Variation of fundamental constants and white dwarfs

Susana J. Landau[1,2]

[1]Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires and IFIBA, Ciudad Universitaria - Pab. I, Buenos Aires 1428, Argentina
email: slandau@df.uba.ar

[2]CONICET, Godoy Cruz 2290, 1425 Ciudad Autónoma de Buenos Aires, Argentina

Abstract. Theories that attempt to unify the four fundamental interactions and alternative theories of gravity predict time and/or spatial variation of the fundamental constants of nature. Different versions of these theories predict different behaviours for these variations. As a consequence, experimental and observational bounds are an important tool to check the validity of such proposals. In this paper, we review constraints on the possible variation of the fundamental constants from astronomical observations and geophysical experiments designed to test the constancy of the fundamental constants of nature over different timescales. We also focus on the limits that can be obtained from white dwarfs, which can constrain the variation of the constants with the gravitational potential.

Keywords. quasars: absorption lines, cosmology: theory, cosmic microwave background, supernovae: general , stars: white dwarfs

---

1. Introduction

Our present knowledge of the properties of nature is based on two fundamental theories: General Relativity (GR) which describes gravitational interaction and the Standard Model of Elementary Particles which depicts the electromagnetic, strong and weak interactions. In this way, all physical phenomena at low energies can be described by the solutions of the equations of both theories. However, there is something missing in this picture, namely, there are 20 parameters in these equations that are not given by the theories and must be determined by experiments. These parameters are the ones that we call the fundamental constants and examples of them are the masses of elementary particles like quarks and leptons; the gauge coupling constants of the electromagnetic, weak and strong interactions; the gravitational constant G_N, the velocity of light c and the Higgs vacuum expectation value $< v >$. In turn, the principle of equivalence on which GR is based, requires the invariance of these quantities against changes in position, time and reference system. The gauge coupling constants of $U(1)$, $SU(2)$ and $SU(3)$ are related to the fine structure constant α, the QCD energy scale Λ_{QCD} and the Fermi coupling constant G_F through the following equations:

$$\alpha^{-1}(E) = \frac{5}{2}\alpha_1^{-1}(E) + \alpha_2^{-1}(E) \tag{1.1}$$

$$\Lambda_{QCD} = E \exp\left[-\frac{2\pi}{7}\alpha_3^{-1}(E)\right] \tag{1.2}$$

$$G_F = \frac{\pi\,\alpha_2\,(M_W)}{\sqrt{2}M_W^2} \tag{1.3}$$

where M_W refers to the mass of the W boson. Furthermore, gyromagnetic factors of atoms g_i are also considered fundamental constants and for the purpose of this paper we will consider the proton's mass m_p as a fundamental constant.

Since the Large Number hypothesis was formulated by Dirac in 1937, the variation of the fundamental constants has been the subject of numerous research papers. Dirac (1937) observed a remarkable coincidence: the dimensionless ratio between the gravitational and the electromagnetic force between a proton and an electron is of order 10^{-39} while the time it takes for the light to pass through a hydrogen atom is just 3 orders of magnitude smaller. This remarkable coincidence led him to formulate the Large Number Hypothesis according to which one or more constants are simple functions of the age of the universe. Furthermore, Dirac proposed that the variation of G_N is of the form: $G_N \sim t^{-1}$. Simultaneously, Milne (1937) also considered a possible variation in the G_N but with a different dependence with time : $G_N \sim t$. Following Dirac's proposal, Teller (1948) suggested that the fine structure constant is also a function of cosmological time and in particular he proposed the following dependence: $\alpha \sim (\log t)^{-1}$. Later, Pochoda & Schwarzschild (1964), showed that Dirac's proposal cannot explain the Sun's observed age and luminosity. In an attempt to rescue Dirac's idea, Gamow (1967) proposed that while G_N remains constant, α varies as a linear function of time. However, Dyson (1967) ruled out Gamow's proposal using the abundance of long lived β decayers in meteorites. At the same time, Bahcall & Schmidt (1967) reached the same conclusion using quasar absorption system spectra. The interest in time and spatial variation of fundamental constants got renewed when the attempt to unify the four interactions of nature resulted in the development of multidimensional theories such as Kaluza-Klein theories (Kaluza 1921; Klein 1926; Marciano 1984), string theories (Maeda 1988; Barr & Mohapatra 1988; Damour & Polyakov 1994; Damour *et al.* 2002) and related brane theories (Youm 2001; Palma *et al.* 2003). In Kaluza-Klein theories, the variation of the fundamental constants is related to the cosmological evolution of the radius of the extra dimensions, while in the case of superstrings and branes, the variation of the vacuum expectation value of a non-massive scalar field (for example the dilaton in string theories) is responsible for such variation. On the other hand, scalar-tensor theories of gravity are natural frameworks to study the variation of G_N. In these theories, a scalar field couples to the Ricci scalar and its dynamics provides the physical mechanism for the variation in G_N (Jordan 1959; Brans & Dicke 1961). More recently, Moffat (2006) proposed an alternative theory of gravity, which attempts to explain observational data from a large number of astrophysical scenarios without the need to include dark matter. In the latter, G_N is also allowed to vary with space and/or time. Following a different path of research, Bekenstein (1982) proposed a theoretical framework to study the fine structure constant variability based on general assumptions: covariance, gauge invariance, causality and time-reversal invariance of electromagnetism, as well as the idea that the Planck-Wheeler length $\left(10^{-33}\text{cm}\right)$ is the shortest scale allowable in any theory. Similar phenomenological frameworks based on first principles were also proposed by other authors (Barrow *et al.* 2002; Olive & Pospelov 2002; Chamoun *et al.* 2001; Barrow & Magueijo 2005). In these phenomenological frameworks, the physical mechanism responsible for such variation is a scalar field which is added to the theory.

