BOOK REVIEWS

KLEENE, S. C. AND VESLEY, R. E., *The Foundations of Intuitionistic Mathematics* (North-Holland Publishing Company, 1965), viii+206 pp., 80s.

In this mongraph, Kleene and one of his former students apply metamathematical and model theoretic methods to L. E. J. Brouwer's constructive version of mathematics. The result is a detailed formal treatment of intuitionistic number theory and analysis which is likely to be more acceptable than the mystical approach of Brouwer, at least to mathematicians not completely and exclusively committed to the philosophy of intuitionism.

The first chapter is devoted to a new formalisation of intuitionistic analysis. This is followed by an account of Kleene's interpretations of the formal theory within the theory of recursive functions, which includes modifications and extensions of previously published work. In the third chapter, Vesley develops the intuitionistic theory of the continuum within the Kleene system and in the final chapter Kleene investigates the claims made by Brouwer in a paper on the properties of the intuitionistic continuum. There is no account of the recent work of other mathematicians in the field.

The book is written in typical Kleene style on the assumption of familiarity with Chapters I-XII of Kleene's earlier book *Introduction to Metamathematics*. Proofs are presented using an elaborate system of cross references (which refer to this earlier book as well as to the book under review), and this involves anyone trying to follow them in a good deal of hard work. Certainly this is an important and original contribution to the literature on intuitionistic mathematics, but it is not the book for the reader wanting a first introduction to the subject.

A. A. TREHERNE

NACHBIN, L., *The Haar Integral* (D. van Nostrand, London, 1965), xii+156 pp., 51s.

The Haar integral (that is, to a first approximation, the generalisation of the Lebesgue integral to locally compact topological groups) has established itself as one of the most useful tools in analysis. In the past, its treatment has usually been relegated to an appendix or to a (necessarily somewhat condensed) chapter in a treatise on some related larger topic; examples are provided by two other volumes in the present series. Here we have for the first time a text devoted primarily to the Haar integral itself.

The author's aim is to provide an elementary introduction rather than an exhaustive account. Accordingly, he requires relatively little background of the reader. The necessary parts of integration theory in locally compact spaces are developed in the first chapter, together with much of the relevant topology. The reader is assumed to know what a Hausdorff space is but not—for example—to know anything about compactness. The first chapter provides a convenient and readable introduction to the Bourbaki theory of integration. In the second chapter the Haar integral itself is discussed; the existence proofs of Weil and Cartan are both presented. In a short third chapter the basic existence and uniqueness theorem s extended to locally compact homogeneous spaces.

Throughout, there is careful motivation, and there are many interesting illustrative examples, usually discussed in detail. The book should be easily accessible to any recent graduate, or enterprising final-year undergraduate, in mathematics. The reviewer's only adverse criticism—and that a minor one—is that the author has perhaps been too severe in limiting the scope of the book. Some further developments and indications of how the integral can be used (for instance, in constructing the L_1-algebra of a group) would have been welcome. But what the author has given us is certainly an attractive and useful account of the basic facts of the subject. His aim of arousing the reader's interest and stimulating him to further study should undoubtedly be achieved. J. H. WILLIAMSON

HELSON, H., *Lectures on Invariant Subspaces* (Academic Press Inc., New York), $5.00.

The material of this book is part classical analysis and part functional analysis; it is also in close relation with other branches of mathematics (for instance, stochastic processes). The basic problem is to characterise, for the space of square-integrable functions on the unit circle, those subspaces that are invariant under multiplication either by exp ix and exp $-ix$ or by exp ix alone. Previous treatments of this and similar problems have for the most part been function-theoretic in character (involving the interior of the unit circle). The author aims to develop methods that involve as few excursions from the unit circle as possible—to replace function-theory by harmonic analysis, essentially—and then to exploit these methods as far as he can. The appropriate tools are found in Hilbert space theory; elementary general theorems about subspaces, together with a little extra information about the special case under consideration, readily yield the required results. The treatment is an elegant example of what can be done by abstract methods in a concrete situation.

Of the eleven "lectures" the first four are devoted to the classical problem described above, and some further related matters. The fifth lecture deals with the similar problem in which the unit circle is replaced by the real line; here the results are similar, but the proofs required are different (and more complicated). The remainder of the book deals again with functions on the unit circle, but now taking values in a separable Hilbert space. Appropriately formulated analogues of the scalar theorems can be proved. Special mention should perhaps be made of the topic discussed in the tenth lecture; this is the general invariant subspace problem for a bounded operator in a separable Hilbert space, a subject of considerable current interest. It is shown that the general problem is equivalent to an apparently more special problem, in spaces of the type considered.

The writing is pleasantly informal, and there is no excess of detail to obscure the main lines of the argument. The book is to be welcomed as a well presented account of some recent research in an interesting and important field. J. H. WILLIAMSON

HERMES, H., *Enumerability, Decidability, Computability. An Introduction to the Theory of Recursive Functions* (Mathematischen Wissenschaften Band 127, Springer-Verlag, 1965), x+245 pp., DM39.

There are several textbooks on recursive functions available in English and this translation from the German covers much the same material as in *Computability and Unsolvability* by M. Davis. The book begins with some introductory reflections on algorithms, then defines computable functions, recursive functions and shows their equivalence, and ends with some undecidability results and miscellaneous topics. The treatment of recursive functions is good.

The translation is good, the few faults are of little consequence. One of the good points of the text is that illustrative examples are introduced at an early stage. The result is a very readable book. R. M. DICKER