1 illustrates the per member annual administrative cost implied by the parameters of Table 1; we assume that all scores and control dummies are zero and that the share of retired members is set to 40% for simplicity's sake in this figure. This cost decreases as the number of members increases, capturing economies of scale in the pension plan's administrative fees.

The investment cost assumption is simpler: we set the cost to 0.5% of assets under management, consistent with the 2010–2020 average cost across the 38 member countries of the Organization for Economic Cooperation and Development (see OECD, 2022, for more details):

$$\text{Investment Cost}_t = 0.005 \times A_t, \quad (3)$$

where A_t is the value of the plan's assets at time t . In the recent literature, there is limited evidence for economies of scale in investment costs (see, e.g., Bikker & Meringa, 2022).

¹⁶See Bégin *et al.* (2023a) for more details on annuity pricing in the context of age-period-cohort models.

¹⁷Refer to Bikker *et al.* (2012) for more details on the service quality and complexity scores.

¹⁸Some pension funds offer occupational plans for the public sector (national, provincial, or municipal governments), teachers, other school employees, police officers, firefighters, and other employees of public safety agencies, workers covered by a corporate or an industry-wide collective agreement, and workers in other occupational categories.

¹⁹The exchange rate used in this study is set to the 2005 average rate as given by the Canada Revenue Agency; that is, 1.5090.

Table 1. Administrative cost of Canadian pension fund

	Parameter	Standard error
Intercept, α	6.098	(1.969)
Number of members, β	0.945	(0.147)
Service quality score, γ	0.063	(0.018)
Complexity score, θ	0.038	(0.017)
Single pension plan offered	−0.129	(0.084)
Share of retired members	−0.003	(0.005)
Share of deferred members	−0.014	(0.003)
National government	−0.006	(0.096)
State or provincial government	0.728	(0.193)
Municipality	0.093	(0.048)
Teacher	−0.119	(0.073)
Other school employees	0.026	(0.049)
Police and other public safety workers	0.052	(0.038)
Other collective agreement groups	−0.212	(0.076)
Corporate pension fund	−0.092	(0.073)
Industry pension fund	0.011	(0.048)

Note: This table reports the parameters estimated by Bikker *et al.* (2012). The parameters were adjusted for inflation and converted to Canadian dollars. The exchange rate is set to the 2005 average rate as given by the Canada Revenue Agency, that is, 1.5090. The total increase in the inflation index between 2005 and 2021 was 28.4%. The parameter standard errors are reported in parentheses.

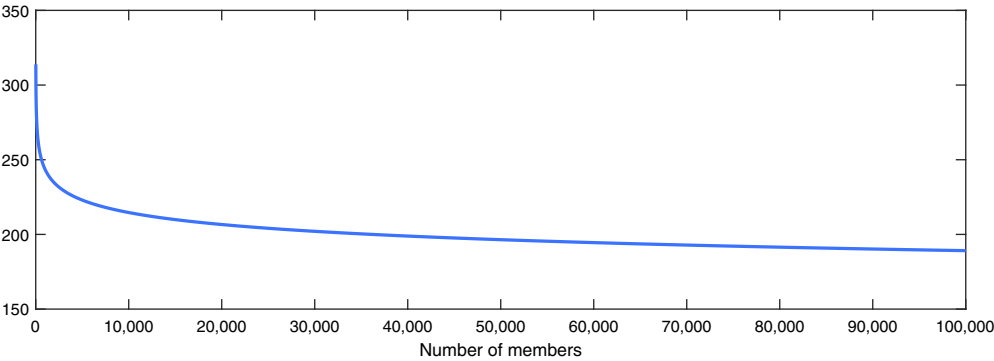


Figure 1. Per member annual administrative cost of Canadian pension fund.

Note: This figure reports the per member administrative cost as a function of the plan membership using the parameters of Table 1. We assume that all scores and control dummies are set to zero and that the share of retired members is set to 40%.

3. Assets, liabilities, and pension fund operation

Based on the ESG, the mortality model, and the administrative and investment costs described in Section 2, we can generate the (random) evolution of the pension fund’s assets and liabilities. These two values – along with current membership information – are useful to understand the plan’s funding status and the normal costs of future benefits. The asset returns depend on the asset allocation strategy, which is also explained in this section. The last part of Section 3 discusses how the plan operation is modeled.

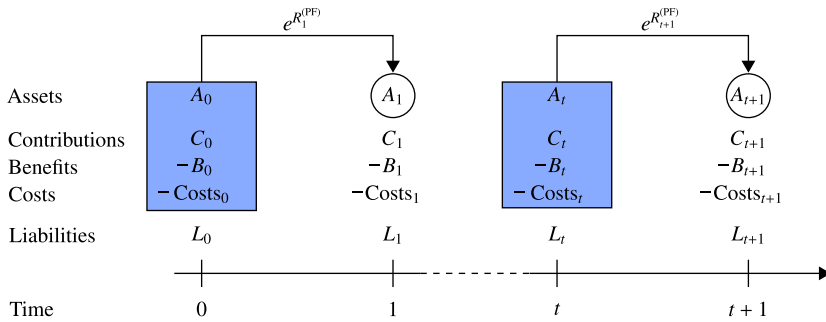


Figure 2. Assets, contributions, benefits, costs, and liabilities timing.

Note: This figure shows the timing of the assets, contributions, benefits, costs, and liabilities.

3.1 Assets, liabilities, and normal cost calculation

We start by assuming that we know the time- t total assets and show how to obtain the contributions, benefits, and normal costs at time t as well as the total assets and total liabilities at time $t + 1$. We assume that the contributions, benefits, administrative cost, and investment cost are paid at the beginning of the year for simplicity's sake. Fig. 2 shows the timing of the various cash flows of interest.

Let the time- t (continuously compounded) valuation rate used to compute the liabilities be denoted by v_t (see Section 3.2 for more details on this rate). Let us further assume that $\mathcal{L}_t$ represents the set of members (actives and retirees) at time t ; that is, $k \in \mathcal{L}_t$ if and only if the k^{th} member is alive at time t . The age of the k^{th} life is denoted by $x_k + t$ at time t . We divide the membership into two subsets: $\mathcal{A}_t$ and $\mathcal{R}_t$ are subsets of $\mathcal{L}_t$ representing the active members and retired members at time t , respectively

As commonly done for DB plans, we use a formula to determine the benefits. Specifically, throughout this study, we assume that the k^{th} member's annual benefit at retirement is calculated as 1.5% of their final salary, multiplied by the number of years of service.²⁰

Their estimated pension based on past service is given by:

$$B_{k,t} = \begin{cases} 0.015 \times \text{Projected Final Salary}_{k,t} \times \text{Years of Service}_{k,t} & \text{if } x_k + t < R \\ 0.015 \times \text{Final Salary}_k \times \text{Years of Service}_{k,t} & \text{if } x_k + t \geq R \end{cases}, \quad (4)$$

where R is the retirement age and Final Salary_k is the salary of member k in their final year of employment. Salary is projected until retirement based on the (continuously compounded) unconditional wage inflation average $\bar{w}$ and the (continuously compounded) merit-based salary increases m set to 0.5% in this study:

$$\text{Projected Final Salary}_{k,t} = \text{Salary}_{k,t} \times e^{m(R-x_k-t)} \times e^{\bar{w}(R-x_k-t)},$$

where $\text{Salary}_{k,t}$ is the time- t salary of member k .

The time- t liability associated with this specific member is thus given by:

$$L_{k,t} = \begin{cases} B_{k,t} \times {}_{R-x_k-t}|\ddot{a}_{x_k+t,t} & \text{if } x_k + t < R \\ B_{k,t} \times \ddot{a}_{x_k+t,t} & \text{if } x_k + t \geq R \end{cases},$$

²⁰A final salary plan in a multiemployer context may pose moral hazard; for example, an employer could sharply increase salaries shortly before retirement to maximize members' benefits while keeping contributions low. Although there is some evidence of such behavior in US multiemployer plans (see, e.g., Fitzpatrick, 2017), this has not typically been an issue in JSPPs. In any case, the problem can easily be addressed from a design perspective, by shifting to a benefit formula based on career average earnings instead of final salary.

where ${}_n|\ddot{a}_{x_k+t,t}$ is a time- t n -year deferred annuity using rate v_t for discounting and $\ddot{a}_{x_k+t,t}$ is a time- t annuity-due using rate v_t for discounting.²¹ Again, notice that the superscripts i_1 , i_2 , and i_3 are removed from the annuity factors for convenience's sake.

The total current liability is then the sum of each member's liability; that is,

$$L_t = \sum_{k \in \mathcal{L}_t} L_{k,t}. \quad (5)$$

Each member's normal cost is based on their (one-year-ahead) projected liability at time $t + 1$, which is a function of the information available at time t . Specifically, the projected benefit is given by:

$$\tilde{B}_{k,t+1} = 0.015 \times \text{Projected Final Salary}_{k,t} \times (\text{Years of Service}_{k,t} + 1).$$

The k^{th} member's (one-year-ahead) projected liability is²²

$$\tilde{L}_{k,t+1} = \begin{cases} \tilde{B}_{k,t+1} \times {}_{R-x_k-t-1}|\ddot{a}_{x_k+t+1,t} & \text{if } x_k + t < R \\ \tilde{B}_{k,t+1} \times \ddot{a}_{x_k+t+1,t} & \text{if } x_k + t \geq R \end{cases}.$$

The normal cost is thus given by the difference between the (discounted) projected liability at time $t + 1$ and that at time t :

$$\text{Normal Cost}_{k,t} = \begin{cases} \tilde{L}_{k,t+1} e^{-v_t} p_{x_k+t,t} - L_{k,t} & \text{if } k \in \mathcal{A}_t \\ 0 & \text{if } k \in \mathcal{R}_t \end{cases},$$

where $p_{x_k+t,t} = 1 - q_{x_k+t,t}$ is the (random) one-year survival probability.²³ The total contribution at time t is given by the sum of the normal costs across the active members, the various administrative and investment costs, and special payments that smooth the surpluses and deficits arising over time:

$$C_t = \sum_{k \in \mathcal{A}_t} \text{Normal Cost}_{k,t} + \text{Administrative Cost}_t + \text{Investment Cost}_t + \kappa(L_t - A_t), \quad (6)$$

where κ is the smoothing factor (set to 20% here) used to determine the special payments.²⁴

The total benefit paid for year t is obtained by summing up all benefits paid to each retired member, that is,

$$B_t = \sum_{k \in \mathcal{R}_t} B_{k,t}, \quad (7)$$

where $B_{k,t}$ is given in Equation (4). From the time- t total contributions and benefits from Equations (6) and (7), respectively, we can obtain the value of the assets at time $t + 1$ recursively:

²¹We assume that current members stay in the plan and survive until retirement; that is, there are no deferred members in the plan and no deaths before age R .