The experimental research can be grouped into astronomical and local methods. The latter ones include geophysical methods such as the natural nuclear reactor that operated about $1.8 \; 10^9$ years ago in Oklo, Gabon, and laboratory measurements such as detailed comparisons of several atomic clock frequencies with different atomic number. The astronomical methods are based mainly in the analysis of spectra from high-redshift quasar absorption systems. Moreover, data from type Ia supernovae also provide limits on the possible variation in α. Besides, primoridal nucleosyntesis and the Cosmic Microwave

Backdground (CMB) provide constraints on the the variation of the fundamental constants in the early universe. Moreover, white dwarf spectra and the mass-radius relation in white dwarfs also provide stringent constraints on the variation in α and the proton to electron mass $\mu = \frac{m_p}{m_e}$. It was mentioned above that in most of the theories that predict variation of the fundamental constants, the physical mechanism for such variation is the dynamics of a scalar field. Near massive objects, like a white dwarf, the effect of the scalar field can change. Therefore, constraints on the fundamental constants from white dwarfs, can also be regarded as limits on those variations in terms of a variation in the gravitational potential. In such a way, the constraints on $\frac{\Delta \alpha}{\alpha}$ can be translated in terms of a dependence on a dimensionless gravitational potential:

$$\frac{\Delta \alpha}{\alpha} \simeq k_\alpha^{(1)} \Delta \Phi + k_\alpha^{(2)} (\Delta \Phi)^2 \tag{1.4}$$

Similar, constraints on the possible variation of the proton to electron mass μ can be expressed as follows:

$$\frac{\Delta \mu}{\mu} \simeq k_\mu^{(1)} \Delta \Phi + k_\mu^{(2)} (\Delta \Phi)^2 \tag{1.5}$$

In Equations 1.4 and 1.5 Φ refers to the gravitational potential and the coefficients $k_\alpha^{(i)}$, $k_\mu^{(i)}$ can be determined by experimental or observational data. In atomic clock experiments, the difference in the gravitational potential is of order $\Delta \Phi \sim 10^{-10}$, while the difference between the gravitational potential in a white dwarf and the respective one on Earth is $\Delta \Phi \sim 10^{-4} - 10^{-5}$. This is the main reason to affirm that white dwarf spectra are the ideal probe for a relationship between the fundamental constants like α and μ and strong gravitational fields. In this article, we will describe bounds on the fundamental constants from geophysical experiments and astronomical observations with a focus on those that can be obtained from white dwarfs. In section 2 we describe the bounds from geophysical data like the Oklo nuclear reactor and atomic clocks. We also discuss the bounds on the relation between the fundamental constants and the gravitational potential that can be obtained with this method. In section 3 we review the bounds from quasar absorption systems and discuss the status of the claimed variation in α from the Many Multiplet Method. Besides, bounds from primordial nucleosynthesis and the Cosmic Microwave Background are discussed in Section 4. Furthermore, bounds from type Ia supernovae data are described in Section 5. A detailed discussion on the bounds that can be obtained from white dwarf data on the possible variation of α, μ and G_N is described in Section 6. We also provide a description on the bounds on G_N that can be obtained from the Lunar Laser Ranging experiment (see Section 7) and helioseismology (see Section 8). Finally, in Section 9 we discuss our conclusions.

2. Bounds from geophysical data

2.1. *The Oklo Phenomenon*

One of the most stringent limits on time variation of the fine structure constant α follows from an analysis of isotope ratios in the natural uranium fission reactor that operated 1.8×10^9 years ago at the present day site of the Oklo mine in Gabon, Africa. From an analysis of nuclear and geochemical data, the operating conditions of the reactor could be reconstructed and the thermal neutron capture cross sections of several nuclear species measured. In particular, a shift in the lowest lying resonance energy level in $^{149}Sm : \Delta = E_r^{149(Oklo)} - E_r^{149(now)}$ can be derived from a shift in the neutron capture cross section of the same nucleus (Damour & Dyson 1996). The shift in Δ can be translated

into a bound on a possible difference between between the values of α and G_F during the Oklo phenomenon and their value now:

$$\Delta = 10^6 \text{eV} \frac{\Delta\alpha}{\alpha} + 5.6 \text{eV} \frac{\Delta G_F}{G_F} \tag{2.1}$$

Several authors have analyzed the Oklo data to put constraints on the possible variation of the fundamental constants (Petrov *et al.* 2006; Davis & Hamdan 2015). The most stringent limit was established by Davis & Hamdan (2015):

$$\frac{\Delta\alpha}{\alpha} < 1.1 \times 10^{-8} \tag{2.2}$$

2.2. *Atomic Clocks*

Atomic clocks provide the most stringent bounds on the variation of fundamental constants. The method consists of comparing atomic clock frequencies of different transitions, which in turn have different dependence with the fundamental constants. For example, the dependence of a hyperfine transition frequency with the fundamental constants can be expressed as follows:

$$\nu_{\text{Hyp}} \sim \alpha^2 g_i \frac{m_e}{m_p} R_\infty c F_{REL}(\alpha Z) \tag{2.3}$$

where g_i is gyromagnetic factor, R_∞ is Rydberg's constant, m_e and m_p are the electron's and proton's mass respectively and F_{REL} is the relativistic contribution to the energy. In such way, clocks based on hyperfine transitions in alkali atoms with different atomic number Z can be used to set bounds on $\alpha^k \frac{\mu_{A_1}}{\mu_{A_2}}$ where k depends on the frequencies measured and μ_{A_i} refers to the nuclear magnetic moment of each atom (Prestage *et al.* 1995; Marion *et al.* 2003; Guéna *et al.* 2012).

On the other hand, an optical transition frequency has the following dependence on α:

$$\nu_{opt} \sim R_\infty B F_i(\alpha) \tag{2.4}$$

where B is a numerical constant assumed not to vary in time and $F_i(\alpha)$ is a dimensionless function of α that takes into account level shifts due to relativistic effects. Thus, comparing an optical transition frequency with a hyperfine transition frequency can be used to set bound on $\alpha^k \frac{m_e}{m_p} \frac{\mu_A}{\mu_B}$ (Le Targat *et al.* 2013; Huntemann *et al.* 2014; Falke *et al.* 2014). Furthermore, the most stringent bound on α variation using this method was obtained by Rosenband *et al.* (2008) and it follows from a comparison between two optical transition frequencies. On the other hand, the energy difference between two adjacent rotational levels in a diatomic molecule is proportional to $\frac{1}{Mr^2}$, r being the bond length and M the reduced mass. Moreover, the vibrational transition of the same molecule has, in first approximation, a $\sqrt{M}$ dependence. In this way, it follows that the vibro-rotational molecular transitions are proportional to $\frac{1}{\sqrt{M}}$. Therefore, comparing vibro-rotational transitions with an hyperfine transition gives information about the variaton of $g_{\text{hyp}} \sqrt{\frac{m_e}{m_p}} \alpha^k$. Shelkovnikov *et al.* (2008) used this latter method to compare the vibro-rotational transition of the SF_6 molecule with the ^{133}Cs hyperfine transition. Their results can also be used to constrain $k_\mu^{(1)}$ from Eq. 1.5. In this experiment, the difference in the gravitational potential in the Earth due to the Sun between aphelion and perihelion was estimated to be $\Delta\Phi = 10^{-10}$, and therefore the bound on $\frac{\Delta\mu}{\mu}$ yields:

