Linear programming and pediatric dietetics

BY CIRO COLAVITA

Pediatric Division, S. Paolo Hospital, Naples 80122, Italy

AND RENATO D'ORSI

Polytecnic, University of Naples, Naples 80122, Italy

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The composition of 500 foods has been stored in a computer in order to analyse a child's diet. The methodology of operations research is applied to a very simple problem: a diet with only two foods. The geometrical representation of the 'feasible region' and of the 'objective function' is illustrated. One of the analytical methods employable with many variables (foods) is considered. This method was used in trying to find diets allowing for the preferential use of selected foods while respecting recommended dietary allowances, the tastes of the child and other constraints. The theoretical difficulty of transferring this methodology to pediatric dietetics was examined. We solved a simple case utilizing this procedure.

Linear programming and dietetics: Computer-aided dietetics

The content in g of the constituents of 500 foods commonly used in Europe have been expressed per 100 g edible portion in accordance with current literature (Paul & Southgate, 1978; Fidanza & Liguori, 1984; Luke, 1984), and have been stored in a computer. The contents of proteins, carbohydrates, lipids, saturated and polyunsaturated fatty acids, cholesterol, sodium, potassium, calcium, phosphorus and iron have been placed in this archive. Other information regarding the presence of milk protein and of gluten, the type of carbohydrates, and the age-dependent minimal and maximal helpings has also been included. A computer program, whose principal menu is shown in Table 1, permits the handling of this information and the compilation of the diet.

| | Paediatric Dietetics | Main menu |
|------------|----------------------|-----------|
| A. | Food composition | |
| B . | Patient data | |
| C. | Diet input | |
| D. | Display | |
| E. | Modifications | |
| F. | Analysis | |
| G. | Personalizations | |
| H. | Linear programming | |
| I. | Printing | |
| L. | Exit | |
| Select | t option | |

Table 1. Main menu

A, food's data base interrogation; B, patient's anagraphic data input; C, patient's diet input; D, patient's diet display (see Table 2); E, patient's diet manipulation; F, analysis of the diet (see Table 3); G, choice allowing the modification of the maximal and minimal helpings and the assignment of a score to the selected foods; H, linear programming option; I, printing options.

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| | Food | Amount (g or ml) | |
|---------------|-------------|------------------|---------|
| Breakfast | Milk | 100 | ······· |
| | Sugar | 15 | |
| | White bread | 20 | |
| | Jam | 15 | |
| | Butter | 10 | |
| Lunch | Pasta | 40 | |
| | Tomatoes | 80 | |
| | Olive oil | 5 | |
| | Sole | 50 | |
| | Olive oil | 10 | |
| | White bread | 40 | |
| | Oranges | 150 | |
| Snack | White bread | 30 | |
| | Butter | 10 | |
| Dinner | Pizza | 100 | |
| | Bacon | 30 | |
| | White bread | 20 | |

Table 2. A sample diet

Table 3. Analysis of the diet in Table 2

(Name Micòl F, age 10 years, weight 30 kg, height 1.35 m, energy intake recommended 7740 kJ/d)

| | Analysis | Recommended dietary allowance | Ý |
|----------------------------------|----------|----------------------------------|------|
| Energy (kJ) | 5422 | 9623-10460 | Low |
| Protein (% energy) | 0.11 | 0.12-0.15 | Low |
| Carbohydrate (% energy) | 0.48 | 0.45-0.20 | OK |
| Fats (% energy) | 0.41 | 0.38-0.42 | OK |
| Cholesterol (mg) | 134 | < 300 | OK |
| Sodium (mg) | 1578 | 6001800 | OK |
| Potassium (mg) | 993 | 1000-3000 | Low |
| Calcium (mg) | 379 | 700-900 | Low |
| Phosphorus (mg) | 509 | 700900 | Low |
| Iron (mg) | 4.4 | 9–11 | Low |
| Polyunsaturated : saturated fats | 0.25 | 0.9-1.0 | Low |
| Ca:P | 0.74 | 0.9-1.0 | Low |
| Breakfast (% energy) | 0.22 | 0.23-0.25 | Low |
| Lunch (% energy) | 0.34 | 0.28-0.32 | High |
| Snack (% energy) | 0.11 | 0.13-0.15 | Low |
| Dinner (% energy) | 0.33 | 0.28-0.32 | High |

Table 2 illustrates a child's diet, as reported by the mother, or as suggested by the paediatrician. The composition of the diet, of course, can be analysed (see Table 3).