²²This valuation relies on the projected unit credit valuation method that views each year of service as giving rise to an additional year of benefit entitlement and measures each year separately to build up the final obligation.

²³Note that the probabilities of survival and annuity prices are random because they depend on the realized mortality scenarios. To value the liabilities, we use the scenario-dependent future expected values of the survival probabilities and annuity prices (see Bégin *et al.*, 2023a, for details on the calculation of these quantities).

²⁴The selected smoothing factor of 20% can be loosely interpreted as having five years to pay back any deficits or spend down any surpluses in the fund.

$$A_{t+1} = \left(A_t - B_t + C_t - \overbrace{\text{Administrative Cost}_t - \text{Investment Cost}_t}^{\text{Costs}_t} \right) \times \underbrace{\left(\omega_t^{(\text{RF})} e^{R_{t+1}^{(\text{RF})}} + \omega_t^{(\text{IG})} e^{R_{t+1}^{(\text{IG})}} + \omega_t^{(\text{S})} e^{R_{t+1}^{(\text{S})}} + \omega_t^{(\text{P})} e^{R_{t+1}^{(\text{P})}} \right)}_{e^{R_{t+1}^{(\text{PF})}}}, \quad (8)$$

where $\omega_t^{(a)}$ is the portfolio allocation to asset a decided based on information at time t , $R_{t+1}^{(a)}$ is the return on asset a at time $t + 1$, and $a \in \{\text{RF}, \text{IG}, \text{S}, \text{P}\}$. Moreover, $R_{t+1}^{(\text{PF})}$ is the (continuously compounded) total asset return at time $t + 1$.

Finally, from both the total assets and liabilities at time $t + 1$, we can obtain the funded ratio at time $t + 1$ as:

$$F_{t+1} = \frac{A_{t+1}}{L_{t+1}}. \quad (9)$$

3.2 Valuation rate

The valuation rate v_t , introduced at the beginning of Section 3.1, is determined in a building block fashion reminiscent of the methodology proposed by the Canadian Institute of Actuaries in their 2015 educational note (see CIA, 2015, for more details). In this study, we rely on the current long rate and unconditional expected future investment returns for the various asset classes and then combine these estimates to obtain a valuation rate that is aligned with the plan's investment policy and the current economic environment.

We start from the current long rate l_t as our baseline and add to it the weighted average of long-term risk premiums – defined as the unconditional expected future investment returns minus the current long rate – based on the asset mix selected at time $t - 1$. Specifically, the time- t valuation rate is given by:

$$v_t = l_t + \sum_{a \in \{\text{RF}, \text{IG}, \text{S}, \text{P}\}} \omega_{t-1}^{(a)} \left(\bar{R}^{(a)} - l_t \right), \quad (10)$$

where the unconditional expected return on asset a is denoted by $\bar{R}^{(a)}$. This simplifies to $v_t = \sum_{a \in \{\text{RF}, \text{IG}, \text{S}, \text{P}\}} \omega_{t-1}^{(a)} \bar{R}^{(a)}$ in our context – the valuation rate is the long-run investment return based on the asset mix at time $t - 1$.

3.3 Asset allocation

The asset dynamics of Equation (8) depend on the asset allocation

$$\omega_t = \begin{bmatrix} \omega_t^{(\text{RF})} & \omega_t^{(\text{IG})} & \omega_t^{(\text{S})} & \omega_t^{(\text{P})} \end{bmatrix}^T,$$

which has to be selected by the plan. To mimic the process used by the plan to decide on an optimal asset allocation strategy, we use a utility-based approach, as presented in Warren (2019). This is not the first use of utility functions for forming portfolios in the literature (see, e.g. Adler & Kritzman, 2007; Blake *et al.*, 2013; Levy, 2016; Estrada & Kritzman, 2018).

In our context, utility functions are convenient as they provide a mechanism to evaluate all potential outcomes by giving each of them a score; these scores can then be averaged to obtain an expected utility, which is maximized to obtain the optimal allocation. This approach is comprehensive as it captures all information about the range of potential outcomes (Adler & Kritzman,

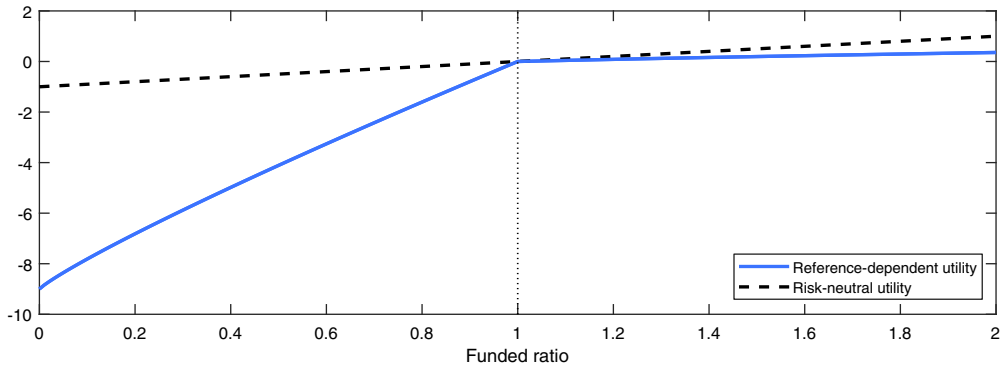


Figure 3. Reference-dependent utility as a function of the funded ratio.

Note: This figure shows the reference-dependent utility function used in this study. The parameters are inspired from Blake *et al.* (2013), and this methodology was recently used by Warren (2019).

2007). A related advantage is that we do not have to use a specific distribution, as the method is flexible enough to accommodate any type of distribution.²⁵

Choosing the *right* utility function is difficult and subjective; yet, some have been used in the pension context thus far, and we build on this past research. Specifically, in this study, we use a reference-dependent utility function to evaluate the outcomes – the future funded ratio F – relative to some reference level – a fully funded plan with a funded ratio of 1:

$$u_A(F) = \begin{cases} w_L(F^{c_L} - 1) & \text{if } F < 1 \\ w_G(F^{c_G} - 1) & \text{if } F \geq 1 \end{cases}, \quad (11)$$

where c_L and c_G are the curvature parameters on losses and gains, respectively, and w_L and w_G are the weighting parameters on the losses and gains, respectively.²⁶ In this article, we use parameters similar to those proposed in Warren (2019): $c_L = 0.88$, $c_G = 0.44$, $w_L = 9$, and $w_G = 1$. The horizon of the future funded ratio h is set to three years as in Warren (2019).²⁷

Fig. 3 reports the shape of the reference-dependent utility. This function is kinked around the reference funded ratio of 1. It reflects the loss aversion of the plan, that is, the penalty applied to deficits is much greater than the reward applied to surpluses, all other things being equal. The function also exhibits curvature, especially as the funded ratio approaches 0.

We find the optimal allocation by maximizing the expected reference-dependent utility:

$$\operatorname{argmax}_{\omega_t} \mathbb{E}_t[u_A(F_{t+h})], \quad (12)$$

where $\mathbb{E}_t$ represents the expected value operator based on the information available at time t . This optimization problem is solved numerically using the Nelder–Mead method (`fminsearch` in Matlab) and by simulating a sample of future funded ratios based on possible realizations of the ESG and the mortality model, thus leading to nested simulations. Specifically, for a given allocation, the expected utility is obtained by first generating values of the future funded ratio at time $t + h$ based on current economic and mortality conditions at time t (see Section 3.1 for more

²⁵The main alternative to using utility functions is based on metrics that summarize the distribution of outcomes (e.g., Sharpe ratio in the classical mean–variance analysis, probability of shortfall, and value-at-risk).

²⁶The subscript A in the utility function refers to asset allocation. This should not be confused with the members' utility function, which will use subscript M .

²⁷As mentioned in Warren (2019), this reference-dependent utility exhibits relatively high aversion to deficit risk and assigns relatively modest value to surpluses, which aligns with the short horizon h of three years selected in this paper. A longer horizon would require a new parametrization of the utility function.

details).²⁸ Then, we evaluate the utility score for each scenario, and we average these values across the scenarios to obtain the expected utility of Equation (12). The optimization routine changes the allocation until it finds the one that maximizes the expected reference-dependent utility given above.

To be consistent with the three-year horizon used in the optimization, we update the optimal asset allocation every three years in our setup (i.e., at time 0, 3, 6, and so forth).

3.4 Pension fund dynamics and plan operation

We start our plan operation dynamics by assuming a given membership structure. Based on it and an exogenous estimate of the initial funded ratio F_0 , we get the total liabilities at time 0, L_0 , and the total assets at time 0, A_0 . Our economic and mortality projections start in 2021, meaning that we assume that 2020 is time 0, and we project the variables of interest for $\tau = 50$ years. Then, for each economic and mortality scenario and for each year, we follow the steps of Algorithm 1 to project the plan operation.

4. Plan solvency, member welfare, and employer risk

This section introduces five measures: two measures for plan solvency, one for member welfare, and two for employer risk. These measures will be used to compare different plans – pre- and post-merger – from different perspectives.