$$k_\mu^{(1)} < 4 \times 10^{-4} \tag{2.5}$$

On the other hand, comparison of radio-frequency transitions between nearly degenerate, opposite parity excited states in atoms allows us to establish bounds on the possible

variation in α due to the large relativistic corrections of opposite sign for the opposite-parity levels. This method was used by Leefer *et al.* (2013) to establish a bound in α from the comparison of frequency transitions in two isotopes of atomic dysprosium (Dy). This bounds can also be used to set constraints on $k_\alpha^{(1)}$ in Equation 1.4. In fact, since the estimated difference in the gravitational potential between aphelion and perihelion for this experiment is $\Delta\Phi = 3.3 \times 10^{-10}$, the bound on $\frac{\Delta\alpha}{\alpha}$ yields:

$$k_\alpha^{(1)} = (-5.5 \pm 5.2) \times 10^{-7} \tag{2.6}$$

Finally, combining all available laboratory data Godun *et al.* (2014) obtain:

$$\frac{\dot{\alpha}}{\alpha} = (0.7 \pm 2.1) \times 10^{-17} \mathrm{yr}^{-1} \tag{2.7}$$

$$\frac{\dot{\mu}}{\mu} = (0.2 \pm 1.1) \times 10^{-16} \mathrm{yr}^{-1} \tag{2.8}$$

3. Bounds from Quasar Absorption Systems

Quasar absorption systems present ideal laboratories in which to test for possible variations of the fundamental constants. The relationship between the wavelength observed in quasar spectra (λ_{obs}) and the ones measured in the laboratory (λ_{lab}) can be expressed:

$$\lambda_{\mathrm{obs}} = \lambda_{\mathrm{lab}} (1 + z). \tag{3.1}$$

Several methods have been developed to test the possible variation of the fundamental constants, among them, it is important to mention the following: i) Alkali Doublet Method, ii) Many Multiplet Method, iii) Comparison of molecular spectra with laboratory spectra, iv) Comparison of radio spectra with optical spectra, v) Comparison of radio spectra with molecular spectra and vi) Conjugate Lines Method. Some of them such as the Akali Doublet Method rely on the comparison between the observed wavelengths in the quasar and the one measured in the laboratory. Others, are based on the comparison of absorption redshifts due to different transitions that happen in the same absorption cloud. We will not discuss all methods in this article. We will focus on i) the Many Multiplet Method which is so far the method that allows us to place the more stringent constraints on $\frac{\Delta\alpha}{\alpha}$ from quasar spectra and ii) the comparison between molecular and optical spectra which provides stringent constraints on the variation of μ.

The Many Multiplet Method was proposed by Webb *et al.* (1999) and relies on the comparison of transitions with different atomic masses in the same absorption cloud. In short, the Many Multiplet Method consist of adjusting the Voigt profiles of several absorption lines, including besides the usual fit parameters: column density, Doppler width, redshift, a possible variation of the fine structure constant. In this way, the method allows us to gain an order of magnitude in sensibility with respect to previously reported data. Using this method and data provided by the Keck telescope, Webb *et al.* (1999) claimed a detection of a variation in α. However, an independent analysis performed with observations made with UVES at the Very Large Telescope (VLT) provided null results (Srianand *et al.* 2004). Contrary to previous results, a third analysis, this time with VLT/UVES archival data also indicated a variation in α, but now with α increasing with redshift (Webb *et al.* 2011; King *et al.* 2012). These results, led the authors to suggest a spatial dipole-type variation in α. The weighted mean of the 293 measurements obtained with data from both Keck and VLT telescopes by the group of Webb et al at $0.3 < z < 3.1$ is:

$$\left(\frac{\Delta\alpha}{\alpha}\right) = -2.16 \pm 0.86 \times 10^{-6}. \tag{3.2}$$

On the other hand, more recently, a reanalysis of systematic errors with new techniques showed that there is no compelling evidence for any variation in α from quasar data

(Whitmore & Murphy 2015). However, it should be noted that in all those mentioned analyses the data acquisition procedures were far from ideal, in particular, regarding the key issue of wavelength calibration. Trying to confirm the above mentioned results was the main motivation for the ESO UVES Large Program, for which key improvements in the data acquisition procedures were implemented (Bonifacio *et al.* 2014). Furthermore, other dedicated measurements of the variation in α with the quasar method were performed recently, by several authors (Agafonova *et al.* 2011; Bainbridge & Webb 2017; Evans *et al.* 2014; Kotuš *et al.* 2017; Molaro *et al.* 2013; Songaila & Cowie 2014). In particular Murphy *et al.* (2016) developed a method to test a variation in α which is not influenced by long-range distortions. The weighted mean of all recent dedicated measurements which span the redshift range $1 < z < 2.4$ is (Martins 2017):

$$\left(\frac{\Delta\alpha}{\alpha}\right) = -0.64 \pm 0.65 \times 10^{-6}, \tag{3.3}$$

showing no variation in α. Nevertheless, Murphy *et al.* (2016) reached the conclusion that their quasar sample is too small to rule out the dipole model. On the other hand, it should be noted that all above mentioned measurements were performed with spectrographs such as UVES, HARPS or Keck-HIRES, which are far from optimal for this type of measurement. Therefore, more precise measurements using the new generation of high-resolution spectrograph, like ESPRESSO for the VLT and E-ELT-HIRES for the E-ELT, are expected to improve significantly the precision of the data.

Varshalovich & Levshakov (1993) developed a method for constraining the variation in $\mu = \frac{m_p}{m_e}$ which is based on the fact that wavelengths of electron-vibro-rotational lines depend on the reduced mass of the molecules, with different dependence for different transitions. In such way, it is possible to distinguish the cosmological redshift of a line from the shift caused by a variation in μ. The wavelength of a molecular line at redshift z_{abs} can be expressed as:

$$\lambda_i = \lambda_i^{\mathrm{lab}} \left(1 + z_{\mathrm{abs}}\right) \left(1 + K_i \frac{\Delta\mu}{\mu}\right) \tag{3.4}$$

where λ_i^{lab} is the wavelength measured in the laboratory and K_i is a coefficient for the molecular band. Several authors have used this method to place stringent constraints on the variation in μ at $0.6 < z < 4.2$ (Kanekar 2011; Bagdonaite *et al.* 2013; van Weerdenburg *et al.* 2011; Rahmani *et al.* 2013; Daprà *et al.* 2015; Albornoz Vásquez *et al.* 2014; Bagdonaite *et al.* 2015). Measurements at low redshift were obtained with radio/mm observations and its weighed mean yields $\frac{\Delta\mu}{\mu} = (-0.24 \pm 0.09) \times 10^{-6}$ (Martins 2017) while the high redshift sample proceeds from UV/optical observations with weighed mean $\frac{\Delta\mu}{\mu} = (2.9 \pm 1.9) \times 10^{-6}$ (In table 2 we show the weighed mean of the complete data set). There is weak evidence for a variation in μ from both samples, even though it should be stressed that the sign of the variation is different at high and low redshift. Besides, Bagdonaite *et al.* (2013b) obtain the most stringent limit from observations of metanol transitions at $z = 0.89 : \frac{\Delta\mu}{\mu} = (-1 \pm 1.8) \times 10^{-7}$.