When the analysis shows marked deviations from requirements, an attempt at solving the problem is generally made by substituting some foods or choosing new helpings, or both. While the previously deviating components are now balanced, the new diet is often unbalanced with respect to other nutrients.

The paediatrician can, of course, use his or her experience, but might best rely on a new conceptual approach in trying to answer the following questions: 'Is it possible to have diets for this child taking into account the constraints I have as a pediatrician at this moment such as the recommended dietary allowances (RDA) and the tastes of the child?'



Fig. 1. The coordinates of the points located within the region delimited by the polygon A_1, B_1, E_5, E_6, D_1 , the 'feasible region', correspond, in a simple diet with two foods X_1 and X_2 to helpings satisfying five constraints. A sheaf of parallel straight lines (Z lines) represent the 'objective function' to be maximized to obtain a 'high quality' diet.

'If such diets do exist, can I find the one diet that allows preferential use of the foods which are most useful to this child?'

In our experience it has been useful to use the methodology of operations research, which is a method used in problems of engineering, economics and management, to obtain an efficient allocation of scarce resources in order to identify the so-called optimal solutions. These solutions, in a specific context, correspond to the lowest cost or the maximal advantages and in general are the most convenient solutions from this point of view.

We may consider, for example, a very simple diet with only two foods (milk and meat) having weights X_1 and X_2 . As it is mandatory to choose reasonable helpings appropriate to various age groups, it will be useful to respect the first four constraints, which could be

$$\begin{array}{ll} X_1 \geqslant \mathbf{A} \,; & X_1 \leqslant \mathbf{B}, \\ X_2 \geqslant \mathbf{C} \,; & X_2 \leqslant \mathbf{D}, \end{array}$$

where A is milk, minimal helping; B is milk, maximal helping; C is meat, minimal helping; D is meat, maximal helping.

These constraints, plotted on a Cartesian plane (Fig. 1), locate a 'feasible region', a set of points satisfying the previously stated conditions. This region is delimited by the quadrilateral A_1, B_1, C_1, D_1 .

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A further constraint can be expressed as:

$$\mathbf{E}_1 X_1 + \mathbf{E}_2 X_2 \leqslant \mathbf{E},\tag{1}$$

where, for example E_1 is g protein/100 g milk, E_2 is g protein/100 g meat, E is maximal allowance of proteins (g/d). A constraint in form 1 can be represented by an equation adding a non-negative variable (SL in the example) which is the slack variable so that the relationship becomes

$$E_1 X_1 + E_2 X_2 + SL = E.$$

The relationship $E_1 X_1 + E_2 X_2 = E$ which derives from (1) with SL = 0, $E_1 > 0$ and $E_2 > 0$ corresponds on a graph to a straight line (Fig. 1) intersecting the abscissa $(X_2 = 0)$ at the point E_3 with abscissa $X_1 = E/E_1$ and the ordinate $(X_1 = 0)$ in the point E_4 with ordinate $X_2 = E/E_2$. As the constraint expressed by the relationship (1) is of the type \leq , the 'feasible region' is reduced to the polygon A_1, B_1, E_5, E_6, D_1 . Additional constraints can either be ineffective or further reduce the region that satisfies all the constraints.