4.1 Funding adequacy via economic capital

The notion of economic capital has been extensively used for the purpose of risk quantification in the financial and insurance sectors. More recently, a few studies considered its application in the pension context (see, e.g., Porteous *et al.* 2012; Yang & Tapadar, 2015; Tapadar *et al.*, 2019; Andrews *et al.*, 2021). Similar to these studies, we adopt the following definition of economic capital.²⁹

Definition 1. *The economic capital of a pension plan is the proportion by which its existing assets would need to be augmented to meet net benefit obligations with a prescribed degree of confidence. A pension scheme's net benefit obligations are all obligations in respect of plan members, including future service and net of future contributions to the plan. (Andrews et al., 2021).*

This definition is very general, and some subjective choices must be made to operationalize the measure. First, we select a very long horizon of 50 years (same as τ above) to ensure that short-term fluctuations do not impact our end results. Second, we focus on a going-concern valuation, which assumes that the pension plan continues into the future indefinitely. This strategy is different than that used by Porteous *et al.* (2012), Yang & Tapadar (2015), Tapadar *et al.* (2019), and Andrews *et al.* (2021); the latter authors use a solvency basis and assume that no new members join the plan.

Based on the quantities of Section 3, we define the time- t net cash outflow of the pension plan (excluding investment returns):

$$X_t = B_t - C_t + \text{Administrative Cost}_t + \text{Investment Cost}_t,$$

²⁸As the valuation rate of Equation (10) relies on the time- t weights, we optimize the optimal asset mix based on the valuation of the previous time step, that is, v_{t-1} .

²⁹Note that we do not suggest that one use economic capital in the context of pension funding. Rather, we only use it as a means to compare different plans before and after a merger.

Algorithm 1. Plan operation dynamics

```

1: for each economic and mortality scenario do
2:   for  $t \in \{0, \dots, \tau - 1\}$  do
3:     compute the time- $t$  administrative and investment costs using Equations (2) and (3), respectively
4:     compute the time- $t$  total contributions and benefits based on Equations (6) and (7), respectively
5:     obtain the optimal asset allocation by solving Equation (12), if needed
6:     update the membership based on some plan-specific assumptions explained in Section 5
       compute death probabilities  $q_{x_k+t,t}$  based on current mortality scenario for each member  $k$ 
       generate deaths using Bernoulli distributions30
       age members who survive
       update salary based on merit scale and wage inflation
       change status of members aged  $R$  to retired
       add new members to the plan
7:   compute the total liabilities at time  $t + 1$  based on Equation (5)
8:   compute the total assets at time  $t + 1$  based on Equation (8)
9:   compute the funded ratio at time  $t + 1$  based on Equation (9)
10: end for
11: end for

```

which accounts for the benefits, the contributions, and the various costs. The time- t profit is then defined as:

$$P_t = L_{t-1} \exp\left(R_t^{(\text{PF})}\right) - X_t - L_t, \quad \text{for } t \in \{1, 2, 3, \dots, \tau\},$$

and the initial profit is given by $P_0 = A_0 - X_0 - L_0$.³¹ The present value of the future profits over the time horizon from 0 to τ is thus given by:

$$\begin{aligned} V_0^{(\tau)} &= \sum_{t=0}^{\tau} P_t \exp\left(-\sum_{s=1}^t R_s^{(\text{PF})}\right) \\ &= A_0 - \sum_{t=0}^{\tau} X_t \exp\left(-\sum_{s=1}^t R_s^{(\text{PF})}\right) - L_{\tau} \exp\left(-\sum_{s=1}^{\tau} R_s^{(\text{PF})}\right). \end{aligned} \quad (13)$$

One of our main aims is to compare pre- and post-merger versions of $V_0^{(\tau)}$; yet, these values would depend on the initial asset values of the respective plans. To remove any impacts related to the initial asset values, we express $V_0^{(\tau)}$ as a percentage of A_0 and denote it by $\check{V}_0^{(\tau)}$. This standardization is helpful as it allows for consistent and comparable values across different plans.

The actual economic capital risk measures can be found in terms of the distribution of $\check{V}_0^{(\tau)}$. As commonly done in the actuarial literature, we rely on the value-at-risk (VaR) and the expected shortfall (ES). The former measure is defined as:

$$\mathbb{P}_0\left(\check{V}_0^{(\tau)} \leq \text{VaR}_p^{\text{Plan}}\right) = p, \quad (14)$$

for a given probability p , and it represents the percentage of initial assets required at time 0 for the plan to meet all future obligations with probability $1 - p$. The latter measure is defined as:

$$\text{ES}_p^{\text{Plan}} = \mathbb{E}_0\left[\check{V}_0^{(\tau)} \mid \check{V}_0^{(\tau)} \leq \text{VaR}_p^{\text{Plan}}\right] \quad (15)$$

³⁰ A member aged $x_k + t$ at time t dies with probability $q_{x_k+t,t}$ and survives with probability $p_{x_k+t,t} = 1 - q_{x_k+t,t}$, which can be modeled by a Bernoulli distribution.

³¹ We use the returns on the assets backing the liabilities to accumulate or discount our various profit cash flows.

for a given probability level p , and it represents the average of all losses (in percentage) that are greater than or equal to the value of $\text{VaR}_p^{\text{Plan}}$ (or the gains that are lesser than or equal to $\text{VaR}_p^{\text{Plan}}$).

4.2 Member's welfare and consumption utility

Comparing pre- and post-merger consumption streams is relevant for determining whether merging is beneficial from the members' perspective. In economics and other related fields, it is common to compare utility (or preferences) instead of dollar amounts as this represents better how members assess their welfare. The utility framework is thus used to mimic this assessment process and compares different options – pre- and post-merger satisfaction.

Plan members' utility functions are assumed to be isoelastic (power utility) in this study, such that

$$u_M(\chi) = \frac{\chi^{1-\eta}}{1-\eta},$$

where χ is the consumption and η is a constant that is positive for risk-averse agents (set to 5 in this study).

From the quantities defined in Section 3, we can determine the actual consumption stream under each economic and mortality scenario. Specifically, a retired member is receiving their benefits as per Equation (4); an active member, on the other hand, is receiving their salary, minus their share of the contributions, which is computed as half of the contribution rate times the member's salary.³² The contribution rate, in turn, is calculated by taking the total contribution C_t and dividing it by the total payroll. In summary, the time- t consumption for the k^{th} member is determined by the following equation:

$$\chi_{k,t} = \begin{cases} \text{Salary}_{k,t} - \frac{1}{2} \left(\frac{C_t}{\sum_{l \in \mathcal{A}_t} \text{Salary}_{l,t}} \right) \times \text{Salary}_{k,t} & \text{if } x_k + t < R \\ B_{k,t} & \text{if } x_k + t \geq R \end{cases}.$$

Because members receive streams of consumption – and not just lump sums – we need to aggregate their consumption over their future lifetimes. In the present study, we use the expected discounted utility model to combine the consumption payments:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} e^{-\delta t} u_M \left(\underbrace{\chi_{k,t} e^{-\sum_{s=1}^t q_s}}_{\text{Real consumption}} \right) {}_t p_{x_k} \right], \quad (16)$$

where δ is the (continuously compounded) rate associated with the member's subjective discount factor (set to 2% in this study).³³ We consider consumption in real terms to make it comparable from one year to the next; this can be easily done in our framework by using each scenario's realization of price inflation. We also consider mortality by multiplying the utility function by the t -year survival probability for member k , ${}_t p_{x_k}$; note that this probability is also random and depends on the realized mortality scenario.

The expected discounted utility is by itself not informative unless it is transformed into a meaningful quantity. For this reason, many authors rely on the certainty equivalent consumption (CEC)

³²We assume that half of the contribution is paid by the employer and the other half by the members, as is customary in a JSPP.

³³Expected utility theory is used in this study to provide a general idea of the members' preferences based on a representative member. We recognize, however, that this theory can only provide general conclusions that apply to many but not all members, as their actual utility function might differ from that of the representative member.

to compare consumption streams. In our case, the k^{th} member's CEC is defined as the solution to the following equation:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} e^{-\delta t} u_M(\text{CEC}_k) {}_t p_{x_k} \right] = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} e^{-\delta t} u_M(\chi_{k,t} e^{-\sum_{s=1}^t q_s}) {}_t p_{x_k} \right]. \quad (17)$$

The CEC can be interpreted as the guaranteed consumption in real terms that a member receives in exchange for their random consumption stream once risk and preferences are accounted for.

4.3 Employer risk and contributions

The employer is an important stakeholder in the context of pension plan mergers. Comparing pre- and post-merger employer contributions (and their risk) allows us to quantify the financial benefits derived from the merger.

To assess contribution risk from the employer's perspective, we first aggregate the employer's contributions by taking their present value over time horizon τ :

$$\zeta_0^{(\tau)} = - \sum_{t=0}^{\tau-1} \frac{C_t}{2} \exp \left(- \sum_{s=1}^t R_s^{(\text{PF})} \right),$$

where, again, we assume that half of the contributions are paid by the members and the other half by the employer; note the negative sign reflects that contributions are losses from the employer's perspective. Then, because comparing between pre- and post-merger plans is difficult as the total dollar contribution depends on the size of the respective plans, we express the present value of the contributions as a percentage of initial liabilities, L_0 . This standardization allows us to compare the various plans under study; we denote the standardized present value of the contributions by $\check{\zeta}_0^{(\tau)}$.

Finally, to understand the risk, we again rely on VaR and ES. In the context of employer's contributions, the first measure is defined as:

$$\mathbb{P}_0 \left(\check{\zeta}_0^{(\tau)} \leq \text{VaR}_p^{\text{Employer}} \right) = p, \quad (18)$$

for a given probability p . This measure loosely represents the percentage of initial liabilities required at time 0 to pay for all future employer contributions with probability $1 - p$. The second measure is defined as:

$$\text{ES}_p^{\text{Employer}} = \mathbb{E}_0 \left[\check{\zeta}_0^{(\tau)} \mid \check{\zeta}_0^{(\tau)} \leq \text{VaR}_p^{\text{Employer}} \right] \quad (19)$$

for a given probability level p . Similar to $\text{ES}_p^{\text{Plan}}$, it represents the average of all contributions (in percentage terms) that are greater than or equal to the value of $-\text{VaR}_p^{\text{Employer}}$.