4. Bounds from the early universe

4.1. *Cosmic Microwave Background*

The Cosmic Microwave Background (CMB) radiation is one of the best tools to study the early universe because it provides information about the physical conditions in the Universe just before decoupling of matter and radiation. Possible variation in α and the

electron mass m_e affect the Thompson scattering cross section $\sigma_T = \frac{8\pi}{3} \frac{\hbar^2 c^2}{m_e^2} \alpha^2$ and the ionization fraction. The main effect in the ionization fraction can be expressed as follows:

$$x_e \simeq \left(\frac{m_e}{k_B T} \right)^{\frac{3}{2}} \exp - \left(\frac{B_1}{k_B T} \right) \tag{4.1}$$

where $B_1 = \frac{\alpha^2 m_e c^2}{2}$ is the Hydrogen binding energy. The effect of a possible variation in α and/or m_e on the CMB doppler peaks is a shift in the position of the peaks and modification in the height of the peaks. From the comparison of the theoretical prediction with observational data obtained by the Planck Collaboration *et al.* (2015), bounds on the possible variation in α and/or m_e can be obtained. Assuming indepedent variations in α and m_e, Hart & Chluba (2018) obtain:

$$\left(\frac{\Delta\alpha}{\alpha} \right) = (0.7 \pm 2.5) \times 10^{-3} \quad \text{and} \quad \left(\frac{\Delta m_e}{m_{e0}} \right) = (3.9 \pm 7.4) \times 10^{-3} \tag{4.2}$$

where the last bound was obtained considering also Baryon Acoustic Oscilation (BAO) data in the statistical analysis. On the other, hand if joint variations of α and m_e are assumed and BAO data are considered in the analysis, the limits obtained are:

$$\left(\frac{\Delta\alpha}{\alpha} \right) = (1.1 \pm 2.6) \times 10^{-3} \quad \text{and} \quad \left(\frac{\Delta m_e}{m_{e0}} \right) = (5.6 \pm 8) \times 10^{-3} \tag{4.3}$$

4.2. *Big Bang Nucleosynthesis (BBN)*

The study of the light nuclei production during the first three minutes of the universe is another important tool to study the early universe. In short, the abundances of light nuclei depend on the following physical parameters: freeze-out time of the weak interactions, neutron-proton mass diference, binding energies of the light nuclei and cross sections of the reactions producing D, ^{4}He, ^{7}Li, ^{6}Li, Tritium, ^{3}He, and ^{7}Be, which, in turn are functions of the fundamental constants α, the Higgs vacuum expectation value $<v>$ and G_N. It should be stressed that unlike other methods described in this paper, it is necessary to asume a theoretical model for the strong interaction within this analysis. Mosquera & Civitarese (2017) considered the Argonne potential for the nucleon-nucleon interaction. From the comparison with the observed abundances of D, ^{4}He and ^{7}Li they obtained:

$$\frac{\Delta\alpha}{\alpha} = -0.022 \pm 0.006 \quad \text{and} \quad \frac{\Delta <v>}{<v>} = 0.042 \pm 0.01 \tag{4.4}$$

whereas if the Bonn potential is assumed, the analysis yielded:

$$\frac{\Delta\alpha}{\alpha} = -0.022^{+0.003}_{+0.004} \quad \text{and} \quad \frac{\Delta <v>}{<v>} = 0.036 \pm 0.007 \tag{4.5}$$

On the other hand, the value of G_N determines the expansion rate of the universe and thus, the relevant time scales for the weak and nuclear reactions. As a consequence, assuming a variation in G_N at the time of the BBN, translates in turn into a variation of the light element abundances with respect to the ones predicted by the standard cosmological model. Copi *et al.* (2004) and Bambi *et al.* (2005) compared the theoretical predictions for the light nuclei abundances with the present abundances of D, ^{4}He and ^{7}Li and established the following bound:

$$\frac{\dot{G}}{G} = (0.05 \pm 0.35) \times 10^{-12} \text{yr}^{-1} \tag{4.6}$$

5. Bounds from supernovae type Ia

Type Ia supernovae (SNe Ia) are among the most energetic and interesting phenomena in our universe. Furthermore, the spectra and light curves of normal SNe Ia are very homogeneous in such a way that they are considered one of the best standard candles known today. All this makes them suitable astronomical objects to test the possible variation of fundamental constants. The homogeneity of the light curve is basically caused by the homogeneity of the nickel mass produced during the supernova outburst. This effect is primary determined by the value of the Chandrasekhar mass which in turn depends on the value of the fundamental constants G_N, α or the velocity of light c as follows:

$$M_{\rm ch} \sim \left(\frac{\hbar c}{G_N} \right)^{3/2} \sim \left(\frac{e^2}{\alpha^2 G_N} \right)^{3/2} \tag{5.1}$$

In addition, the possible variation of the fundamental constants with cosmological time would also change the cosmic evolution and therefore affect the luminosity distance relation. Gaztañaga et al. (2002) performed an analysis allowing only for the variation in G_N and showed that the latter is typically several times smaller than the change produced by the corresponding variation of the Chandrasekhar mass. Furthermore, these authors used data from the Supernova Cosmology Project to put the following bound:

$$\frac{\dot{G}_N}{G_N} < 10^{-11} {\rm yr}^{-1} \tag{5.2}$$

Garcia-Berro et al. (2006) considered that the variation in G_N follows the predictions of scalar-tensor theories and reached similar conclusions. On the other hand, Kraiselburd et al. (2015) analised the dependence of type Ia supernovae explosions on α, including both the change in the Chandrasekhar mass and the the dependence of the mean opacity of the expanding supernovae photosphere. Furthermore, motivated by the results of the quasar data, they considered a spatial dipole-type variation for α. From the comparison of supernovae data from the Union 2.1 compilation, they obtained:

$$\frac{\Delta \alpha}{\alpha} = (2.1 \pm 0.8) \times 10^{-2} \tag{5.3}$$

In a posterior work, Negrelli et al. (2018) considered the variation in c as the source of the variation in the Chandrasekhar mass. In addition, they also included the change of the energy release during the explosion in their analysis. Comparing the theoretical predictions for a spatial dipole-type variation in c, with the Union 2.1 and JLA supernovae type Ia data, they established:

$$\frac{\Delta c}{c} = (-3.7 \pm 0.8) \times 10^{-2} \tag{5.4}$$

6. Bounds from White Dwarfs

6.1. Bounds on α and μ

Hot white dwarf stars are ideal laboratories to probe for a relationship between the fundamental constants and strong gravitational fields. Hot white dwarfs with masses comparable to the sun and radii comparable to Earth generate strong gravitational fields and are typically bright with numerous absorption lines. The relationship between the laboratory wavelengths ($\lambda_{\rm lab}$) and those observed near a white dwarf ($\lambda_{\rm WD}$) is:

$$\frac{\Delta \lambda}{\lambda} = \frac{\lambda_{\rm WD} - \lambda_{\rm lab}}{\lambda_{\rm lab}} = z_{\rm WD} - Q_\alpha \frac{\Delta \alpha}{\alpha}(1 + z_{\rm WD}), \tag{6.1}$$
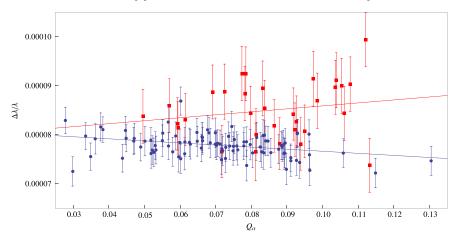
Figure 1. $\frac{\Delta\lambda}{\lambda}$ vs. Q_α for transitions in FeV (blue circles) and NiV (red squares). The slope of the lines give $\frac{\Delta\alpha}{\alpha} = (4.2 \pm 1.6) \times 10^{-5}$ for FeV and $\frac{\Delta\alpha}{\alpha} = (-6.1 \pm 5.8) \times 10^{-5}$ for NiV, respectively. Reprinted with permission from Physical Review Letters 111, 010801 (2013). Copyright(2013) by the American Physical Society.

where $Q_\alpha = 2q/\omega_0$ is the relative sensitivity of the transition frequency to a variation in α. Berengut *et al.* (2013) observed the wavelength shift in 96 quadruply ionized iron and 32 quadruply ionized nickel absorption features from the white dwarf star G191-B2B and derived separate limits for each metal. In all measurements line calibration was the expected culprit for the observed signal. Both results (see Figure 1) are inconsistent with each other at the 1.6 sigma level, the likely reason being related to uncertainties in laboratory wavelength measurements and systematics in the wavelength calibration. More recently, Bainbridge *et al.* (2017) presented preliminary results of a similar analysis incorporating improvements such as; i) the use of new laboratory wavelengths; ii) measurements of a sample of objects rather than a single object ; and iii) the use of robust techniques from quasar absorption systems (the Many Multiplet method) in the data analysis method. Therefore, future measurements with improved data analysis methods could provide more stringent bounds.

On the other hand, a possibe variation in μ would be manifested as shifts in the observed wavelengths of molecular hydrogen (H_2) when compared to laboratory wavelengths as follows

$$\frac{\lambda_i^{WD}}{\lambda_i^{\text{lab}}} = (1 + z_{WD})(1 + \frac{\Delta\mu}{\mu}K_i) \tag{6.2}$$

where λ_i^{WD} represents the H_2 transition wavelength observed in white dwarf spectra, λ_i^{lab} is a corresponding wavelength measured in the laboratory and K_i are the sensitivity coefficients. Bagdonaite *et al.* (2014) applied this method to the spectrum of the white dwarf star G29–38 with a potential of $2 \times 10^4\Phi_{\text{earth}}$ to obtain:

$$\Delta\mu/\mu = (-5.8 \pm 3.8_{\text{stat}} \pm 0.3_{\text{sys}}) \times 10^{-5} \tag{6.3}$$

while for the white dwarf GD133 with a potential of $10^4\Phi_{\text{earth}}$ the resulting bound was:

$$\Delta\mu/\mu = (-2.7 \pm 4.7_{\text{stat}} \pm 0.2_{\text{sys}}) \times 10^{-5} \tag{6.4}$$

We have discussed in Section 1 the theoretical motivation for considering varying fundamental constants and the inclusion of scalar fields as the physical mechanism for that variation in most theoretical models. Therefore, constraints on $\Delta\mu/\mu$ can be interpreted

in terms of a dependence on a dimensionless gravitational potential. Applying Eq. 1.5 to the above bounds yields the following limit:

$$k_\mu^{(2)} < 1 \times 10^3 \tag{6.5}$$

which is several orders of magnitude more stringent than the ones obtained from Earth-based experiments. And we remember the reader that the constraint on $k_\mu^{(1)}$ is obtained from atomic clock experiments (see Section 2.2).

On the other hand, Magano *et al.* (2017) obtained bounds on the possible variation of the fundamental constants from the mass-radius relation in white dwarfs. Unlike the analyses performed for other bounds described in this paper†, it is necessary to assume a theoretical model for the relation between the variation of the electron and nucleon masses and α. For this, the authors assume the expressions given by the generic class of unification models proposed by Coc *et al.* (2007):

$$\frac{\Delta m_e}{m_e} = \frac{1}{2}(1 + S)\frac{\Delta\alpha}{\alpha} \tag{6.6}$$

$$\frac{\Delta m_N}{m_N} = [0.8R + 0.2(1 + S)]\frac{\Delta\alpha}{\alpha} \tag{6.7}$$

where m_N refers to the nucleon masses, and R and S depend of the specific unification model considered. Using these relations, the mass continuity and hidrostatic equations can be expressed for the case of varying constants as follows:

$$\frac{dm'}{dr} = m_0 r^2 x_F^3 \tag{6.8}$$

$$\frac{dx_F}{dr} = -K_1(1 + \beta)\,\frac{m'}{r^2}\frac{\sqrt{1 + x_F^2}}{x_F} \tag{6.9}$$

$$m = K_2(1 + \gamma)\,m' \tag{6.10}$$

where $K_1 = \frac{16}{3\pi(2q)^2}\frac{R_\odot}{m_0}\frac{c^3}{G\hbar^3}\alpha_e\alpha_N$, $K_2 = \frac{8}{3\pi(2q)8}\frac{R_\odot^3}{M_\odot m_0}\frac{c^5}{G^2\hbar}\alpha_e^{\frac{3}{2}}\alpha_N^{\frac{1}{2}}$, $\alpha_i = \frac{Gm_i^2}{\hbar c}$, $x_F = \frac{p_F}{m_e c}$, p_F is the Fermi momentum, m_0 is a dimensionless constant and q is the number of electrons per nucleon. In the above equations, β and γ enclose the dependence of the above equations with the fundamental constants and are given by the following expressions:

$$\beta = \left[\frac{9}{5}R + \frac{8}{5}(1 + S)\right]\frac{\Delta\alpha}{\alpha}\,,\,\gamma = \left[\frac{4}{5}R + \frac{23}{10}(1 + S)\right]\frac{\Delta\alpha}{\alpha}\,.$$