As a consequence, we shall have other type 1 relationships and in general a system such as the following:

$$\begin{array}{l} X_1 \geqslant \mathbf{A} \\ \mathbf{X}_1 \leqslant \mathbf{B} \\ \mathbf{X}_2 \geqslant \mathbf{C} \\ \mathbf{X}_2 \leqslant \mathbf{D} \\ \mathbf{E}_1 X_1 + \mathbf{E}_2 X_2 \leqslant \mathbf{E} \\ \mathbf{F}_1 X_1 + \mathbf{F}_2 X_2 \leqslant \mathbf{F} \\ \vdots \\ \mathbf{W}_1 X_1 + \mathbf{W}_2 X_2 \leqslant \mathbf{W}, \end{array}$$

where for example E_1 is g protein/100 g milk, E_2 is g protein/100 g meat, E is maximal allowance of protein (g/d), F_1 is g carbohydrate/100 g milk, F_2 is g carbohydrate/100 g meat, F is maximal allowance of carbohydrate (g/d), W_1 is (g lipid or Na or K etc.)/100 g milk, W_2 is (g lipid or Na or K etc.)/100 g meat, W is maximal allowance of lipid or Na or K etc. (g/d).

When the constraints are of the type \ge (i.e. $X_1 > A$) it is possible to refer to the previous instances by subtracting a non-negative variable (SU) which is the surplus variable

$$X_1 - SU = A.$$

The 'feasible region' in the previous example (Fig. 2) is delimited by the polygon E_5 , B_1 , W_5 , W_6 , F_6 , D_1 , E_6 . Sometimes the 'feasible region' can be void, as illustrated in Fig. 3. In fact the 'feasible region' delimited by the quadrilateral A_1 , B_1 , C_1 , D_1 can be reduced to the triangle A_1 , F_5 , F_6 by a new constraint represented by the straight line $F_3 F_4$. Furthermore an additional constraint, represented by the line $E_3 E_4$, can delimit a second region (E_5 , B_1 , C_1 , E_6) which has no points in common with the previous triangle. For a further analysis of this topic see specialized texts on linear programming (Ippolito, 1974; Improta, 1975; Bronson, 1982).

The considerations developed so far for just two foods can also be applied to cases having n foods. Analysing whether a feasible region exists means verifying the possibility of combining the selected foods while taking into account constraints important for the child's health (RDA) and for the feasibility of the diet (minimal and maximal helpings appropriate to the various ages).

Examining now the second question we can imagine giving the foods chosen for the diet a particular 'score' directly proportional to their nutritional quality for a particular child.



Fig. 2. The coordinates of the points located within the region delimited by the polygon E_5 , B_1 , W_5 , W_6 , F_6 , D_1 , E_6 correspond, in a simple diet with two foods, to helpings satisfying a larger number of constraints than in Fig. 1.

In the previously considered elementary diet, we can give the score D_1 to the first food $(X_1$ is milk) and the score D_2 to the second $(X_2$ is meat). While respecting the constraints, we want to find X_1 and X_2 values allowing the maximum Z value in the following relationship

$$Z = D_1 X_1 + D_2 X_2$$
(2)

which marks the quality of the diet.

The objective function (2) we want to maximize is represented on a graph by a sheaf of parallel straight lines. In order to locate the direction of this sheaf, we choose the straight line belonging to the sheaf for which Z = 0. This straight line passes through the origin and has an angular coefficient.

$$X_2/X_1 = -D_1/D_2$$

which is a quantity expressing the relationship between the preferences assigned by the pediatrician to the two foods and measured by scores D_1 and D_2 . Obviously a different relationship will bring about a different position of the sheaf of parallel lines on the Cartesian plane. The line for which Z = 0 in Fig. 1 is represented by PQ. If we consider a straight line parallel to PQ, for example P_1Q_1 , the distance between these lines is proportional to Z (Fig. 1). Therefore, if we want to maximize Z, we must shift PQ in the direction allowing Z to grow without extending beyond the feasible region delimited by the pentagon A_1, B_1, E_5, E_6, D_1 . It will then be necessary for the straight line parallel to PQ



Fig. 3. The feasible region in this case is void. The constraints are so arranged that it is not possible to have helpings of the two foods satisfying all the constraints (recommended dietary allowances, etc.).

to pass through E_6 . The coordinates $(X_1 \text{ and } X_2)$ of E_6 give the solution to the problem. In fact, taking X_1 g milk and X_2 g meat, we shall obtain the maximum of the function Z while respecting all the constraints. Therefore if we assign a higher score value to milk among the several available solutions we shall choose the one which contains the maximal amount of milk while staying within the given constraints.