5. Merger of a small plan and a large plan

Inspired by CAAT's example, we apply our framework and the measures described above to a case study of a merger between a small Ontario corporate plan of about 1,000 members and a large Ontario plan of 50,000 members. As mentioned earlier, we set the horizon to $\tau = 50$ years. All calculations rely on 25,000 simulated scenarios based on the ESG and the mortality model.³⁴

³⁴The scenario generation requires nested simulations; that is, outer loops to generate the future economic and mortality conditions and inner loops to find the optimal asset allocation as explained in Section 3.3. We rely on the heterogeneous high-performance computer cluster Cedar, part of the Digital Research Alliance of Canada, to run our calculations. Each scenario is run in parallel to reduce computational times; the total running time for each case is between 17 and 31 hours, on average.

Table 2. Summary of pension plan features and assumptions

	Small plan	Large plan	Merged plan
Plan features			
Initial funded ratio	1	1	–
Service score	0	2	2
Complexity score	0	2	2
Other school employees	0	0	0
Corporate pension fund	1	0	0
All other dummies	0	0	0
Investment-related assumptions			
Investment universe	RF, IG, S	RF, IG, S, P	RF, IG, S, P
Rounding	Yes	No	No
Membership assumptions			
Starting age	25	25	–
Retirement age	65	65	–
Starting salary at time 0	\$40,000	\$40,000	–
Initial membership assumptions			
Numbers	1,000	50,000	–
Plan maturity	Stationarity	Stationarity	–
Proportion of females	50%	50%	–
Province	Ontario	Ontario	–

Note: This table summarizes the various plan features, investment-related, membership, and initial membership assumptions. We study a merger between a small Ontario corporate plan of about 1,000 members and a large Ontario plan of 50,000 members.

In this case study, we assume that the pre-merger initial funded ratio is 1 for both the small and the large plans. The service and complexity scores are 0 for the small plan; they are assumed to be 2 for both the large and merged plans (see Bikker *et al.*, 2012, for more details on these scores). The small, large, and merged plans can all invest in long-term risk-free government bonds, investment grade bonds, and the stock index. The investment universe also includes an alternative asset (i.e., private equity) for the large plan and the merged plan, but the small plan is not able to invest in this asset class.³⁵ We also assume that the small plan's asset allocation is suboptimal; this suboptimality is proxied by allocations that are rounded to the nearest multiple of 5% of its asset mix.³⁶ The plan membership is assumed to be stationary and comprises about half females and half males.^{37,38} We further assume that new members join the plan at 25 years old, earn a starting salary of \$40,000 per year (at time 0) that increases over time with wage inflation, and retire at 65 years old; the number of new members is constant every year (and is chosen in accordance with the total number of members and the stationarity assumption). The different attributes of this case study are summarized in Table 2.

³⁵We assume that large plans can trade some alternative assets (i.e., private equity), whereas small plans might not have access to the same investment opportunities. This assumption is consistent with the results of de Dreu & Bikker (2012) and Gorter & Bikker (2013).

³⁶This assumption is consistent with de Dreu & Bikker's (2012) results on investor sophistication: "most pension funds round strategic asset allocations to the nearest multiple of 5%, similar to age heaping in demographic and historical studies."

³⁷Stationary membership refers to a membership consistent with a plan operating for a very long time.

³⁸Survival and death probabilities are by-products of a member's age, sex, region, and socioeconomic status (proxied by their salary).

5.1 Comparison between the small, large, and merged plans

We begin this comparison exercise with a caveat about the size of the differences: one should not expect massive deviations between the small and merged plans. A merger has the potential to improve solvency and welfare, but the size of these improvements will most likely be modest in the grand scheme of things. Note, however, that even the smallest improvements in percentage terms can be worthwhile as it involves substantial gains in dollar terms.

Before turning to the various solvency and welfare measures of Section 4, we first look at different (long-run) outputs that are relevant to understanding the plans under study. Specifically, we compare the merged plan's behavior to that of the unmerged small and large plans. Unmerged here assumes that the small and large plans continue to operate without merging; these are denoted by "Small plan" and "Large plan" below.

Table 3 reports statistics about the long-run behavior of the small (Panel A), large (Panel B), and merged plans (Panel C); specifically, we report statistics on the distribution of the portfolio weights, the total asset return, the funded ratio, the valuation rate, and the contribution rate, all at the end of the horizon of $\tau = 50$ years.

The asset allocation of the small plan is generally different than that of the merged plan. In addition to rounding allocations to the nearest 5%, the small plan cannot invest in the alternative asset. This implies very different weights, on average. Even though both the small and the merged plans invest about 18% of their assets in long-term government bonds, the allocations are different for the other, riskier assets. On average, the small plan invests about 41% of its assets in investment grade bonds and 41% in the stock index. For the merged plan, it is 35% in investment grade bonds, 31% in the stock index, and 16% in the alternative asset. Access to additional asset classes allows the merged plan to obtain more benefits from diversification, which ultimately leads to an average asset return that is 30 bps higher than that of the small plan. The asset allocation of the large plan, on the other hand, is virtually identical to that of the merged plan – having 1,000 additional members does not impact the plan's allocation strategy. This leads to also having the same average asset return of 6.8% in the merged plan as in the large plan.

The different asset allocation strategy also impacts the valuation rate used to calculate the liabilities. In the case of the small plan, the average valuation rate is about 6.4%; it is 6.8% for the large and merged plans. This difference of 40 bps makes the per-capita liabilities of the merged plan smaller, which explains the higher average funded ratio at the end of the horizon to some extent. Indeed, the average funded ratio increases from 1.066 in the small plan to 1.081 in the merged plan. The contribution rate also drops, from 12.7% to 10.2%, on average. However, these changes in the funded ratio and the contribution rate are only partially explained by the change in the valuation rate. For instance, plan size differences create gaps between the various plans' administrative and investment costs and could affect the long-run dynamics of the plans' funded status and contribution rate. Indeed, administrative and investment costs are 1.3% lower in the merged plan than in the small plan, on average. Moreover, having more (similar) members reduces the risk associated with idiosyncratic mortality, which ultimately increases the funded ratio and decreases the contribution rate.

To assess the merger from the plan's, the members', and the employer's perspective, we use the measures explained in Section 4. The main goal of this comparison is to assess the improvement (or deterioration) associated with the merger for the plan via economic capital risk measures, for members via CECs, and for employers via contribution risk measures.

Fig. 4 shows histograms of the standardized present value of the future profits for the small (top panel), large (middle panel), and merged plans (bottom panel). These distributions are the primary building blocks in obtaining the economic capital risk measures (i.e., from the plan's perspective). Note that the left tail – associated with negative profits – is fatter in the case of the small plans. The tenth percentile of the distribution (equivalent to $\text{VaR}_{0.10}^{\text{Plan}}$) is improved by about

Table 3. Long-run statistics of asset allocation as well as asset- and liability-related quantities

Panel A: Small plan							
	Moments		Percentiles				
	Average	Std. dev.	10 th	25 th	50 th	75 th	90 th
Asset allocation							
Long-term government bond portfolio	0.174	0.164	0.000	0.050	0.100	0.250	0.450
Investment grade bond portfolio	0.414	0.194	0.100	0.250	0.450	0.600	0.650
Stock index	0.412	0.123	0.250	0.350	0.400	0.500	0.550
Alternative asset	—	—	—	—	—	—	—
Asset-related quantity							
Total asset return	0.065	0.081	−0.031	0.019	0.067	0.114	0.159
Liability-related quantities							
Funded ratio	1.066	0.239	0.808	0.909	1.031	1.183	1.366
Valuation rate	0.064	0.006	0.057	0.060	0.064	0.067	0.072
Contribution rate	0.127	0.280	−0.227	−0.032	0.154	0.317	0.452
Panel B: Large plan							
	Moments		Percentiles				
	Average	Std. dev.	10 th	25 th	50 th	75 th	90 th
Asset allocation							
Long-term government bond portfolio	0.180	0.122	0.062	0.087	0.144	0.254	0.359
Investment grade bond portfolio	0.348	0.162	0.125	0.223	0.350	0.463	0.560
Stock index	0.308	0.098	0.193	0.243	0.299	0.365	0.436
Alternative asset	0.165	0.058	0.094	0.125	0.161	0.200	0.242
Asset-related quantity							
Total asset return	0.068	0.087	−0.037	0.017	0.069	0.122	0.173
Liability-related quantities							
Funded ratio	1.082	0.270	0.792	0.902	1.040	1.210	1.421
Valuation rate	0.068	0.007	0.059	0.063	0.067	0.072	0.077
Contribution rate	0.102	0.297	−0.274	−0.061	0.132	0.302	0.444
Panel C: Merged plan							
	Moments		Percentiles				
	Average	Std. dev.	10 th	25 th	50 th	75 th	90 th
Asset allocation							
Long-term government bond portfolio	0.180	0.122	0.062	0.087	0.145	0.254	0.358
Investment grade bond portfolio	0.348	0.162	0.125	0.223	0.350	0.463	0.560
Stock index	0.308	0.098	0.193	0.243	0.299	0.364	0.435
Alternative asset	0.165	0.057	0.094	0.125	0.161	0.200	0.242
Asset-related quantity							
Total asset return	0.068	0.087	−0.037	0.017	0.069	0.122	0.173

Table 3. Continued.