Results of the numerical integration of the above equations from the original work are shown in Figure 2, for the cases: i)$\beta = 0$ and $\gamma = 0$ (standard model), ii) $\beta = \pm 0.01$ and iii)a more simple polytropic model; toghether with present data for the mass-radius relation. Next, a statistical analysis was performed to estimate bounds on α from observational data. For each star i in the catalog, Magano *et al.* (2017) chose a value of $M_\star$ to minimize the following quantity:

$$\chi_i^2(M_\star) = \frac{(M_\star - M_i)^2}{\sigma_{M,i}^2} + \frac{(R_{th}(M_\star) - R_i)^2}{\sigma_{R,i}^2}\,, \tag{6.11}$$

where M_i, $\sigma_{M,i}$, R_i, and $\sigma_{R,i}$ are the mass and radius of the ith star and their respective uncertainties, and $R_{th}(M)$ is the theoretical prediction. Thus, the total value of χ^2 to

† with the exception of primordial nucleosynthesis (see Section 4.2 where a model for the strong interactions has to be assumed)
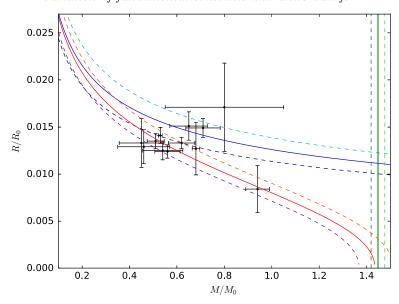
Figure 2. Solid red line for the standard model, with the nearby darker and lighter dashed lines corresponding to $\beta \pm = 0.01$. Solid blue line shows the nonrelativistic limit of the polytropic model, and darker and lighter dashed lines for $x = \pm 0.1$. $x = \left[\frac{4}{3}R + \frac{5}{6}(1+S)\right]\frac{\Delta\alpha}{\alpha}$. The black points with error bars correspond to the data. R_0 and M_0 refer to the Sun's radius and mass respectively. Reprinted with permission from Physical Review D96, 083012 (2017). Copyright(2017) by the American Physical Society.

be calculated was $\chi^2 = \sum_{i=1}^{N}\chi_i^2(\hat{M}_i)$ with $\hat{M}_i$ as the value of M that minimizes the corresponding χ_i^2. From the statistical analysis, the following constraints were obtained:

$$\beta = 0.012 \pm 0.032\,, \tag{6.12}$$

$$\gamma = 0.006 \pm 0.060 \tag{6.13}$$

If the typical values suggested in Coc *et al.* 2007 were assumed: $R \sim 30$ and $S \sim 160$, allowing a 10% uncertainty in each of them, the following bound was established:

$$\frac{\Delta\alpha}{\alpha} = (2.7 \pm 9.1) \times 10^{-5} \tag{6.14}$$

Finally, it should be noted, that improvements in mass and radius measurements, such as those expected from the Gaia satellite, will allow us to obtain more stringent constraints.

6.2. *Bounds on* G_N

White dwarfs can also be used to put independent limits on a possible variation in G_N. The most important reason for this is that white dwarfs have very long evolutionary timescales. A possible variation of G_N over cosmological ages changes the temperature in the central regions of white dwarf progenitors. As a consequence, the thermonuclear rates, luminosities and main sequences lifetimes of these stars are also modified. Furthermore, white dwarf cooling times are also affected by a change in G_N. García-Berro *et al.* (2011) calculated the effects of a varying G_N on the main sequence ages using a stellar evolutionary code. Furthermore, employing modified white dwarf cooling ages in order to account for a varying G_N, they built the white dwarf luminosity function for the old, metal-rich Galactic open cluster NGC 6791. Comparing the theoretical predictions for a

varying G_N with the observational color-magnitude diagram and the white dwarf luminosity function, and using the distance modulus measured with eclipsing binaries, the following bound was obtained:

$$\frac{\dot{G}_N}{G_N} < -1.8 \times 10^{-12} \text{yr}^{-1} \tag{6.15}$$

On the other hand, pulsational properties of white dwarfs can be used to constrain a possible variation in G_N. Córsico *et al.* (2013) compared the theoretical rates of period change of white dwarfs including the effects of a running G_N with the measured rates of change of the periods of two well studied pulsating white dwarfs, G117-B15A and R548. For this, they used a stellar evolutionary code and a pulsational code to compute pulsational properties of variable white dwarfs. Results show that the pulsation periods do not change in a significant way, but the rates of period change are strongly affected when a varying G_N is assumed. Moreover, they obtain the following bound

$$\frac{\dot{G}_N}{G_N} < -1.3 \times 10^{-10} \text{yr}^{-1} \tag{6.16}$$

which is less stringent that the one obtained from the white dwarf luminosity function.

7. Bounds from the Lunar Laser Ranging experiment

The Lunar Laser Ranging (LLR) experiment, has provided high-precision values of the Earth-Moon distance, through measurements of round trip travel times of laser pulses between stations on the Earth and retroreflectors on the Moon. Results of this experiment have been used to test General Relativity, the Strong Equivalence Principle, metric parameters, preferred frame effects and the temporal variation of the gravitational constant G_N. Hofmann *et al.* (2010) updated the Institut fur Erdmessung (IfE) LLR model taking the effect of a fluid lunar core into consideration. In this way, they reduced the uncertainty in the current variation in G_N by a factor of 2, obtaining:

$$\frac{\dot{G}_N}{G_N} = (0.7 \pm 3.8) \times 10^{-13} \text{yr}^{-1} \tag{7.1}$$