When dealing with two variable (two foods) the geometrical representation is clear and instructive.

When examining problems with n variables (foods) and m constraints, analytical methods must be employed, since the geometric representation of a polyhedron delimited by hyperspaces, in which every constraint is a hyperplane, is impossible. One of the analytical methods employed is the 'simplex'. This method in our case may locate the feasible solution which, respecting the constraints, maximizes the chosen objective function and, therefore, allows the maximum amount of the foods having a higher score in the relationship (2).

Often, having selected the foods, their minimum and maximum helpings and some of the nutritional qualities (RDA, etc.) we may be faced with constraints not coherent with each other. The diet we are considering might be unrealistic because some constraints (for example, the need to guarantee the RDA of Ca and P) are inconsistent with other constraints, such as the maximum realistic helpings of the foods we have chosen, which might have a very low content of Ca or P. In these cases, the simplex method, just like the geometrical representation in the case of two foods (Fig. 3) can provide additional useful information. The latter can, among other things, help to understand which constraint

hinders the maximization of the objective function for reasons relative to the area of the feasible solutions determined by the chosen constraints.

Some of this information is found in the analysis of the slack and surplus variables that are associated with each constraint.

A negative slack or surplus variable is associated with each constraint that contributes to the impossibility of solving the problem. If, by applying the simplex method to our example $(X_1 - SU = A)$, we found the solution to be impossible and SU to be negative, we should have to deduce that in order to accommodate the other constraints, the system cannot avoid suggesting a helping of milk smaller than the one normally used. Similar considerations can be made for the slack variables. Further useful considerations can be deduced from the analysis of the shadow cost that the method provides. In effect, with each problem of linear programming there can be associated a dual problem, which derives directly from the first and which also serves to define the value we must attribute to resources used optimally. This value is defined as the shadow cost. In our case the resources are represented by the foods and by the sum of the individual components of the foods (proteins, lipids, Ca, etc.). A shadow cost associated with each resource reflects changes in the diet's quality (that is, its objective function) based on unitary variations of the resources. We can, therefore, also say that it reflects how the objective function would change according to variations of the known terms of the constraints (RDA, minimal and maximal helpings, etc.). The shadow cost can also be helpful in modifying the constraints whenever required conditions lead to impractical cases.

The choice of studying a diet on a daily basis has led us to analyse peculiar aspects of this problem that had not been previously examined (Stiegler, 1945; Balintfy, 1964, 1974; Smith, 1959; Bassham *et al.* 1984). The objective of these reports was generally to define minimum cost diets for populations during extended periods (i.e. weeks) while taking into account certain nutritional standards (RDA, etc.).

Smith (1959) in fact succeeded in enhancing the acceptability of the diets by enlarging the choice of the foods and by constraining the system not to choose some foods (i.e. flour or pasta), unless they were appropriately combined with other foods (i.e. yeast, oil, etc.) normally used for their preparation or cooking. Some of these concepts seemed adaptable to our context. If in fact foods that are not very appetizing are included, the program allows for the concomitant inclusion of other foods (oil, yeast, etc.) that are normally used in their preparation. The child's acceptance of a diet in our system is also conditioned partly by how we construct the objective functions, as we shall explain later.

DISCUSSION

Now we consider further what we have defined as the 'food's score'. The crucial theoretical difficulty encountered in transferring the techniques of the simplex method to paediatric dietetics is, in our opinion, the difficulty of coherently defining the coefficients of the objective function. In transport problems the transport times are usually the coefficients of the objective function (2); in the economic field the coefficients are the cost of labour or materials, or both; this usually allows a coherent and satisfactory definition of the problem.