Panel C: Merged plan							
	Moments		Percentiles				
	Average	Std. dev.	10 th	25 th	50 th	75 th	90 th
Liability-related quantities							
Funded ratio	1.081	0.270	0.792	0.902	1.039	1.209	1.420
Valuation rate	0.068	0.007	0.059	0.063	0.067	0.072	0.077
Contribution rate	0.102	0.297	−0.273	−0.061	0.133	0.303	0.443

Note: This table reports the long-run average and standard deviation (std. dev. in the table) as well as the 10th, 25th, 50th, 75th, and 90th percentiles of the weights invested in each asset class, the total asset return, the funded ratio, the valuation rate, and the contribution rate. Long-run quantities are taken at a horizon of 50 years, at which time the random quantities have converged to their long-run distributions. This is done for the small plan (Panel A), the large plan (Panel B), and the merged plan (Panel C). Note that the allocations sum to one in all scenarios.

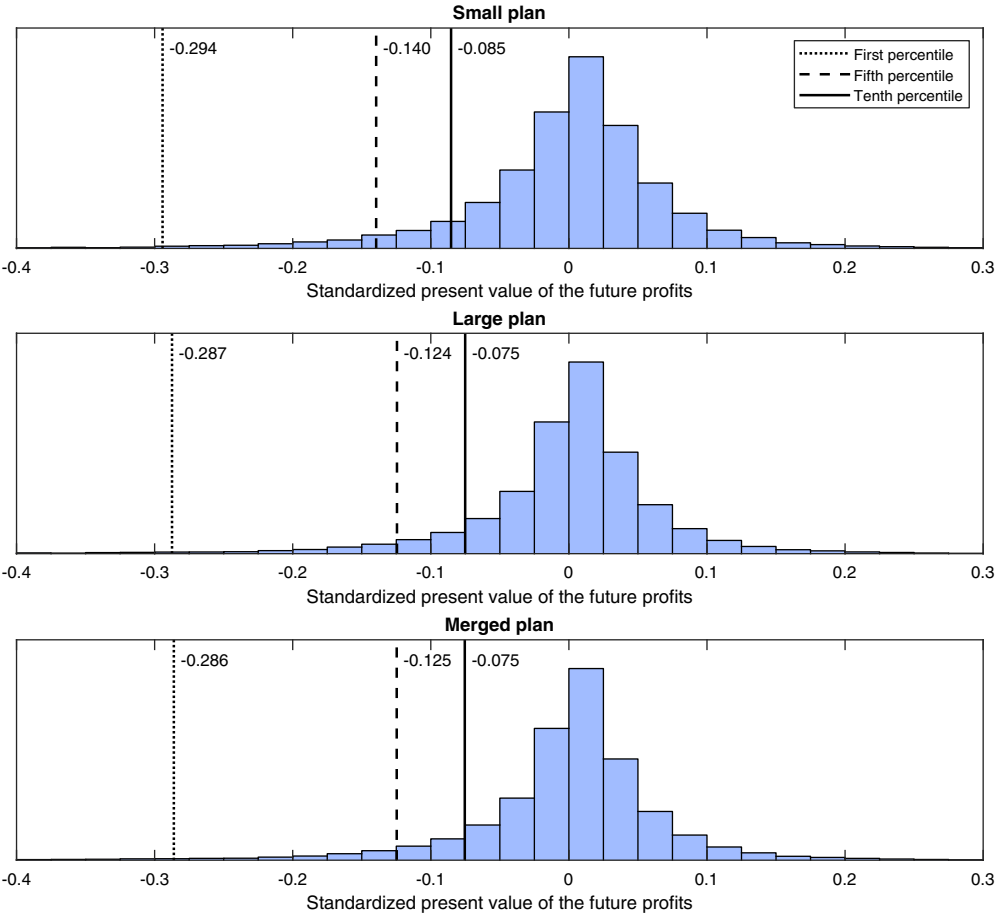


Figure 4. Distribution of standardized present value of the future profits.
Note: This figure reports the distribution of the standardized present value of the future profits as given by Equation (13) for the small plans in the top panel and for the merged plan in the bottom panel. The first, fifth, and tenth quantiles of both distributions are reported with dotted, dashed, and solid lines, respectively.

Table 4. Economic capital risk measures and improvements for the small plan

	$\text{VaR}_p^{\text{Plan}}$			$\text{ES}_p^{\text{Plan}}$		
	0.01	0.05	0.10	0.01	0.05	0.10
Base	−0.294 (3%)	−0.140 (11%)	−0.085 (12%)	−0.449 (5%)	−0.244 (7%)	−0.177 (8%)
Initial funded ratio at 0.8	−0.378 (24%)	−0.180 (30%)	−0.111 (32%)	−0.573 (25%)	−0.314 (27%)	−0.228 (29%)
Initial funded ratio at 1.2	−0.236 (−21%)	−0.112 (−11%)	−0.067 (−11%)	−0.369 (−15%)	−0.198 (−14%)	−0.142 (−13%)
Less mature plan	−0.311 (9%)	−0.147 (15%)	−0.088 (15%)	−0.468 (9%)	−0.257 (11%)	−0.185 (12%)
No retirees at inception	−0.432 (33%)	−0.203 (37%)	−0.120 (37%)	−0.663 (35%)	−0.360 (37%)	−0.258 (37%)
British Columbia	−0.293 (2%)	−0.138 (10%)	−0.084 (11%)	−0.438 (3%)	−0.241 (6%)	−0.175 (7%)
Nova Scotia	−0.311 (7%)	−0.148 (15%)	−0.090 (16%)	−0.470 (9%)	−0.257 (11%)	−0.186 (13%)
Higher starting salary	−0.287 (1%)	−0.135 (7%)	−0.081 (7%)	−0.431 (1%)	−0.235 (3%)	−0.170 (5%)
Lower starting salary	−0.299 (4%)	−0.144 (14%)	−0.088 (14%)	−0.455 (6%)	−0.249 (9%)	−0.181 (10%)
More female	−0.287 (1%)	−0.138 (10%)	−0.084 (10%)	−0.438 (3%)	−0.239 (5%)	−0.173 (6%)
More male	−0.301 (5%)	−0.141 (12%)	−0.085 (11%)	−0.460 (7%)	−0.248 (8%)	−0.178 (9%)
Retirement at 70 years old	−0.307 (7%)	−0.138 (10%)	−0.085 (12%)	−0.507 (15%)	−0.258 (12%)	−0.183 (11%)
Larger plan	−0.286 (0%)	−0.125 (0%)	−0.076 (0%)	−0.436 (2%)	−0.230 (1%)	−0.163 (1%)
Smaller plan	−0.296 (3%)	−0.142 (12%)	−0.088 (14%)	−0.443 (4%)	−0.245 (7%)	−0.178 (9%)
No rounding	−0.295 (3%)	−0.138 (10%)	−0.085 (11%)	−0.443 (4%)	−0.242 (6%)	−0.175 (8%)

Note: This table reports economic capital risk measures for the small plan and the post-merger improvements (in percentage) in parentheses. The value-at-risk and the expected shortfall are computed using Equations (14) and (15), respectively, for probability levels of 1%, 5%, and 10%. In addition to the base case, the risk measures are also computed for all relevant robustness cases: a small plan initial funded ratio of 0.8 and 1.2, a less mature small plan for which members and retirees are younger than 71 years old, a small plan with no retirees at inception (only active members), a small plan for which members are coming from British Columbia (i.e., lower mortality rates), a small plan for which members are coming from Nova Scotia (i.e., higher mortality rates), a small plan with a higher starting salary of \$80,000, a small plan with a lower starting salary of \$20,000, a small plan with 90% of members that are female, a small plan with 90% of members that are male, a small plan for which members retire at 70 years old, a larger small plan with 5,000 members, a smaller small plan with 500 members, and a small plan that does not need to round its asset allocation.

12% (jumping from −0.085 to −0.075). Similar improvements are observed for other probability levels p as well as for ES measures (see first rows denoted by “Base” in Table 4).

The large plan does not seem to be impacted much by the merger when the counterpart is small in comparison: most percentiles in Fig. 4 are the same for both the large and the merged

plan. Indeed, improvements in economic capital risk measures (and all the other metrics considered in this study as a matter of fact) are virtually nil for the large plan; see Section SM.C of the Supplementary Material for more details. We note that, although a single merger between a large and small plan may bring negligible benefits to the large plan, multiple successive mergers produce material benefits. This may justify some large plans' aggressive agenda for expansion through repeated mergers.³⁹ For the rest of this section, we focus on the impacts of mergers from the perspective of small plans.

The first rows of Table 5 report relevant CEC measures for male and female members aged 25, 35, 45, and 55 years for the base case.⁴⁰ Overall, the CECs are noticeably larger for the merged plan, meaning that merging is beneficial from the members' perspective; for instance, 25-year-old males (females) get an additional \$265 (\$252) per annum in terms of CEC if the plans merge. Differences in the contribution rates can explain the discrepancy between the small and merged plan CECs – lower contribution rates translate into better average consumption profiles.

Lower contribution rates also impact the employer's measures. The first rows of Table 6 document the decreases in the present value of the standardized employer's contributions. The median of the distribution (i.e., $\text{VaR}_{0.50}^{\text{Employer}}$) is 24% lower for the merged plan when compared to the small plan. We obtain similar decreases for other probability levels p and for the employer's ES measures.

The lower contributions, which affect both members' and the employer's welfare, are explained by various factors in our setting. As noted earlier, the small plan's suboptimal investment strategy due to rounding and its lack of access to alternative investments makes its asset returns lower, on average, which also decreases the valuation rate. The reduced asset returns then need to be compensated by higher contributions from the members and the employer. Second, investment and administrative costs are higher for small plans, on average; this additional cost requires extra contributions from the employer and the members. Finally, having more members in the merged plan creates less uncertainty in the plan's benefits due to better mortality pooling; the standard deviation of annual benefit payments is 1% lower for the merged plan than that observed in the small plans, on average.

All in all, merging plans is beneficial for both small plan's members and employers, which is ultimately reflected in the plan's perspective via lower economic capital measures in Table 4.