8. Bounds from Helioseismology

Guenther *et al.* (1998) considered solar models including the possible variation of the gravitational constant G_N during the solar lifetime. Furthermore, they compared the p-mode oscilation spectra of those models with observations from the Global Oscillation Network Group (GONG) instrument and Birmingham Solar Oscillation Network (BiSON). As a result of their research, they obtain the following bound:

$$\frac{\dot{G}_N}{G_N} < 1.6 \times 10^{-12} \text{yr}^{-1} \tag{8.1}$$

9. Summary and Conclusions

In this paper we have described the most important present bounds on the possible variaton of the fundamental constants α, μ and G_N. We also have mentioned the dependence of some observables on other fundamental constants such as the velocity of light c, the Fermi constant G_F and the Higgs vaccum expectation value $<v>$. In order to have a comprehensive picture, Tables 1, 2 and 3 show a summary of the bounds that can be obtained with each of the methods described in this review, together with the time interval for which the change in the fundamental constants was measured. It is important to stress, that bounds on the variation in α and μ from white dwarf spectra, are

Table 1. Bounds on $\frac{\Delta\alpha}{\alpha}$. Columns: (1) Method considered (2) mean value and the corresponding $1-\sigma$ error, (3) the time interval for which the variation was measured, 4) Reference. The bound obtained from white dwarf data constraints the possible variation in α with the gravitational potential.

Source	$\frac{\Delta\alpha}{\alpha} \pm \sigma$	Δt (yr)	Reference
Atomic Clocks	$(2.1 \pm 6.3) \times 10^{-17}$	13.8×10^9	Godun *et al.* (2014)
Oklo	$< 1.1 \times 10^{-8}$	1.8×10^9	Davis & Hamdan (2015)
Quasars	$(-0.64 \pm 0.65) \times 10^{-6}$	$(8-11) \times 10^9$	Martins (2017)
CMB	$(0.7 \pm 2.5) \times 10^{-3}$	13.8×10^9	Hart & Chluba (2018)
BBN	$(-2.2 \pm 0.6) \times 10^{-2}$	13.8×10^9	Mosquera & Civitarese (2017)
Supernovae type Ia	$(2.1 \pm 0.8) \times 10^{-2}$	$(0.1-9) \times 10^{10}$	Kraiselburd *et al.* (2015)
White Dwarfs	$(2.7 \pm 9.1) \times 10^{-5}$		Magano *et al.* (2017)

Table 2. Bounds on $\frac{\Delta\mu}{\mu}$. Columns: (1) Method considered (2) mean value and the corresponding $1-\sigma$ error, (3) the time interval for which the variation was measured, 4) Reference. The bound obtained from white dwarf data constraints the possible variation in μ with the gravitational potential.

Source	$\frac{\Delta\mu}{\mu} \pm \sigma$	Δt (yr)	Reference
Atomic Clocks	$(0.2 \pm 2.1) \times 10^{-16}$	$2-4$	Godun *et al.* (2014)
Quasars	$(-0.23 \pm 0.1) \times 10^{-6}$	$(6.4-12.4) \times 10^9$	Martins (2017)
CMB	$(3.9 \pm 7.4) \times 10^{-3}$	13.8×10^9	Hart & Chluba (2018)
White Dwarfs	$(-5.8 \pm 4.1) \times 10^{-5}$		Bagdonaite *et al.* (2014)

Table 3. Bounds on $\frac{\dot{G}_N}{G_N}$. Columns: (1) Method considered (2) mean value and the corresponding $1-\sigma$ error, (3) the time interval for which the variation was measured, 4) Reference.

Source	$\frac{\dot{G}_N}{G_N} \pm \sigma$ in units of 10^{-12}	Δt (yr)	Reference
BBN	(0.05 ± 0.35)	13.8×10^9	Copi *et al.* (2004); Bambi *et al.* (2005)
LLR	(0.07 ± 0.38)	40	Hofmann *et al.* (2010)
Supernovae	< 10	5×10^9	Gaztañaga *et al.* (2002)
Helioseismology	< -1.6	4.6×10^9	Guenther *et al.* (1998)
White Dwarfs	< -1.8	7.7×10^9	García-Berro *et al.* (2011)

constraints of changes with the gravitational potential (which could also be regarded as a spatial variation), but they do not refer under any point of view to a time variation of fundamental constants. Finally, we would like to emphasize, the most important aspects that were discussed in this review:

(*a*) There is no evidence for a time or space variation of the fundamental constants.

(*b*) The most stringent limits on variation of the fundamental constants are provided by atomic clocks experiments:

(*c*) White dwarf stars are ideal laboratories to test for a relationship between the fine-structure constant or the proton-to-electron-mass and strong gravitational fields. Current constraints are derived from molecular hydrogen measurements and the mass-radius relationship while future measurements of Fe V and Ni V spectra in white dwarfs could provide more stringent bounds.

Acknowledgments

References

Agafonova, I. I., Molaro, P., Levshakov, S. A., *et al.* 2011, *A&A*, 529, A28

Albornoz Vásquez, D., Rahmani, H., Noterdaeme, P., *et al.* 2014, *A&A*, 562, A88

Bagdonaite, J., Daprà, M., Jansen, P., *et al.* 2013, *Physical Review Letters*, 111, 231101

Bagdonaite, J., Jansen, P., Henkel, C., *et al.* 2013, *Science*, 339, 46

Bagdonaite, J., Salumbides, E. J., Preval, S. P., *et al.* 2014, *Physical Review Letters*, 113, 123002

Bagdonaite, J., Ubachs, W., Murphy, M. T., *et al.* 2015, *Physical Review Letters*, 114, 071301

Bahcall, J. N. & Schmidt, M. 1967, *Physical Review Letters*, 19, 1294

Bainbridge, M. B. & Webb, J. K. 2017, *MNRAS*, 468, 1639

Bainbridge, M., Barstow, M., Reindl, N., *et al.* 2017, *Universe*, 3, 32

Bambi, C., Giannotti, M., & Villante, F. L. 2005,*Physical Review D*, 71, 123524

Barr, S. M. & Mohapatra, P. K. 1988, *Physical Review D*, 38, 3011

Barrow, J. D. & Magueijo, J. 2005, *Physical Review D*, 72, 043521

Barrow, J. D.,Sandvik, H. B. & Magueijo, J. 2002, *Physical Review D*, 65, 063504

Bonifacio, P., Rahmani, H., Whitmore, J. B., *et al.* 2014, *Astronomische Nachrichten*, 335, 83

Brans, C. & Dicke, R. H. 1961, *Physical Review*, 124, 925

Bekenstein, J. D. 1982, *Physical Review D*, 25, 1527

Berengut, J. C., Flambaum, V. V., Ong, A., *et al.* 2013, *Physical Review Letters*, 111, 010801