Could we choose, in paediatric dietetics, the cost of the foods? The maximization of the costs in order to obtain the optimal diet may not correspond to a realistic optimal solution since we cannot claim that there is a direct relationship between cost and food quality. (Advertising plays a distorting role in this regard.) On the other hand, cost reduction would be useful only when compiling diets for communities (Stiegler, 1945) or in very poor families. We therefore decided to rank foods according to the pediatrician's high or poor

Table 4. Linear programming diet breakdown*

(Weights are expressed in g and as the edible portion)

 $(1) \ 100X_1 + 100X_2 + 100X_3 + 100X_4 + 100X_5 + 100X_6 + 100X_7 + 100X_8 + 200X_9 + 100X_{10} + 100X_{11} + 200X_{12} + 100X_{12} + 100X_{13} + 100X_{14} + 100X_{15} + 1$ $+10X_{13}^{\dagger}$ $\begin{array}{l} (2) \ X_{1}^{''} < = 1 \cdot 5^{+}_{+} \ (3) \ X_{2} < = 0 \cdot 1; \ (4) \ X_{3} < = 2 \cdot 8; \ (5) \ X_{4} < = 0 \cdot 3; \ (6) \ X_{5} < = 0 \cdot 1; \ (7) \ X_{6} < = 0 \cdot 7; \\ (8) \ X_{7} < = 1; \ (9) \ X_{8} < = 0 \cdot 35; \ (10) \ X_{9} < = 1 \cdot 17; \ (11) \ X_{10} < = 1 \cdot 5; \ (12) \ X_{11} < = 0 \cdot 1; \ (13) \ X_{12} < = 1 \cdot 5; \end{array}$ (14) $X_{13} < = 0.5$ (15) $X_1 > = 1.2$; (16) $X_2 > = 0.05$; (17) $X_3 > = 0.8$; (18) $X_4 > = 0.05$; (19) $X_5 > = 0.05$; (20) $X_6 > = 0.3$; (21) $X_7 > = 0.4$; (22) $X_8 > = 0.1$; (23) $X_9 > = 0.72$; (24) $X_{10} > = 0.97$; (25) $X_{11} > = 0.05$; (26) $X_{12} > = 0.8$; (27) $\mathring{X}_{13} > = 0.4$ $\begin{array}{l} (26) \ X_{12} > = 0.8; \ (27) \ X_{13} > = 0.4 \\ (28) \ 3.2X_1 + 7.8X_3 + 0.6X_4 + 0.5X_5 + 10.8X_6 + 0.3X_7 + 15.9X_9 + 0.6X_{10} + 0.5X_{11} + 8.2X_{12} + 22.8X_{13} < = 69 \\ (29) \ 3.2X_1 + 7.8X_3 + 0.6X_4 + 0.5X_5 + 10.8X_6 + 0.3X_7 + 15.9X_9 + 0.6X_{10} + 0.5X_{11} + 8.2X_{12} + 22.8X_{13} > = 55 \\ (30) \ 4.6X_1 + 104.5X_2 + 49.7X_3 + 58.3X_4 + 0.6X_5 + 77.4X_6 + 3.4X_7 + 0.9Y_9 + 6.4X_{10} + 0.6X_{11} + 31.5X_{12} < = 231 \\ (31) \ 4.6X_1 + 104.5X_2 + 49.7X_3 + 58.3X_4 + 0.6X_5 + 77.4X_6 + 3.4X_7 + 0.9Y_9 + 6.4X_{10} + 0.6X_{11} + 31.5X_{12} < = 231 \\ (32) \ 3.7X_1 + 1.7X_3 + 83.1X_5 + 0.3X_6 + 0.2X_7 + 100X_8 + 