5.2 Robustness tests

The results above depend on the assumptions selected at the beginning of this section. To verify the robustness of our conclusions, we consider some modifications that create heterogeneity in the merged plan:

- We first consider cases where the small plan is underfunded (with funded ratios of 0.8) or overfunded (with funded ratios of 1.2) before the merger.
- We assess the impact of the small plan having younger members and populations that have not reached stationarity at time 0. We look at a case where the small plan has no retirees at time 0 (i.e., “No retirees at inception” throughout the article) and another case where the small plan has retirees but no members older than 71 years of age (i.e., “Less mature plan” hereafter).⁴¹

³⁹To test this assumption, we consider a merger of a large plan of 25,000 members similar to that of Table 2 with 25 small plans of 1,000 members each. The results, shown in Table SM.6 of the Supplementary Material, report an overall benefit for both large and small plans, thus supporting multiple successive mergers with smaller plans.

⁴⁰For retired members aged 65 years and above, calculating CECs is futile as there is no difference between small and merged plans' benefit payments by virtue of all plans sharing the same benefit formula.

⁴¹We assume that the less mature plan has liabilities at time 0 that are halfway between the liabilities of the base case and those of the “No retirees” case. The age of 71 years has been selected as it generates such liabilities.

Table 5. Certainty equivalent consumptions and improvements for the small plan

	Male				Female			
	25	35	45	55	25	35	45	55
Base	34,755 (265)	32,150 (139)	29,045 (65)	25,682 (27)	34,512 (252)	31,788 (129)	28,584 (58)	25,179 (23)
Initial funded ratio at 0.8	34,036 (977)	31,753 (532)	28,849 (259)	25,591 (116)	33,826 (931)	31,421 (492)	28,409 (231)	25,102 (99)
Initial funded ratio at 1.2	35,133 (-106)	32,350 (-57)	29,141 (-29)	25,726 (-17)	34,871 (-100)	31,972 (-52)	28,669 (-26)	25,217 (-14)
Less mature plan	34,820 (198)	32,196 (94)	29,077 (33)	25,702 (6)	34,574 (188)	31,830 (86)	28,612 (29)	25,197 (5)
No retirees at inception	34,945 (80)	32,265 (27)	29,112 (9)	25,719 (1)	34,693 (76)	31,893 (25)	28,643 (8)	25,211 (1)
British Columbia	34,745 (256)	32,130 (134)	29,016 (62)	25,648 (26)	34,505 (244)	31,773 (124)	28,562 (56)	25,155 (22)
Nova Scotia	35,100 (166)	32,524 (89)	29,442 (43)	26,078 (19)	34,836 (158)	32,128 (81)	28,935 (38)	25,521 (16)
Higher starting salary	69,333 (545)	64,065 (283)	57,792 (131)	51,055 (52)	68,920 (522)	63,435 (263)	56,991 (117)	50,184 (45)
Lower starting salary	17,366 (151)	16,088 (79)	14,549 (37)	12,873 (15)	17,241 (143)	15,896 (73)	14,304 (33)	12,605 (12)
More female	34,738 (281)	32,142 (147)	29,043 (67)	25,684 (25)	34,496 (267)	31,780 (135)	28,582 (59)	25,181 (21)
More male	34,908 (115)	32,231 (60)	29,082 (29)	25,697 (12)	34,658 (109)	31,862 (56)	28,617 (25)	25,192 (10)
Retirement at 70 years old	39,590 (-108)	37,730 (-68)	34,905 (-38)	31,437 (-21)	39,445 (-105)	37,452 (-65)	34,480 (-35)	30,906 (-19)
Larger plan	34,936 (87)	32,245 (47)	29,088 (23)	25,699 (10)	34,685 (83)	31,875 (43)	28,622 (20)	25,194 (8)
Smaller plan	34,646 (374)	32,094 (196)	29,020 (90)	25,676 (33)	34,409 (356)	31,736 (181)	28,562 (80)	25,174 (28)
No rounding	34,762 (258)	32,154 (136)	29,046 (64)	25,682 (26)	34,519 (245)	31,791 (125)	28,585 (56)	25,180 (22)

Note: This table reports certainty equivalent consumptions (CECs) for the small plan and their post-merger improvements (in dollar) in parentheses. The CECs are computed using Equation (17) for males and females aged 25, 35, 45, and 55 years old. In addition to the base case, the CECs are also computed for all the robustness cases mentioned in the caption of Table 4.

- We change the demographic profile of the plan, thus impacting the mortality and longevity of our pool; that is, the province of residence (British Columbia or Nova Scotia instead of Ontario), the starting salary at time 0 as a proxy for the socioeconomic group (\$80,000 in the case of “High starting salary” and \$20,000 in the case of “Low starting salary”), and the sex (90% of members being female in “More female” and 90% of members being male in “More male”).
- We investigate the impact of members retiring later in life – at 70 years old.

Table 6. Employer contribution risk measures and improvements for the small plan

	VaR _p ^{Employer}			ES _p ^{Employer}		
	0.05	0.10	0.50	0.05	0.10	0.50
Base	−3.113 (11%)	−2.596 (13%)	−1.316 (24%)	−3.832 (9%)	−3.329 (10%)	−2.115 (15%)
Initial funded ratio at 0.8	−3.341 (17%)	−2.858 (21%)	−1.607 (37%)	−4.049 (14%)	−3.564 (16%)	−2.389 (25%)
Initial funded ratio at 1.2	−2.918 (5%)	−2.356 (4%)	−1.001 (1%)	−3.725 (6%)	−3.167 (6%)	−1.855 (4%)
Less mature plan	−4.174 (33%)	−3.427 (34%)	−1.636 (38%)	−5.193 (32%)	−4.474 (33%)	−2.756 (35%)
No retirees at inception	−6.054 (54%)	−4.921 (54%)	−2.197 (54%)	−7.762 (54%)	−6.586 (54%)	−3.893 (54%)
British Columbia	−3.101 (10%)	−2.579 (12%)	−1.300 (23%)	−3.811 (8%)	−3.307 (10%)	−2.097 (15%)
Nova Scotia	−3.212 (13%)	−2.673 (15%)	−1.342 (25%)	−3.966 (12%)	−3.440 (13%)	−2.175 (18%)
Higher starting salary	−3.011 (8%)	−2.509 (10%)	−1.257 (20%)	−3.718 (6%)	−3.225 (7%)	−2.039 (12%)
Lower starting salary	−3.261 (15%)	−2.727 (17%)	−1.402 (29%)	−4.005 (13%)	−3.484 (14%)	−2.231 (20%)
More female	−3.071 (10%)	−2.554 (11%)	−1.269 (21%)	−3.806 (8%)	−3.295 (9%)	−2.073 (14%)
More male	−3.061 (9%)	−2.540 (11%)	−1.237 (19%)	−3.824 (9%)	−3.299 (9%)	−2.051 (13%)
Retirement at 70 years old	−2.498 (−11%)	−1.980 (−14%)	−0.772 (−28%)	−3.324 (−5%)	−2.766 (−8%)	−1.537 (−16%)
Larger plan	−2.923 (5%)	−2.404 (6%)	−1.128 (11%)	−3.683 (5%)	−3.158 (5%)	−1.927 (7%)
Smaller plan	−3.263 (15%)	−2.737 (17%)	−1.428 (30%)	−3.979 (12%)	−3.475 (14%)	−2.245 (20%)
No rounding	−3.107 (11%)	−2.593 (13%)	−1.309 (23%)	−3.822 (9%)	−3.321 (10%)	−2.110 (15%)

Note: This table reports employer contribution risk measures for the small plan and their post-merger improvements in percentage (in parentheses). The value-at-risk and the expected shortfall are computed using Equations (18) and (19), respectively, for probability levels of 5%, 10%, and 50%. In addition to the base case, the employer contribution risk measures are also computed for all the robustness cases mentioned in the caption of Table 4.

- We modify the size of the small plan from 1,000 members to 500 (i.e., “Smaller plan” hereafter) and to 5,000 members (i.e., “Larger plan” henceforth). In the former, the small plan represents about 1% of the large plan’s membership; in the latter, it is about 10%.
- We consider a small plan that does not round its asset allocation to verify whether this assumption has a material impact on our results.

Table 4 reports the economic capital risk measures – taken from the plan’s perspective – for the base case of Section 5.1 and each robustness test. Tables 5 and 6 do the same from the members’ and the employer’s perspectives by reporting CECs and risk measures based on the present value of standardized contributions, respectively. Overall, the conclusion that the merger of a small and

a large plan is beneficial remains robust to the aforementioned changes; the rest of this section is devoted to explaining these results.

5.2.1 Over and underfunded plans

Having an over or underfunded small plan can substantially impact the merger. On the one hand, merging a fully funded large plan with an underfunded small one is beneficial for the small plan as it improves its funded ratio, leading to lower contributions. This translates into significantly greater improvements in the economic capital risk measures, the CECs, and the employer contribution risk measures from the small plan's perspective. Of course, the merger has the opposite effect on the large plan, with economic capital risk measures, CECs, and employer contribution risk measures getting worse in this case. However, due to the size difference between the two plans, this negative effect is considerably diluted so that the overall impact on the large plan becomes negligible after the merger. For example, the CEC of a 25-year-old member of the large plan decreases by only \$10 per year due to the merger. See Section SM.D of the Supplementary Material for more details.

On the other hand, when an overfunded small plan is combined with a fully funded large plan, the advantages of merging (in terms of economies of scale) are outweighed by the loss of contribution holidays in the early years. This has a direct negative impact on members in the small plan, making the merger unattractive in terms of CEC, and especially so for younger members. The effect is more muted from the employer's perspective, which weighs contribution levels and contribution risk differently than members do. As a result, when the small plan's funded ratio is 1.2, overall, there is still a small advantage to be drawn by the employer from merging. From the plan's perspective (in terms of economic capital), the merger is no longer beneficial.