Chamoun, N., Landau, S. J. & Vucetich, H. 2001, *Physics Letters B*, 504, 1

Coc, A., Nunes, N. J., Olive, K. A., *et al.* 2007, *Physical Review D*, 76, 023511

Copi, C. J., Davis, A. N., & Krauss, L. M. 2004, *Physical Review Letters*, 92, 171301

Córsico, A. H., Althaus, L. G., García-Berro, E., *et al.* 2013, *JCAP*, 2013, 032

Damour, T. & Dyson, F. 1996, *Nuclear Physics B*, 480, 37

Damour, T., Piazza, F., & Veneziano, G. 2002, *Physical Review Letters*, 89, 081601

Damour, T. & Polyakov, A. M 1994, *Nuclear Physics B*, 95, 10347

Daprà, M., Bagdonaite, J., Murphy, M. T., *et al.* 2015, *MNRAS*, 454, 489

Davis, E. D. & Hamdan, L. 2015, *Physical Review C*, 92, 014319

Dirac, P. A. M. 1937, *Nature*, 139, 323

Dyson, F. 1967, *Physical Review Letters*, 19, 1291

Evans, T. M., Murphy, M. T., Whitmore, J. B., *et al.* 2014, *MNRAS*, 445, 128

Falke, S., Lemke, N., Grebing, C., *et al.* 2014, *New Journal of Physics*, 16, 073023

Gamow, G. 1967, *Physical Review Letters*, 19, 759

Garcia-Berro, E., Kubyshin, Y., Loren-Aguilar, P., *et al.* 2006, *International Journal of Modern Physics D*, 15, 1163

García-Berro, E., Lorén-Aguilar, P., Torres, S., *et al.* 2011, *JCAP*, 2011, 021

Gaztañaga, E., García-Berro, E., Isern, J., *et al.* 2002, *Physical Review D*, 65, 023506

Godun, R. M., Nisbet-Jones, P. B. R., Jones, J. M., *et al.* 2014, *Physical Review Letters*, 113, 210801

Guéna, J., Abgrall, M., Rovera, D., *et al.* 2012, *Physical Review Letters*, 109, 080801

Guenther, D. B., Krauss, L. M., & Demarque, P. 1998, *ApJ*, 498, 871

Hofmann, F., Müller, J., & Biskupek, L. 2010, *A&A*, 522, L5

Huntemann, N., Lipphardt, B., Tamm, C., *et al.* 2014, *Physical Review Letters*, 113, 210802

Hart, L. & Chluba, J. 2018, *MNRAS*, 474, 1850

Jordan, P. 1959, *Zeitschrift fur Physik*, 157, 112

Kaluza, Th. 1921, *Sitzungber. Preuss. Akad. Wiss.K*, 1, 966

Kanekar, N. 2011, *ApJ Letters*, 728, L12

King, J. A., Webb, J. K., Murphy, M. T., *et al.* 2012, *MNRAS*, 422, 3370

Klein, O. 1926, *Z. Phys.*, 37, 895

Kotuš, S. M., Murphy, M. T., & Carswell, R. F. 2017, *MNRAS*, 464, 3679

Kraiselburd, L., Landau, S. J., Negrelli, C., *et al.* 2015, *Astrophysics Space and Science*, 357, 4

Leefer, N., Weber, C. T. M., Cingöz, A., *et al.* 2013, *Physical Review Letters*, 111, 060801

Le Targat, R., Lorini, L., Le Coq, Y., *et al.* 2013, *Nature Communications*, 4, 2109

Maeda, K. 1988, *Modern Physics. Letters A*, 3, 243

Marciano, W. J. 1984, *Physical Review Letters*, 52, 489

Magano, D. M. N., Vilas Boas, J. M. A., & Martins, C. J. A. P. 2017, *Physical Review D*, 96, 083012

Marion, H., Pereira Dos Santos, F., Abgrall, M., *et al.* 2003, *Physical Review Letters*, 90, 150801

Martins, C. J. A. P. 2017, *Reports on Progress in Physics*, 80, 126902

Milne, E. 1937, *Proceeding Royal Society A*, 158, 324

Moffat, J. W. 2006, *JCAP*, 2006, 004

Molaro, P., Centurión, M., Whitmore, J. B., *et al.* 2013, *A&A*, 555, A68

Mosquera, M. E. & Civitarese, O. 2017, *Physical Review C*, 96, 045802

Murphy, M. T., Malec, A. L., & Prochaska, J. X. 2016, *MNRAS*, 461, 2461

Negrelli, C., Kraiselburd, L., Landau, S., *et al.* 2018, *International Journal of Modern Physics D*, 27, 1850099

Olive, K. A. & Pospelov, M. 2002, *Physical Review D*, 65, 085044

Palma, G. A.,Brax, P., Davis, A. C. & van de Bruck, C. 2003, *Physical Review D*, 68, 123519

Petrov, Y. V., Nazarov, A. I., Onegin, M. S., *et al.* 2006, *Physical Review C*, 74, 064610

Planck Collaboration, Ade, P. A. R., Aghanim, N., *et al.* 2015, *A&A*, 580, A22

Prestage, J. D., Tjoelker, R. L., & Maleki, L. 1995, *Physical Review Letters*, 74, 3511

Pochoda, P. & Schwarzschild, M. 1964, *ApJ*, 139, 587

Rahmani, H., Wendt, M., Srianand, R., *et al.* 2013, *MNRAS*, 435, 861

Rosenband, T., Hume, D. B., Schmidt, P. O., *et al.* 2008, *Science*, 319, 1808

Songaila, A., & Cowie, L. L. 2014, *ApJ*, 793, 103

Shelkovnikov, A., Butcher, R. J., Chardonnet, C., *et al.* 2008, *Physical Review Letters*, 100, 150801

Srianand, R., Chand, H., Petitjean, P., *et al.* 2004, *Physical Review Letters*, 92, 121302

Teller, E. 1948, *Physical Review D*, 73, 801

van Weerdenburg, F., Murphy, M. T., Malec, A. L., *et al.* 2011, *Physical Review Letters* , 106, 180802

Varshalovich, D. A. & Levshakov, S. A. 1993, *Soviet Journal of Experimental and Theoretical Physics Letters*, 58, 237

Webb, J. K., Flambaum, V. V., Churchill, C. W., *et al.* 1999, *Physical Review Letters*, 82, 884

Webb, J. K., King, J. A., Murphy, M. T., *et al.* 2011, *Physical Review Letters*, 107, 191101

Whitmore, J. B. & Murphy, M. T. 2015, *MNRAS*, 447, 446

Youm, D. 2001, *Physical Review D*, 64, 085011