1.7X_9 + 83.1X_{11} + 9.3X_{12} + 38.1X_{13} < = 86 \\ (33) \ 3.7X_1 + 1.7X_3 + 83.1X_5 + 0.3X_6 + 0.2X_7 + 100X_8 + 1.7X_9 + 83.1X_{11} + 9.3X_{12} + 38.1X_{13} > 78 \\ (34) \ 0.05X_1 + 0.01X_2 + 0.54X_3 + 0.009X_4 + 0.011X_5 + 0.028X_6 + 0.043X_7 + 0.127X_9 + 0.002X_{10} + 0.011X_{11} \\ + 0.026Y_1 + 2.733Y_1 < = 1.8 \end{array}$ $+0.036X_{12} + 2.733X_{13} < = 1.8$ $(35) \ 0.05X_1 + 0.001X_2 + 0.54X_3 + 0.009X_4 + 0.011X_5 + 0.028X_6 + 0.043X_7 + 0.127X_9 + 0.002X_{10} + 0.011X_{11} + 0.011X_{11} + 0.011X_{12} + 0.011X_{12} + 0.001X_{12} + 0.00X_{12} + 0$ $+0.036X_{12} + 2.733X_{13} > = 0.6$ $(36) \quad 0.136X_1 + \tilde{0}.003X_2 + \tilde{0}.21X_3 + 0.086X_4 + 0.006X_5 + 0.311X_6 + 0.076X_7 + 0.33X_9 + 0.15X_{10} + 0.006X_{11} + 0.0006X_{11} + 0.0006X_{11}$ $+0.088X_{12}+0.323X_{13} < = 3$ $(37) \quad 0.136X_1 + \tilde{0}.003X_2 + \tilde{0}.21X_3 + 0.086X_4 + 0.006X_5 + 0.311X_6 + 0.076X_7 + 0.33X_9 + 0.15X_{10} + 0.006X_{11} + 0.006X_{12} + 0.006X_{13} + 0.006X_{14} + 0.006X_{15} + 0.0006X_{15} + 0.0006X_{15}$ $+0.088X_{12}+0.323X_{13} > = 1$ $(38) \quad 0.124X_1 + \bar{0} \cdot 1X_3 + 0.0\bar{1} \cdot 8X_4 + 0.017X_5 + 0.017X_6 + 0.009X_7 + 0.012X_9 + 0.031X_{10} + 0.017X_{11} + 0.08X_{12} + 0.017X_{11} + 0.08X_{12} + 0.017X_{11} + 0.017X_{12} + 0.017X_{12} + 0.017X_{13} + 0.017X_{14} + 0.000X_{14} + 0.00X_{14} + 0.0$ $+0.021X_{13} < = 0.9$ $(39) \quad 0.124X_1 + 0.1X_3 + 0.018X_4 + 0.017X_5 + 0.017X_6 + 0.009X_7 + 0.012X_9 + 0.031X_{10} + 0.017X_{11} + 0.08X_{12} + 0.017X_{11} + 0.08X_{12} + 0.017X_{11} + 0.017X_{12} + 0.017X_{12} + 0.017X_{13} + 0.017X_{14} + 0.017X_{14} + 0.017X_{14} + 0.017X_{14} + 0.017X_{14} + 0.000X_{14} + 0.00$ $+0.021X_{13} > = 0.7$ $(40) \ 0.091X_1 + 0.097X_2 + 0.011X_4 + 0.021X_5 + 0.165X_6 + 0.022X_7 + 0.195X_6 + 0.018X_{10} + 0.021X_{11} + 0.106X_{12} + 0.018X_{10} + 0.021X_{11} + 0.001X_{12} +$ $+0.178X_{13} < = 0.9$ $(41) \quad 0.091X_1 + 0.097X_3 + 0.011X_4 + 0.021X_5 + 0.165X_6 + 0.022X_7 + 0.195X_9 + 0.018X_{10} + 0.021X_{11} + 0.106X_{12} + 0.018X_{10} + 0.021X_{11} + 0.016X_{12} + 0.018X_{10} + 0.002X_{11} + 0.002X_{12} +$ $+0.178X_{13} > = 0.7$

* The meaning of the relationship is illustrated in Table 5.