We note that the merger can still be beneficial (or close to neutral) for all parties, even when the small plan is slightly overfunded relative to the large plan before the merger. Under our assumptions, the threshold is around an initial funded ratio of 105% for the small plan.

5.2.2 Younger small plans

A younger small plan's economic capital risk measures improve after the merger, and this improvement is more pronounced than in the base case. This is a consequence of the pattern of contributions under the younger (unmerged) small plan when compared to the contributions under the merged plan.

The younger small plan invests heavily in the stock index over the first ten years or so – a by-product of the longer duration of the liabilities – after which the allocation to risky assets is reduced to reach a level similar to that of the large and merged plans. For instance, the small (merged) plan invests 30% (50%) of its assets in government and investment grade bonds, whereas 70% (50%) of its assets are in riskier investments like the stock index and the alternative asset (when available). A larger initial allocation to risky assets means a higher valuation rate, resulting in lower contributions in the early years under the younger small plan than under the base case. However, these contributions are also more volatile.

Over time, as the membership gets older, the asset allocation tilts toward safer assets like bonds, triggering a corresponding decline in the valuation rate. Each drop in the valuation rate increases the liabilities and the normal costs, giving rise to increasing contributions in the medium term. In the very long run, the small plan reaches maturity and settles on a similar asset allocation and a similar distribution of contributions regardless of its maturity at inception.

From the plan's perspective, what matters is the left tail of the profit distribution. Accordingly, the higher volatility of the early contributions leads to worse economic capital measures for a younger small plan than in the base case. As a result, the positive stabilizing effect of the merger is amplified, so the benefit of the merger is greater for a younger small plan than it is in the base case.

From the employer's perspective, similar dynamics are at play: the present value of the standardized contributions tends to be worse (i.e., higher) in a younger small plan than in the base case. Thus, the corresponding improvements in Table 6 on account of the merger are also more pronounced than those observed in the base case.

The story is slightly different from the members' perspective: although the merger is still beneficial for a younger small plan (the changes in CECs in the corresponding rows of Table 5 are positive across the board), the improvements are much less significant than in the base case. In fact, if the small plan is very immature (i.e., has no retirees at the time of the merger), the benefit of the merger in terms of CECs for members over the age of 40 years is negligible.

The difference in the members' perspective, on the one hand, and the plan's and the employer's perspective, on the other hand, is primarily due to the different emphasis that the various measures place on different parts of the distribution of contributions. The VaR and ES-based measures of the plan and of the employer focus on extreme outcomes, so the negative impact of higher contribution volatility in the early years of the younger small plan outweighs the positive impact of the lower contribution levels, making a younger (unmerged) small plan look worse than the base case. This amplifies the apparent benefit of the merger for a younger small plan. By contrast, the CECs calculated from the members' perspective focus on expected values, so the positive impacts of the lower contributions make the younger (unmerged) small plan look better than the base case. This makes the merger significantly less attractive, although still beneficial.

5.2.3 *Impact of demographic profile*

Generally speaking, it seems beneficial to merge plans, even if there is heterogeneity in the membership in terms of demographic profile. All improvement measures are positive for all mortality-related robustness cases (see cases from "British Columbia" to "More male" in Tables 4, 5, and 6).

From the plan's perspective, merging a small plan with lower mortality rates (e.g., with members from British Columbia, with a higher starting salary, or plans with a majority of female members) leads to less advantages than those obtained with a plan with higher mortality (e.g., members from Nova Scotia, with a smaller starting salary, or plans with a majority of male members).

From the members' and the employer's perspectives, we obtain a similar conclusion: merging is more beneficial for groups with worse mortality than groups with better mortality. This is explained by the difference in asset allocation: a lower life expectancy, on average, means that the plan needs to invest more heavily in safe assets such as bonds, reducing the valuation rate and, in turn, increasing the normal costs. This ultimately affects the contributions: plans with worse mortality have higher contribution rates than those with better mortality, on average.

5.2.4 *Older retirees in the small plan*

Merging a small plan in which members retire later in life (i.e., at age 70 years) into a large plan in which members retire at age 65 years is disadvantageous for the small plan members and their employer – the CECs and standardized employer contribution improvements are negative. This is partly caused by the different asset allocation strategies between small and merged plans: the small plan invests more aggressively in risky assets (i.e., stock index and private equity) as its investment horizon is longer because of the higher retirement age. In the merged plan, the asset allocation tilts more toward bonds. This difference in asset allocation therefore impacts the contribution rate via the valuation rate: the small plan has a higher valuation rate because of its asset allocation, leading to smaller normal costs and, ultimately, contributions. The small plan's members (who retire at age 70 years and have a long-run contribution rate of 8.8%), in effect, subsidize the cost of pension

benefits for the large plan's members (who retire at age 65 years and have a long-run contribution rate of 10.2%).

From the plan's perspective, it still seems beneficial to merge; economic capital improvements are similar to those reported with the base case. These improvements are explained by lower (per member) costs, lower fund return and contribution volatility, as well as better mortality pooling leading to fewer extreme cases in the profit streams.

5.2.5 Size of the small plan

The size of the plan matters: a small plan that is closer in size to the large plan gets fewer benefits from merging. A small plan that is very small, on the other hand, gets more benefit from the merger, with all improvement factors being significantly higher than those of the base case.

5.2.6 More optimal small plan allocations

This robustness test considers a small plan that does not round its asset weights, leading to more optimal allocation. In this case, the benefits of merging are slightly less than those observed in the base case. The difference is, nonetheless, very small: the effect of rounding (or not rounding) clearly has only a second-order effect on the final results, and merging is still beneficial in this case.

5.3 Merger of three small-sized plans as a robustness test

As an additional robustness test of our results, we apply our framework and metrics to a second case study – a merger involving three small-sized Ontario universities of about 2,000 members each, inspired by UPP's experience. This case is distinct from the previous one as the plans merging are more similar in nature. The features of this second case study are summarized in Table SM.7 of the Supplementary Material. Detailed assumptions and results are presented in Section SM.D of the Supplementary Material. Overall, the merger of three small-sized plans is beneficial for all parties involved – the plan, the members, and the employers – under all circumstances considered.

6. Impacts of asset allocation and cost modeling assumptions

With respect to the merger between the small and large plans, the previous section investigated changes to the plan's population and funding status. In contrast, this section assesses the impact on our base case of changing some modeling assumptions relating to the small plan's asset allocation and costs:

1. First, we change the parameters of the reference-dependent utility function of Equation (11) to reduce the small plan's allocation to riskier asset classes.
2. We also modify the administrative and investment cost assumptions to impact the costs of running the small plan.

Tables 7, 8, and 9 report the impact of changing the asset allocation and cost modeling assumptions on economic capital risk measures, on certainty equivalent consumptions, and on employer contribution risk measures, respectively.

6.1 Asset allocation

Smaller plans tend to be more conservative than larger plans with their asset allocation. To capture this possibility, we consider different reference-dependent utility parameter values in Equation (11); specifically, we change the small plan's loss curvature parameter c_L and the loss weighting

Table 7. Impact of changing the asset allocation and cost modeling assumptions on economic capital risk measures and improvements for the small plan

	VaR _p ^{Plan}			ES _p ^{Plan}		
	0.01	0.05	0.10	0.01	0.05	0.10
High loss curvature parameter	−0.293 (2%)	−0.140 (11%)	−0.089 (15%)	−0.429 (0%)	−0.232 (2%)	−0.172 (6%)
High loss weighting parameter	−0.292 (2%)	−0.140 (11%)	−0.089 (15%)	−0.431 (1%)	−0.232 (2%)	−0.172 (6%)
High loss curvature and weighting parameters	−0.293 (2%)	−0.142 (12%)	−0.092 (18%)	−0.427 (0%)	−0.227 (0%)	−0.170 (5%)
Doubled administrative cost	−0.294 (3%)	−0.140 (11%)	−0.085 (12%)	−0.449 (5%)	−0.244 (7%)	−0.177 (8%)
Tripled administrative cost	−0.294 (3%)	−0.140 (11%)	−0.085 (12%)	−0.449 (5%)	−0.244 (7%)	−0.177 (8%)
Quadrupled administrative cost	−0.294 (3%)	−0.140 (11%)	−0.085 (12%)	−0.449 (5%)	−0.244 (7%)	−0.177 (8%)
Doubled investment cost	−0.294 (3%)	−0.140 (11%)	−0.085 (12%)	−0.449 (5%)	−0.244 (7%)	−0.177 (8%)
Tripled investment cost	−0.294 (3%)	−0.140 (11%)	−0.085 (12%)	−0.449 (5%)	−0.244 (7%)	−0.177 (8%)
Quadrupled investment cost	−0.294 (3%)	−0.140 (11%)	−0.085 (12%)	−0.449 (5%)	−0.244 (7%)	−0.177 (8%)

Note: This table reports the impact of changing the asset allocation and cost modeling assumptions on economic capital risk measures for the small plan and the post-merger improvements (in percentage) in parentheses. The risk measures are computed for the following cases: a high loss curvature parameter in the reference utility function (i.e., $c_L = 2.2$), a high loss weighting parameter in the reference utility function (i.e., $w_L = 18$), high loss curvature and weighting parameters in the reference utility function (i.e., $c_L = 2.2$ and $w_L = 18$), higher administrative costs (2, 3, and 4 times the cost of Equation (2)), and higher investment costs (2, 3, and 4 times the cost of Equation (3)). See the caption of Table 4 for additional details.

parameter w_L to assess the impact of a more risk-averse allocator. We consider three cases: (1) a high loss curvature parameter of $c_L = 2.2$, (2) a high loss weighting parameter of $w_L = 18$, and (3) a combination of the two (i.e., high curvature and weighting parameters).

Changing these parameters significantly impacts the long-run asset allocation of the small plan. For instance, the allocation to the stock index is between 32% and 36% in the three modifications considered in this section; it was about 41% in the base case. These lower allocations to the stock index are compensated by larger allocations to the long-term government bond portfolio (increasing from 17% in the base case to 19–21%) and the investment grade bond portfolio (increasing from 41% in the base case to 44–46%).

Having a small plan allocate more of its assets to conservative asset classes enhances the merger’s overall benefits, aligning favorably with the interests of the plan, its members, and the employer. These enhancements typically surpass the outcomes achieved in the base scenario outlined in Section 5. For the three cases considered and on average, the improvements in CECs attributable to the merger are nearly double when compared to the base case, whereas the employer contribution risk measures are elevated by 57% in comparison to the baseline case. Merging allows the small plan members and employer to reduce the average contribution level – a by-product of having higher asset returns, on average, in the merged plan – which improves their risk–reward profiles. From the plan’s perspective, we report a similar behavior to that of the base case of Section 5, meaning that it is still beneficial for the small plan to merge.

Table 8. Impact of changing the asset allocation and cost modeling assumptions on certainty equivalent consumptions and improvements for the small plan

	Male				Female			
	25	35	45	55	25	35	45	55
High loss curvature parameter	34,546 (475)	32,036 (253)	28,989 (121)	25,657 (52)	34,313 (451)	31,683 (234)	28,534 (107)	25,158 (44)
High loss weighting parameter	34,573 (447)	32,051 (238)	28,996 (114)	25,660 (49)	34,339 (425)	31,696 (220)	28,541 (101)	25,161 (41)
High loss curvature and weighting parameters	34,408 (613)	31,961 (329)	28,952 (158)	25,641 (68)	34,181 (583)	31,613 (303)	28,501 (140)	25,144 (58)
Doubled administrative cost	34,698 (322)	32,121 (169)	29,032 (78)	25,677 (31)	34,458 (306)	31,761 (156)	28,572 (69)	25,176 (26)
Tripled administrative cost	34,641 (379)	32,091 (199)	29,019 (92)	25,673 (36)	34,404 (360)	31,733 (183)	28,560 (81)	25,172 (30)
Quadrupled administrative cost	34,522 (498)	32,029 (261)	28,991 (119)	25,664 (45)	34,291 (473)	31,676 (241)	28,536 (106)	25,164 (38)
Doubled investment cost	34,593 (427)	32,067 (223)	29,009 (101)	25,670 (38)	34,358 (406)	31,711 (205)	28,552 (90)	25,169 (33)
Tripled investment cost	34,422 (598)	31,978 (312)	28,970 (140)	25,657 (51)	34,195 (569)	31,629 (288)	28,517 (125)	25,159 (44)
Quadrupled investment cost	34,242 (778)	31,882 (407)	28,928 (182)	25,644 (65)	34,023 (741)	31,540 (376)	28,480 (162)	25,147 (55)

Note: This table reports the impact of changing the asset allocation and cost modeling assumptions on certainty equivalent consumptions (CECs) for the small plan and their post-merger improvements (in dollar) in parentheses. The CECs are computed for the following cases: a high loss curvature parameter in the reference utility function (i.e., $c_L = 2.2$), a high loss weighting parameter in the reference utility function (i.e., $w_L = 18$), high loss curvature and weighting parameters in the reference utility function (i.e., $c_L = 2.2$ and $w_L = 18$), higher administrative costs (2, 3, and 4 times the cost of Equation (2)), and higher investment costs (2, 3, and 4 times the cost of Equation (3)). See the caption of Table 5 for additional details.

6.2 Administrative and investment costs

The small plan's higher administrative and investment costs could also impact the plan's stakeholders in nontrivial ways. Accordingly, we consider additional tests in which we raise these costs. Specifically, we investigate administrative costs that are doubled, tripled, and quadrupled, as well as investment costs that are doubled, tripled, and quadrupled with respect to the base case.

Overall, increasing the small plan's investment or administrative cost makes the merger significantly more beneficial from the members' and the employer's perspectives. For instance, doubling the administrative cost increases the impact of the merger on CECs by about 20% when compared to the base case, on average. Doubling the investment cost – from 0.5% to 1.0% of the small plan's assets – also increases the improvements in the CECs by about 55%, on average.

The employer risk measures exhibit noteworthy enhancements in comparison to the base case scenario outlined in Section 5. On average, the improvement factors surge by 17% when the administrative cost is doubled. Similarly, a substantial increase is observed when the investment cost is increased, surpassing the improvements obtained under the base case.

From the plan's perspective, the improvements are neither better nor worse when compared to the base case of Section 5. This is expected as the administrative and investment costs are absorbed by plan members and the employer. Indeed, the stream of profits do not change as a function of these expenses.

To conclude this section, considering more risk-averse small plans or higher costs tends to make the merger more beneficial for members and the employer.

Table 9. Impact of changing the asset allocation and cost modeling assumptions on employer contribution risk measures and improvements for the small plan

	$\text{VaR}_p^{\text{Employer}}$			$\text{ES}_p^{\text{Employer}}$		
	0.05	0.10	0.50	0.05	0.10	0.50
High loss curvature parameter	−3.318 (16%)	−2.830 (20%)	−1.564 (36%)	−3.977 (12%)	−3.514 (15%)	−2.354 (24%)
High loss weight parameter	−3.287 (15%)	−2.792 (19%)	−1.531 (35%)	−3.946 (11%)	−3.480 (14%)	−2.317 (23%)
High loss curvature and weight parameters	−3.485 (20%)	−2.994 (24%)	−1.704 (41%)	−4.110 (15%)	−3.661 (18%)	−2.506 (29%)
Doubled administrative cost	−3.188 (13%)	−2.664 (15%)	−1.365 (27%)	−3.917 (11%)	−3.407 (12%)	−2.176 (18%)
Tripled administrative cost	−3.264 (15%)	−2.730 (17%)	−1.413 (29%)	−4.001 (13%)	−3.485 (14%)	−2.236 (20%)
Quadrupled administrative cost	−3.412 (19%)	−2.865 (21%)	−1.510 (34%)	−4.170 (16%)	−3.641 (18%)	−2.357 (24%)
Doubled investment cost	−3.369 (18%)	−2.824 (20%)	−1.479 (32%)	−4.122 (15%)	−3.595 (17%)	−2.320 (23%)
Tripled investment cost	−3.626 (23%)	−3.052 (26%)	−1.642 (39%)	−4.411 (21%)	−3.860 (22%)	−2.524 (29%)
Quadrupled investment cost	−3.878 (28%)	−3.281 (31%)	−1.806 (44%)	−4.701 (26%)	−4.126 (27%)	−2.728 (34%)

Note: This table reports the impact of changing the asset allocation and cost modeling assumptions on employer contribution risk measures for the small plan and their post-merger improvements in percentage (in parentheses). The risk measures are computed for the following cases: a high loss curvature parameter in the reference utility function (i.e., $c_L = 2.2$), a high loss weighting parameter in the reference utility function (i.e., $w_L = 18$), high loss curvature and weighting parameters in the reference utility function (i.e., $c_L = 2.2$ and $w_L = 18$), higher administrative costs (2, 3, and 4 times the cost of Equation (2)), and higher investment costs (2, 3, and 4 times the cost of Equation (3)). See the caption of Table 6 for additional details.

7. Concluding remarks

Understanding the impact of a full pension merger on all parties is critical: in the absence of a thorough quantitative assessment, the resulting disadvantages may outweigh the advantages. In order to promote sound decision-making by all stakeholders and to reduce the threat of botched mergers, the assessment ought to consider a variety of factors, including different levels of administrative and investment costs, differences in asset allocation strategies, differences in the investment universe, and the efficiency of mortality pooling. They should also take into account potential heterogeneity in the financial and demographic profiles of the merging plans.

This study demonstrated how such assessments can be conducted, by quantifying the solvency and welfare impacts of pension mergers under a variety of circumstances using a comprehensive framework that takes into account both asset- and liability-side changes. The framework included an ESG, a stochastic mortality model that considers multiple subpopulations, a cost model with economies of scale, and a dynamic asset allocation methodology. From this framework, three groups of measures were constructed: plan-related risk measures assessing profits from an economic capital perspective, CECs to understand the members' perspective, and contribution-based metrics capturing the risk from the employer's viewpoint.

Using this framework, the present study investigated a hypothetical merger. It involved a small corporate plan of about 1,000 members and a large plan of 50,000 members. Overall, we found

solid evidence of the benefits of consolidation. We reported gains from the plan's, the members', and the employer's perspectives. Importantly, the results were robust to most changes in our assumptions – the merger was still beneficial for small plans in the majority of the cases investigated, even with reasonably heterogeneous mortality expectations and different plan maturity levels. Two notable exceptions are when plans have very different initial funded statuses and when plans have different retirement ages. Pension practitioners should bear in mind these cases and potentially advise against such mergers. Additional tests showed that merging might be even more beneficial for a small plan when it is more risk-averse or when its administrative or investment costs are higher.

Interestingly, the large plan's benefits from merging with a small plan were marginal according to our results. Yet, several large plans are currently looking to attract new groups of members. As shown in Section 5, consolidating multiple small plans increases the benefits of the merger for large plans, making the gains from mergers more material for them – a positive-sum game for all stakeholders. More members (and more assets) could also allow large plans to invest in other alternative investments – for instance, infrastructure, real estate, riskier bonds, hedge funds, cryptocurrencies, and commodities – which could explain why they are so eager to merge with smaller plans. This was not modeled in the present study. Preliminary results show, however, that including such alternative assets could create a worthwhile benefit for large plans. We leave this question for future research.

There are also other benefits that we have not modeled in this study. For instance, mergers could improve governance. This aspect, even though hard to quantify, can positively affect various pension plan stakeholders – including members and employers. Further study of this issue is left for future work.

Data availability statement. The code that supports the findings of this study is available on the corresponding author's personal website. The raw data used to estimate the models are not publicly available due to proprietary reasons.

Supplementary material. The supplementary material for this article can be found at <http://doi.org/10.1017/S1748499524000150>.

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