[†] Variable X_1 represents milk which is the first food listed in the diet described in Table 2, variable X_2 represents sugar and so on. The first line is the objective function, the others the various constraints to be respected. The simplex method will assign certain values to variable X_1, X_2, \ldots so that the objective function will be maximized while all the other constraints will be respected.

 $\ddagger 1 = 100.$

opinion of them but other methods, for example, 'preferential scores' based on acceptability of foods or better still setting priorities for different nutritional targets such as meeting iron needs, could equally be employed. The paediatrician expresses his or her opinion by assigning a 'score' to each of the foods chosen.

Three possible food scores might be: 10, 100 or 200. The paediatrician may possibly leave the food score, which is normally 100, unchanged. In this case the simplex method will give the possible solution on this basis. He may, on the other hand, decide to give preference to some foods by giving them a score of 200 or express a low opinion of other foods by reducing their score to 10, or both. This facility for changing the coefficient of the objective function (2) is a powerful means of dietetic personalization. The simplex method, while respecting the constraints, will find in this case the maximal possible apportionment of foods with the highest scores.

Case report

We report the case of a child, Micòl F. (Table 2), who was given a hypoenergetic diet slightly deviating from normal standards (Table 3). The parents were concerned about the girl's lack of appetite. The patient's clinical condition was satisfactory. Having learned that

Table 5. Detailed explanation in dietetic terms of the linear programming mathematicalexpressions listed in Table 4

| Line | Interpretation | |
|-------|---|--|
| 2-14 | Constraints reflecting maximal helpings allowed. | |
| 15-27 | Constraints reflecting minimal helpings required. | |
| 28 | Constraint required to avoid an excessive protein intake.* | |
| 29 | Constraint required to ensure a minimum protein intake. | |
| 30 | Constraint required to avoid an excessive carbohydrate intake. | |
| 31 | Constraint required to ensure a minimum carbohydrate intake. | |
| 32 | Constraint required to avoid an excessive lipid intake. | |
| 33 | Constraint required to ensure a minimum lipid intake. | |
| 34 | Constraint required to avoid an excessive sodium intake. [†] | |
| 35 | Constraint required to ensure a minimum sodium intake. | |
| 36 | Constraint required to avoid an excessive potassium intake. | |
| 37 | Constraint required to ensure a minimum potassium intake. | |
| 38 | Constraint required to avoid an excessive calcium intake. | |
| 39 | Constraint required to ensure a minimum calcium intake. | |
| 40 | Constraint required to avoid an excessive phosphorus intake. | |
| 41 | Constraint required to ensure a minimum phosphorus intake. | |

* The foods' protein content, in 100 g edible portion, is represented by the numerical coefficients of the variables X_1, X_2, \ldots . The minimal and maximal intakes of proteins, carbohydrates and lipids are calculated taking into account the need to obtain a diet having approximately 7740 kJ, in which 0.12-0.15 of the total energy is derived from proteins, 0.45-0.50 from carbohydrates and 0.38-0.42 from lipids.

[†] The sodium, potassium, calcium and phosphorus minimal and maximal intakes that must be respected by the constraints derive from RDA.

she liked fish, (particularly sole) and pizza, we decided to give a higher score to these two foods while giving a lower one to bacon, which she disliked. Having received the informed consent of the parents, we employed the simplex method to find a better diet.

Table 4 shows the objective function and forty-nine constraints necessary to obtain a diet of 7740 kJ with 0.12-0.15 of the total energy derived from proteins, 0.45-0.50 derived from carbohydrates and 0.38-0.42 from lipids. Allowing 7740 kJ seemed an acceptable initial compromise, taking into account the girl's previously mentioned lack of appetite.

Table 5 shows what each constraint represents. (Note that foods that are present in the diet during several meals or various times during a meal (bread, oil, etc.) are actually listed as only one food in the calculations and the minimal and maximal portions are appropriately calibrated.) In our case, therefore, bread corresponds to food X_3 and maize oil to $X_{\rm s}$. The program reflected the existence of two negative surplus variables, one being associated to the 27th constraint (ham, minimal helping) and the other one to the 39th (minimum recommended Ca). We found that the posed constraints were in fact incompatible and that it was impossible to provide a minimal helping of ham and a minimum apportionment of Ca while respecting the other constraints. We therefore agreed to reduce the intake of ham, since the girl disliked it. In spite of that, a negative surplus variable was still associated to the 39th constraint (indicating that selected foods had a low content in Ca). We then analysed the shadow costs. The higher shadow costs noted were associated with the 39th constraint (minimum recommended for Ca), to the 2nd (maximal intake of milk) and to the 27th (minimal intake of ham). Allowing a higher intake of milk and further reducing the minimal helping of ham was sufficient to render the new established constraints coherent and obtain the optimal solution illustrated in Table 6.

Table 7 shows an analysis of this diet. The foods that are listed once in the algorithm even though eaten during several meals are redistributed interactively by the program aiming at

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| | Food | Amount (g or ml) | |
|-----------|-------------|------------------|---|
| Breakfast | Milk | 250 | _ |
| | Sugar | 10 | |
| | White bread | 50 | |
| | Jam | 30 | |
| | Butter | 10 | |
| Lunch | Pasta | 30 | |
| | Tomatoes | 100 | |
| | Olive oil | 20 | |
| | Sole | 200 | |
| | Olive oil | 15 | |
| | White bread | 65 | |
| | Oranges | 200 | |
| Snack | White bread | 50 | |
| | Butter | 10 | |
| Dinner | Pizza | 150 | |
| | Bacon | 10 | |
| | White bread | 50 | |

Table 6. The diet obtained with the simplex method

Table 7. Analysis of the diet in Table 6

(Name Micòl F, age 10 years, weight 30 kg, height 1.35 m, energy intake recommended 7740 kJ/d)

| | | Recommended dietary | Ý |
|----------------------------------|----------|---------------------|------|
| | Analysis | allowances | |
| Energy (kJ) | 8000 | 9623-10460 | OK |
| Protein (% energy) | 0.12 | 0.12-0.12 | OK |
| Carbohydrate (% energy) | 0.48 | 0.45-0.20 | OK |
| Fats (% energy) | 0.39 | 0.38-0.42 | OK |
| Cholesterol (mg) | 180 | < 300 | OK |
| Sodium (mg) | 1789 | 600-1800 | OK |
| Potassium (mg) | 1681 | 1000-3000 | OK |
| Calcium (mg) | 727 | 700-900 | OK |
| Phosphorus (mg) | 895 | 700-900 | OK |
| Iron (mg) | 7.0 | 9–11 | Low |
| Polyunsaturated : saturated fats | 0.78 | 0.9–1.0 | Low |
| Ca:P | 0.81 | 0.9-1.0 | Low |
| Breakfast (% energy) | 0.25 | 0.23-0.22 | OK |
| Lunch* (% energy) | 0.37 | 0.28-0.32 | High |
| Snack (% energy) | 0.10 | 0.13-0.12 | Low |
| Dinner (% energy) | 0.28 | 0.58-0.35 | OK |

* A higher energy intake was accepted as in Italy lunch is considered the main meal and it is difficult to correct this habit in a short period of time.

a better balance of energy intake between the meals. This balance seemed easier to obtain by this means rather than by a different and more complex manipulation of the constraints or of the objective function and also saved time.

In conclusion we think it is useful to employ these methods in the personalization of the child's diet. Further development of the program will of course be necessary to allow the user lacking mathematical knowledge to use the program more easily. Additional studies will be necessary to clarify the values and limits of the method.

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